Axion string simulations

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Work in progress, in collaboration with
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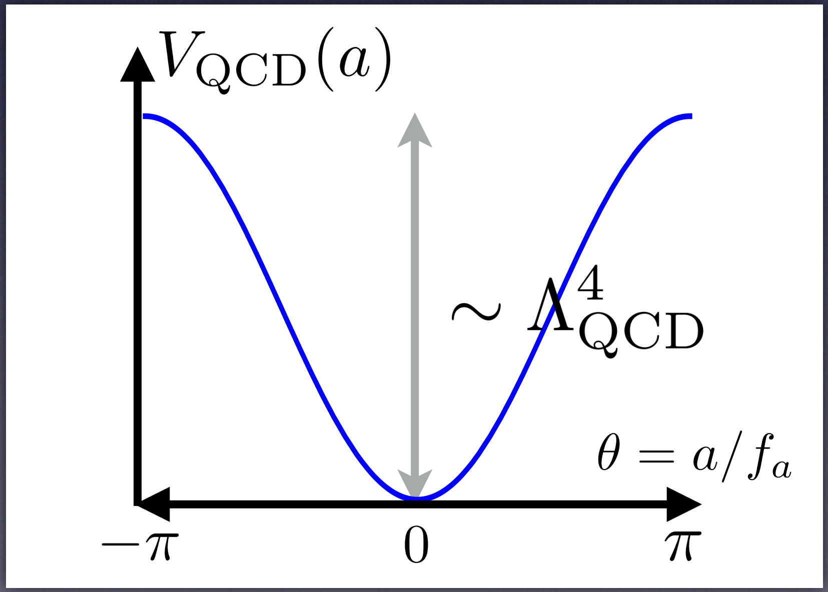
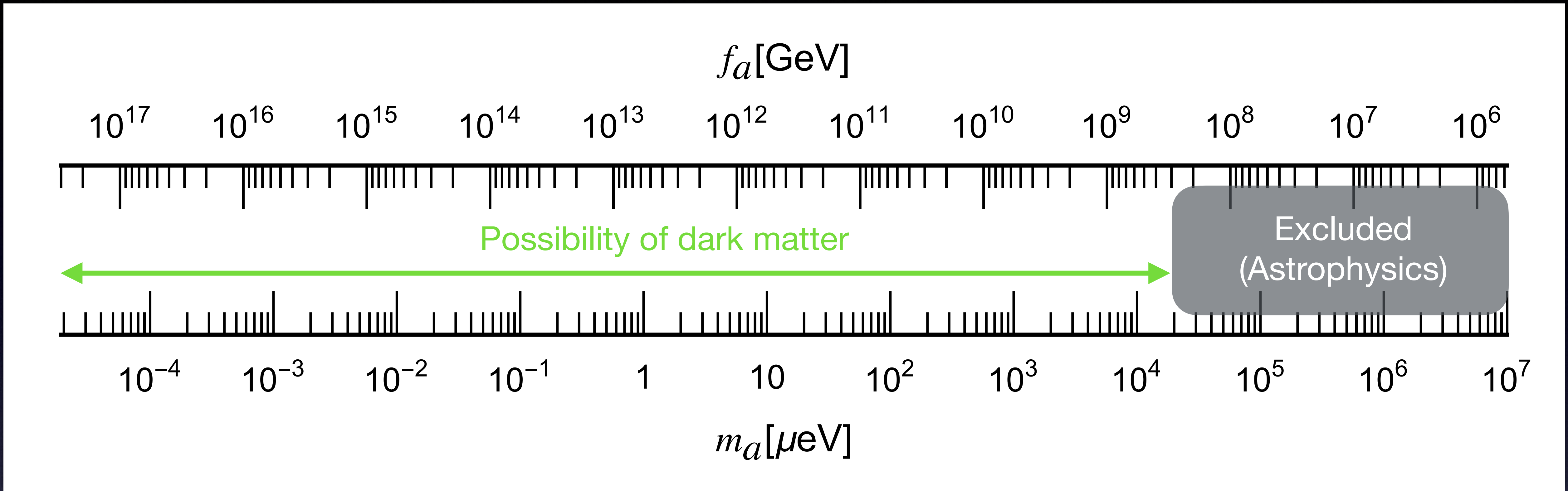
The QCD axion

- The QCD axion
  - Pseudo Nambu-Goldstone boson associated with spontaneous breaking of the global Peccei-Quinn (PQ) symmetry at the scale f_a ("axion decay constant").
  - Solution to the strong CP problem
  - Good candidate of cold dark matter
- Acquires a mass below the QCD scale:

$$m_a \simeq 57\,\mu\mathrm{eV}\left(\frac{10^{11}\,\mathrm{GeV}}{f_a}\right)$$



Axion dark matter mass?



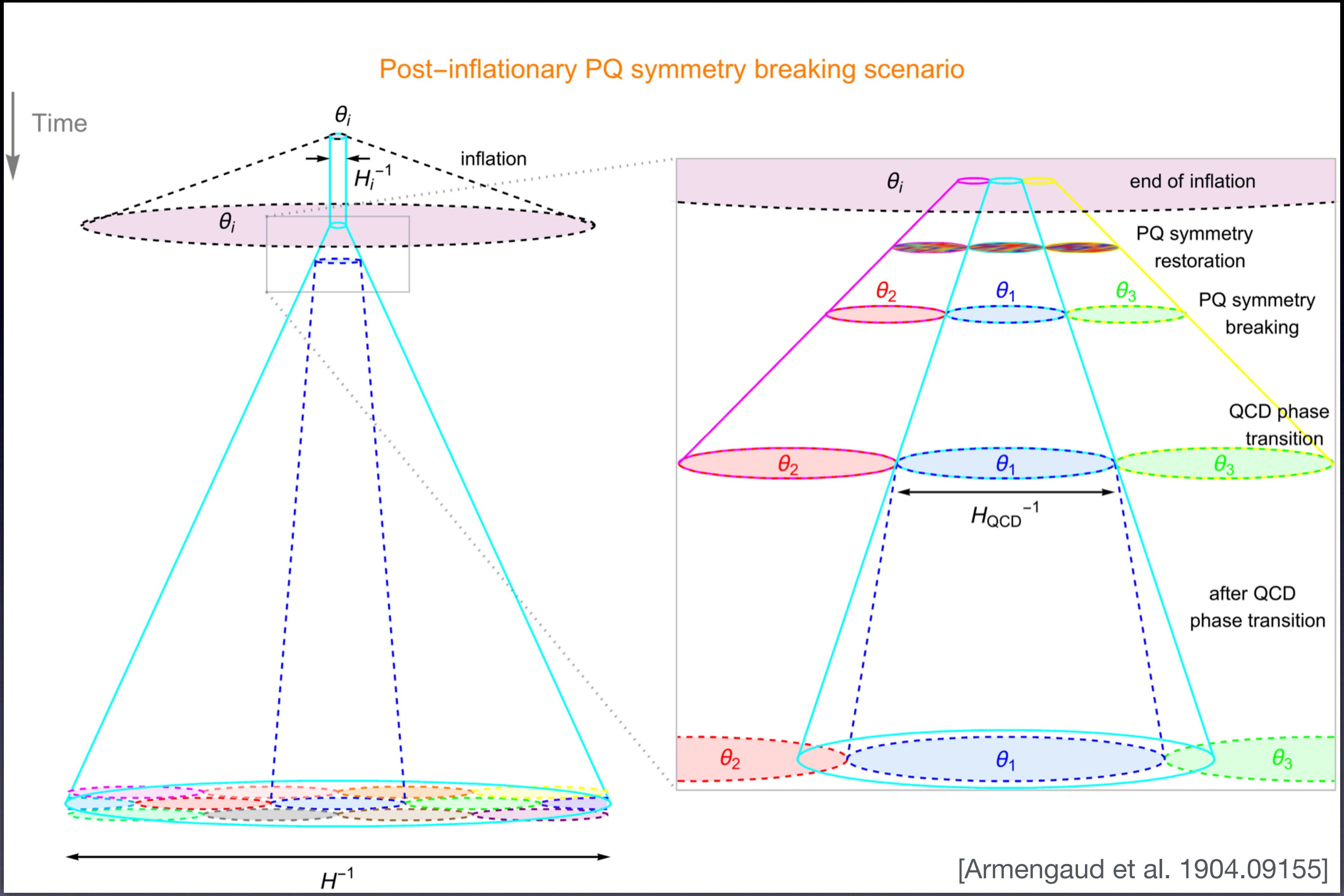
- Relic axion abundance depends on the Peccei-Quinn scale, and hence on the axion mass.

$$\Omega_a = \Omega_a(f_a), \quad m_a \simeq 57\,\mu\mathrm{eV}\left(\frac{10^{11}\,\mathrm{GeV}}{f_a}\right)$$

- One can guess the axion DM mass from its relic density.

$$\Omega_a h^2 = 0.12 \quad \Rightarrow \quad m_a = ???\,\mu\mathrm{eV}$$

Assumption: Post-inflationary PQ symmetry breaking

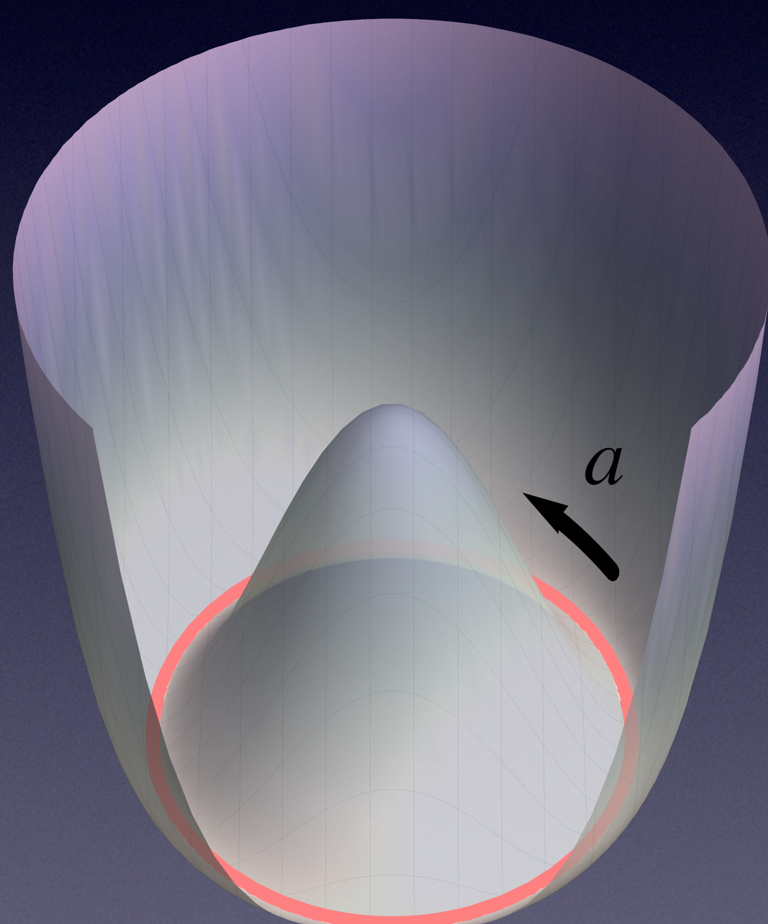


- Axion abundance should be uniquely determined if we precisely know the field configurations around the QCD phase transition ("average over initial angle").
- Complicated stuff: *axions produced from strings.* [Davis (1986)]

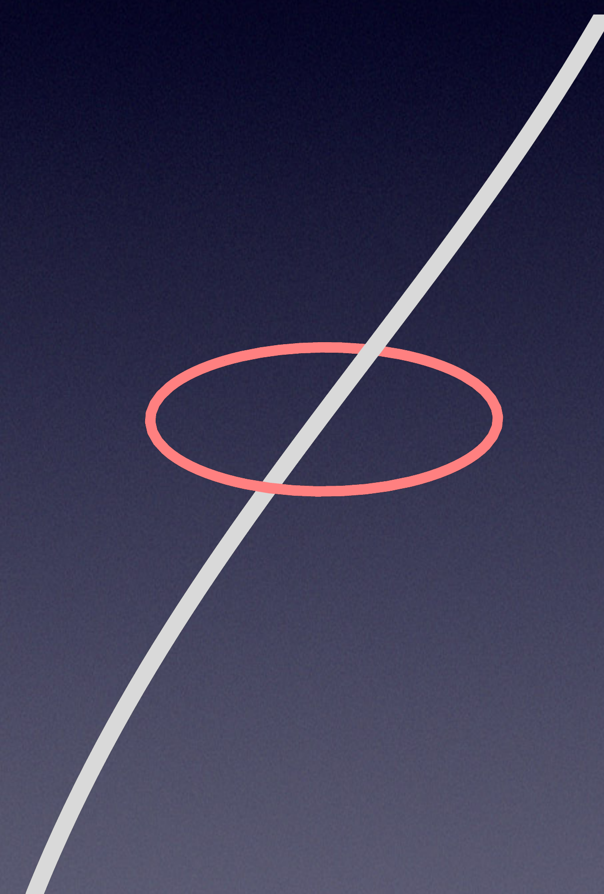
Axion (global) strings

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi), \quad V(\phi) = \lambda \left(|\phi|^2 - \frac{f_a^2}{2} \right)^2$$

$V(\phi)$



Position space

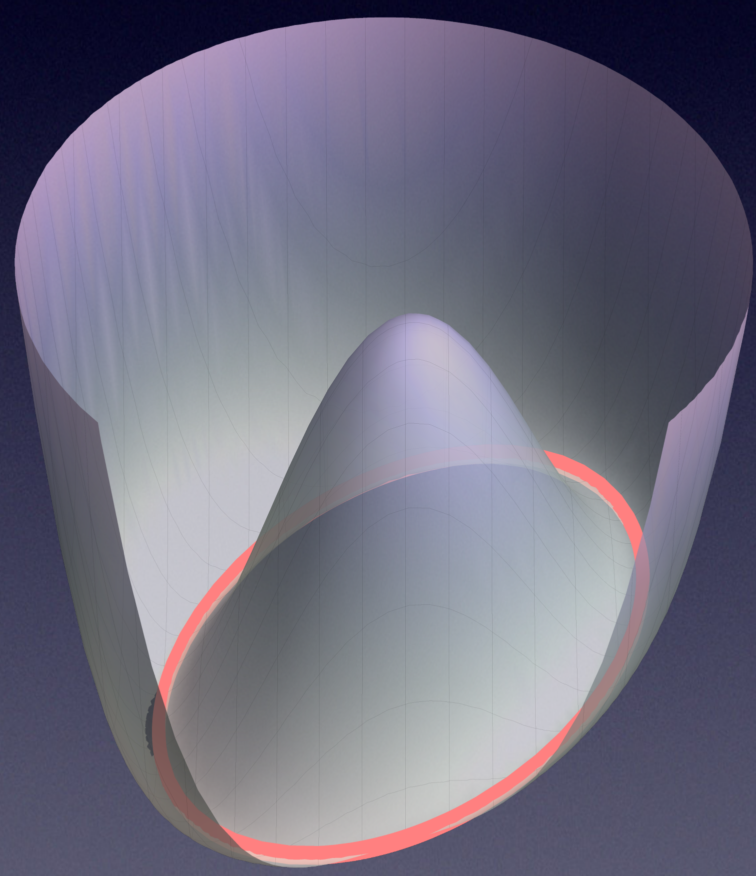


- Form when global $U(1)_{PQ}$ symmetry is spontaneously broken.
- Disappear around the epoch of the QCD phase transition (if $N_{DW} = 1$).

Axion (global) strings

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi), \quad V(\phi) = \lambda \left(|\phi|^2 - \frac{f_a^2}{2} \right)^2 + \chi(T) \left(1 - \cos\left(\frac{a}{f_a} \right) \right)$$

$V(\phi)$



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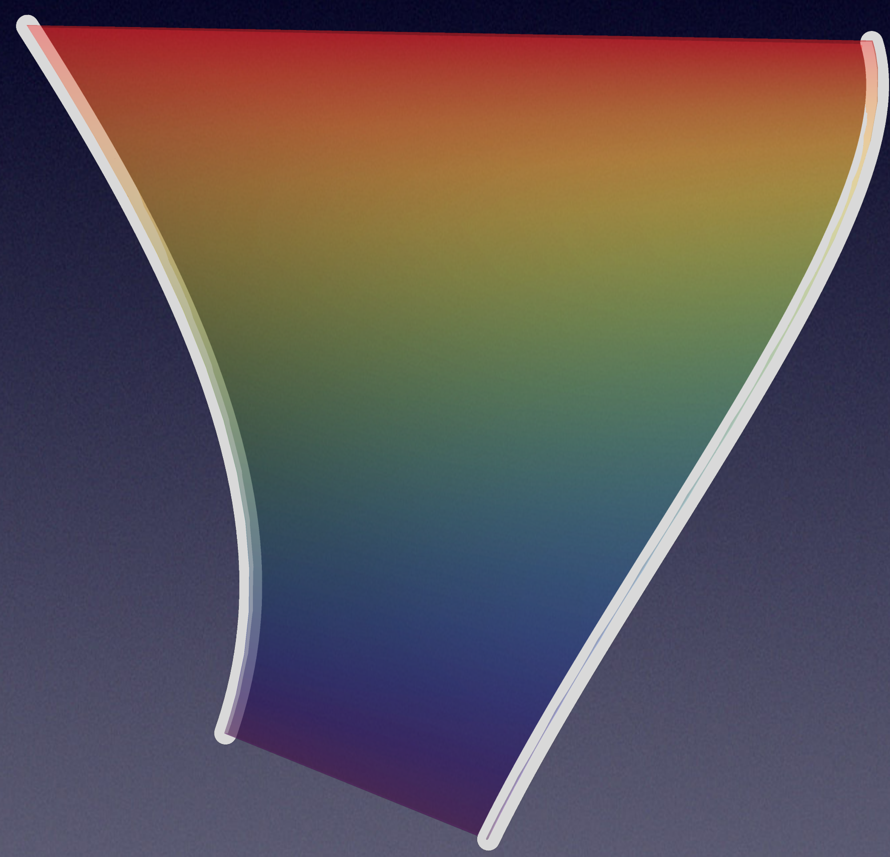

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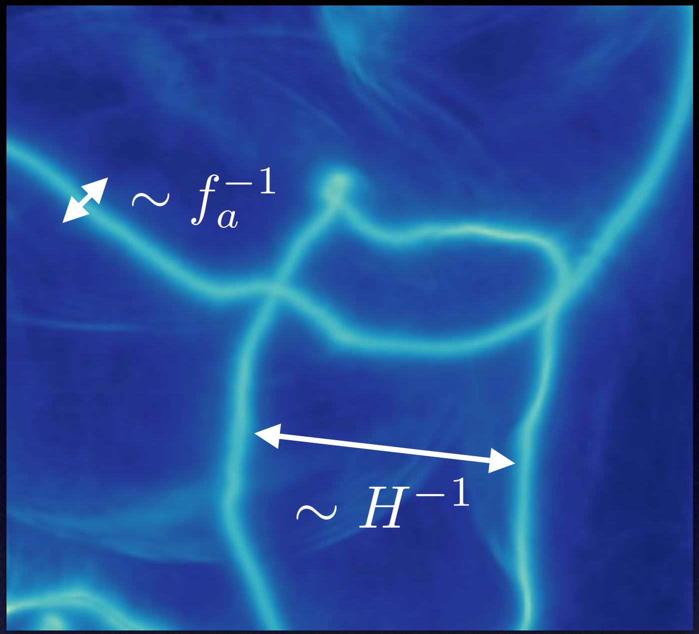
Difficulty in string dynamics

- Two extremely different length scales.
  - String core radius $\sim m_r^{-1} \sim f_a^{-1}$

    m_r : mass of the radial direction

  - Hubble radius $\sim H^{-1}$



- String tension acquires a logarithmic correction.

$$\mu = \frac{\text{energy}}{\text{length}} \simeq \pi f_a^2 \log\left(\frac{m_r}{H}\right)$$

  - Realistic value

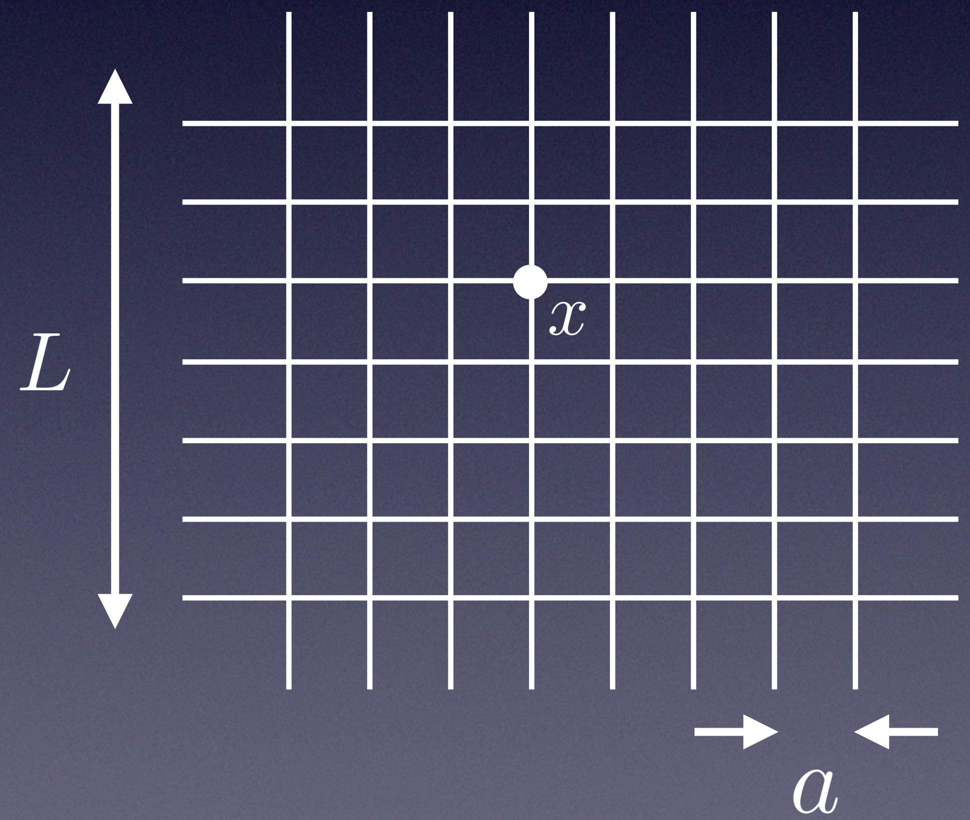
$$f_a/H_{\rm QCD} \sim 10^{30} \quad \Rightarrow \quad \log(m_r/H) \sim 70$$

  - Difficult to reach it in simulations with limited dynamical ranges: Actual strings may be "heavier" than what we observe in simulations. (affects dynamics?)

Lattice simulations

- EOM for a complex scalar field (PQ field) in comoving coordinates

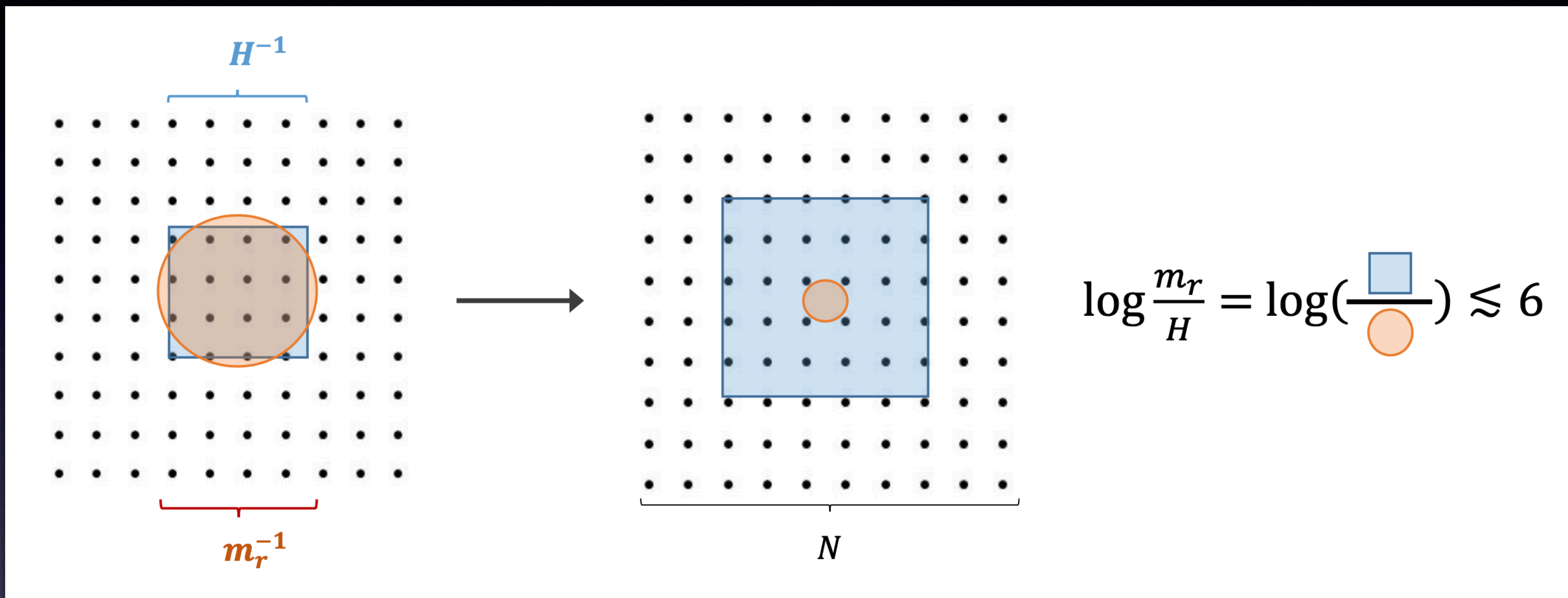
$$\phi_{\tau\tau} - \nabla^2\phi + \lambda\phi(|\phi|^2 - \tau^2) = 0$$



- The PQ field lives on the sites.
- Discretize space and time in a computer.
  - Finite lattice spacing a
  - Finite spatial volume L
  - Finite time step $d\tau$

$a = \frac{L}{N}$ for cubic lattice with size N^3

Log parameter



[Gorghetto, Hardy and Villadoro, 1806.04677]

Large log requires large N.

$$\log\left(\frac{m_r}{H}\right) = \log\left(m_r a \cdot \frac{N}{LH}\right) \sim \log(N)$$

Scaling (attractor) solution

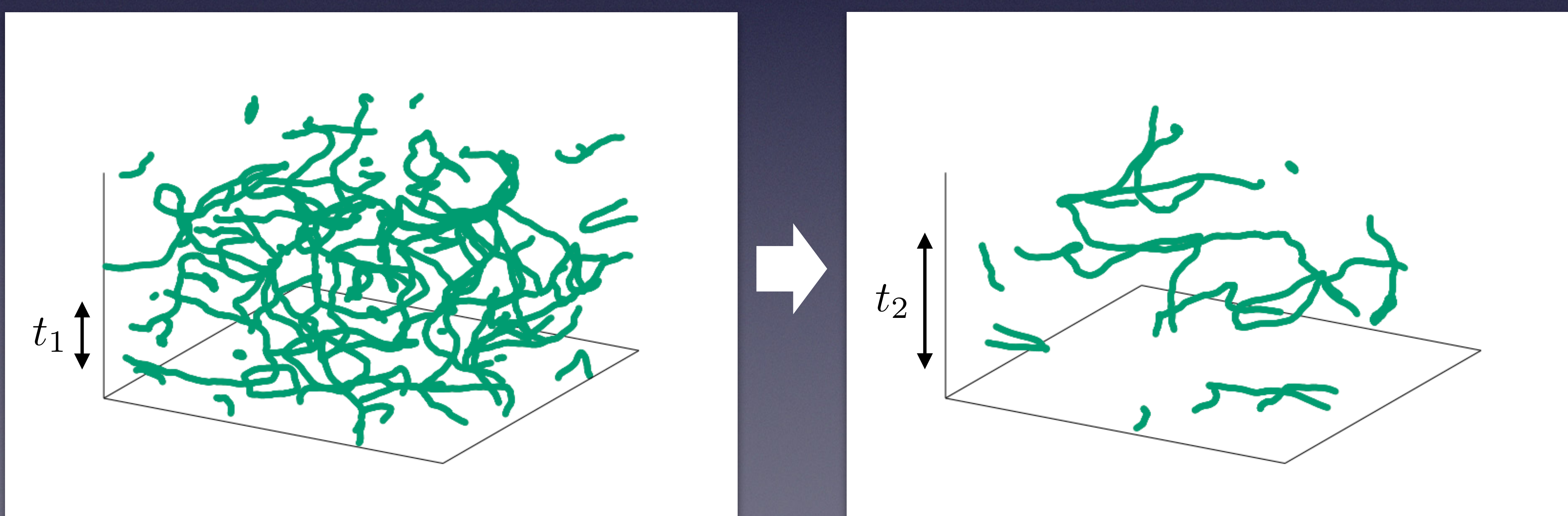
- $\mathcal{O}(1)$ strings per horizon volume:

$$\rho_{\rm string} = \xi \frac{\mu}{t^2} \sim \left.\frac{\mu \ell}{\ell^3}\right|_{\ell \sim H^{-1} \sim t}$$

ξ : dimensionless coeff.

- The net energy density of radiated axions should be the same order.

$$\rho_a \sim \xi \frac{\mu}{t^2} \sim \xi H^2 f_a^2 \log(m_r/H)$$



Issue of large log

- "Scaling" solution suggests that the energy density of the system is of order

$$\rho \sim 8\pi\xi \log(m_r/H) H^2 f_a^2$$

This leads to an enhancement by a factor of $\sim \xi \log$ than typical density $H^2 f_a^2$ at QCD temperatures.

Hereafter we use $\log \equiv \log(m_r/H)$

- *Does it imply an enhancement of the axion abundance (and dark matter mass)?*
- We need to know how this energy is partitioned into radiated axions (axion spectrum).

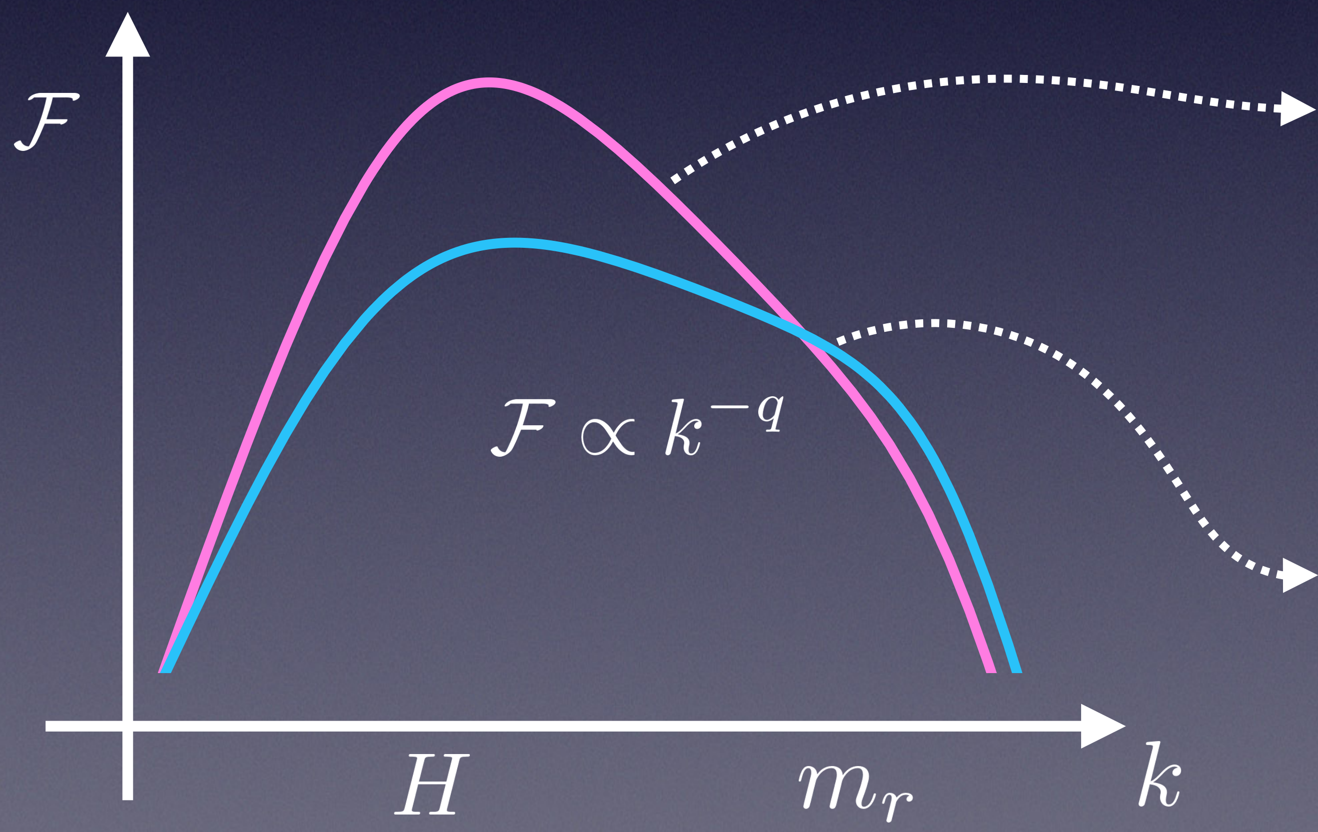
Spectrum of radiated axions

- Differential energy transfer rate

$$\mathcal{F}\left(\frac{k}{RH},\frac{m_r}{H}\right) \equiv \frac{1}{(f_a H)^2}\frac{1}{R^3}\frac{\partial}{\partial t}\left(R^4\frac{\partial \rho_a}{\partial k}\right)$$

R : scale factor

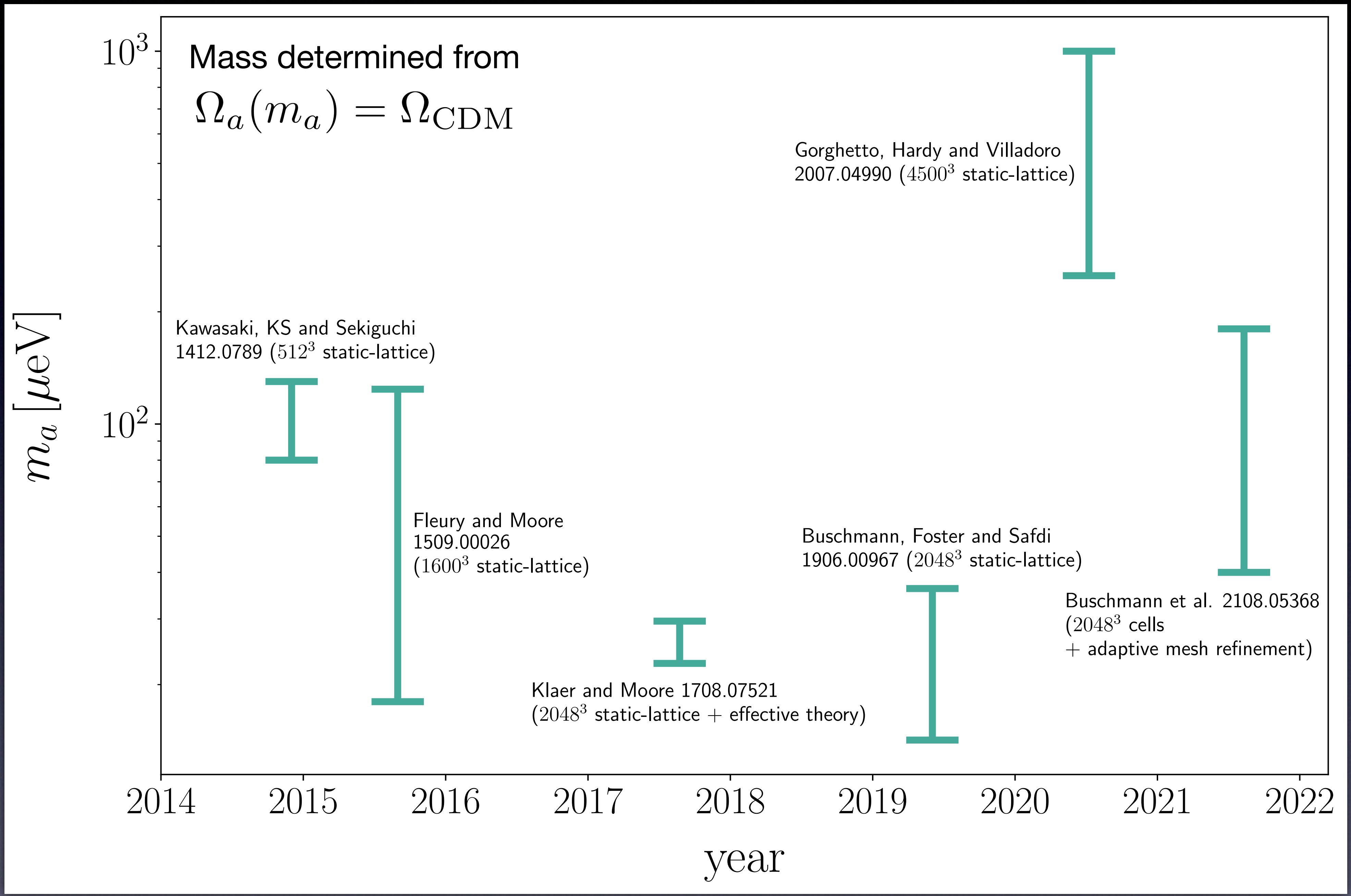
- Slope matters. [Gorghetto, Hardy and Villadoro, 1806.04677]



$q > 1$ (IR modes dominate)
Many soft axions.
Enhancement of DM abundance (?).

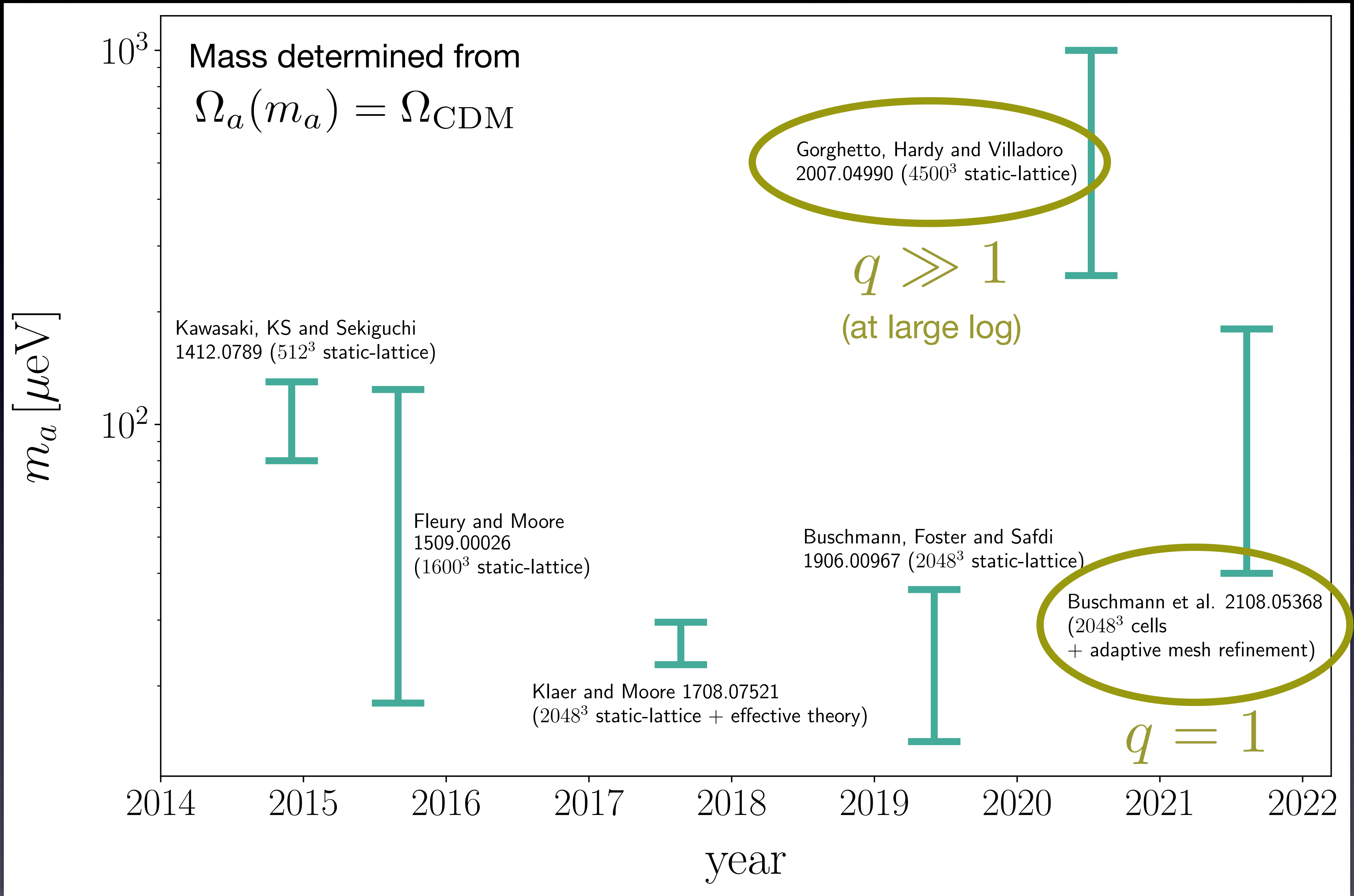
$q < 1$ (UV modes dominate)
Few hard axions.
Suppression of DM abundance.

Controversy on the axion abundance estimation



- Simulations performed by several groups.
- Not only the final result for the axion DM mass but also several outcomes (such as the shape of the radiation spectrum) are disparate.

Controversy on the axion abundance estimation



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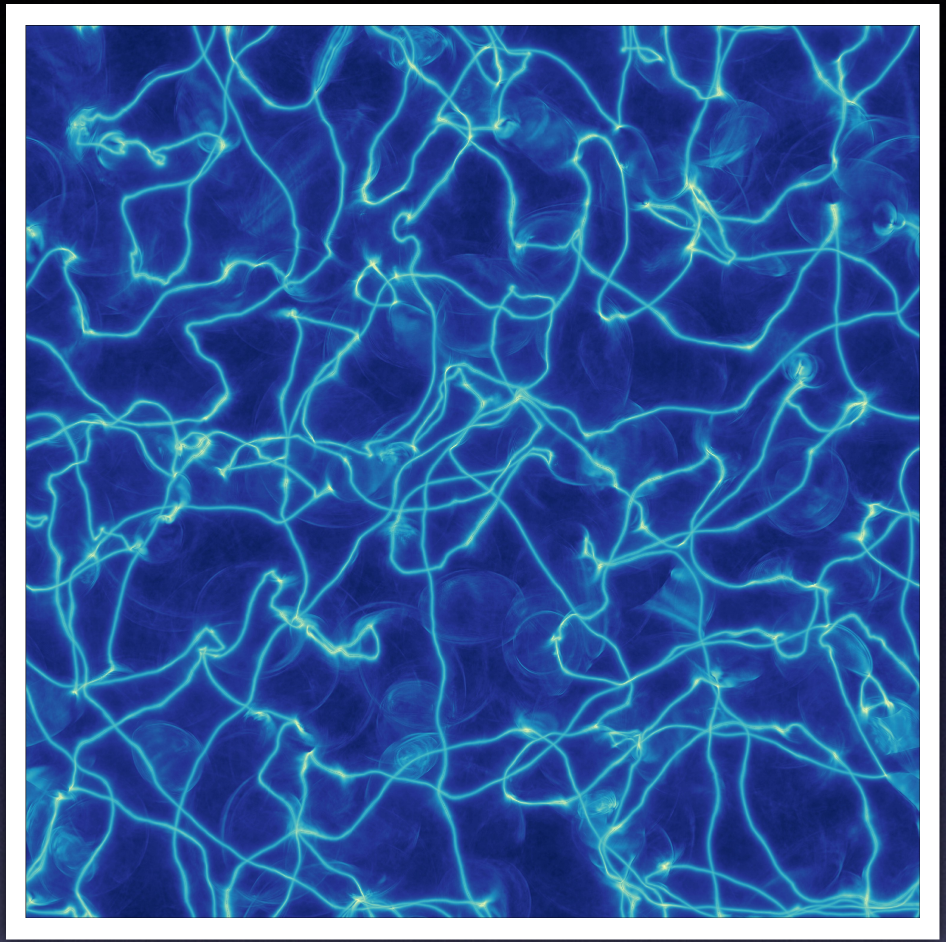
Simulations at MPCDF [Redondo, KS and Vaquero]

- Parallel code specialized to simulate the evolution of the axion field
  - hybrid MPI+openMP parallelization
  - extensive use of advanced vector extensions (AVX, AVX2, AVX512)
  - cache tuner functionality
- Simulations performed at RAVEN and COBRA supercomputers at Max Planck Computing and Data Facility (MPCDF), Garching



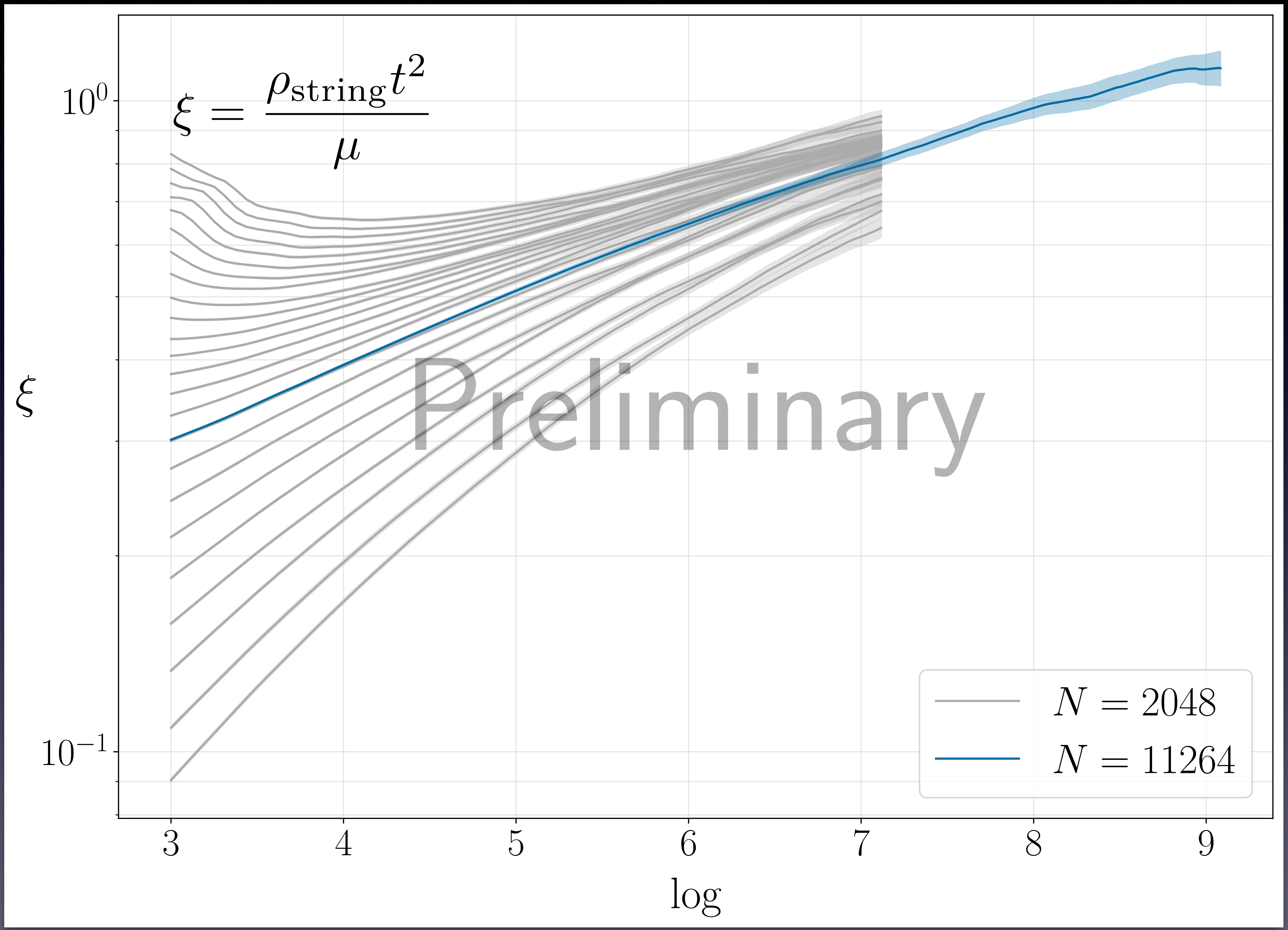
MPCDF MAX PLANCK COMPUTING & DATA FACILITY

https://www.mpcdf.mpg.de

- The number of grids reaches $N^3 = 11264^3$ (RAVEN, 256 nodes, 18432 CPUs)

➡ $\log \lesssim 9$ is feasible.

String density

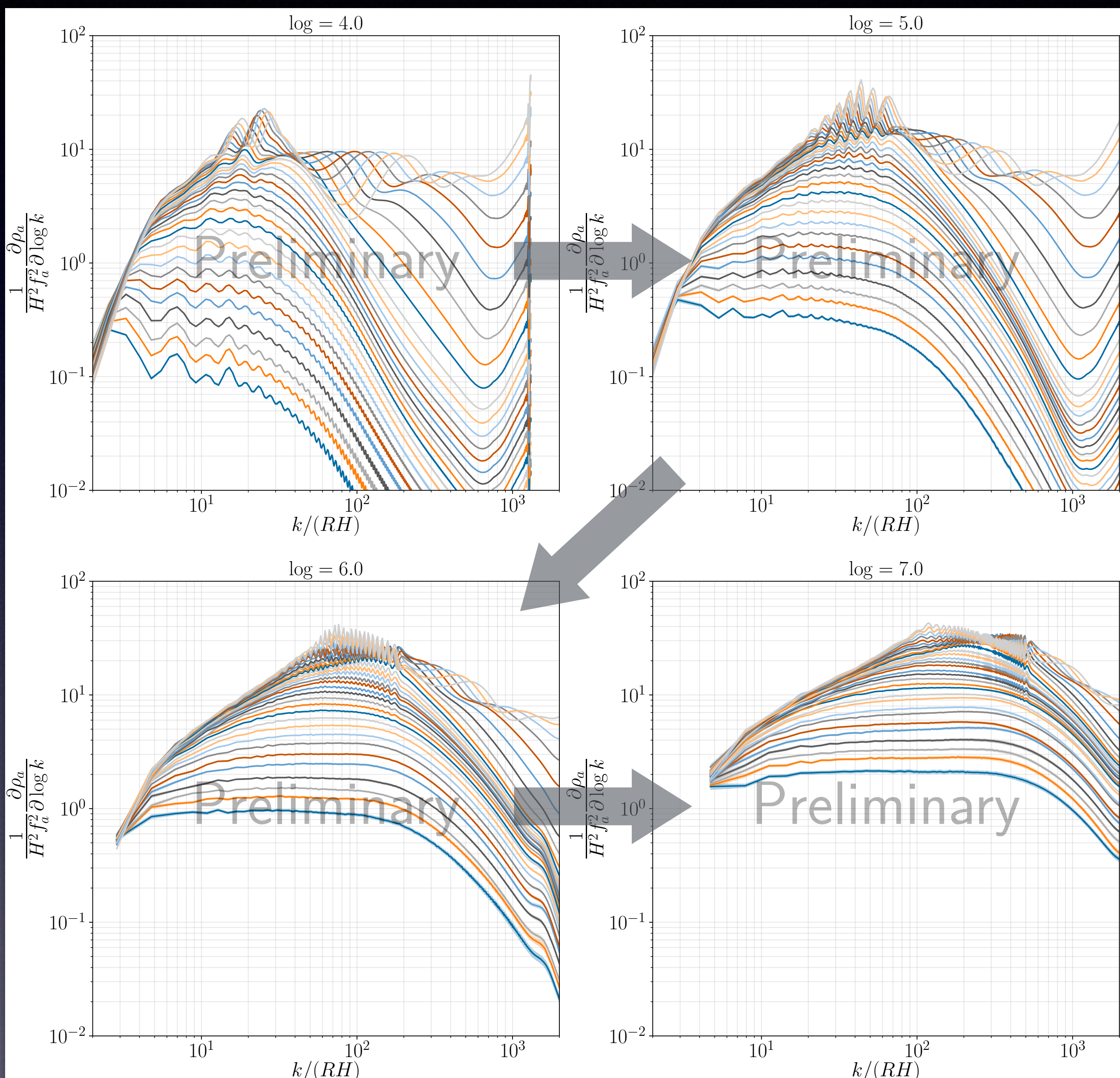


Logarithmic growth and "attractor" behavior compatible with previous findings.

[Fleury and Moore, 1509.00026]
[Gorghetto, Hardy and Villadoro, 1806.04677; 2007.04990]
[Kawasaki, Sekiguchi, Yamaguchi and Yokoyama, 1806.05566]

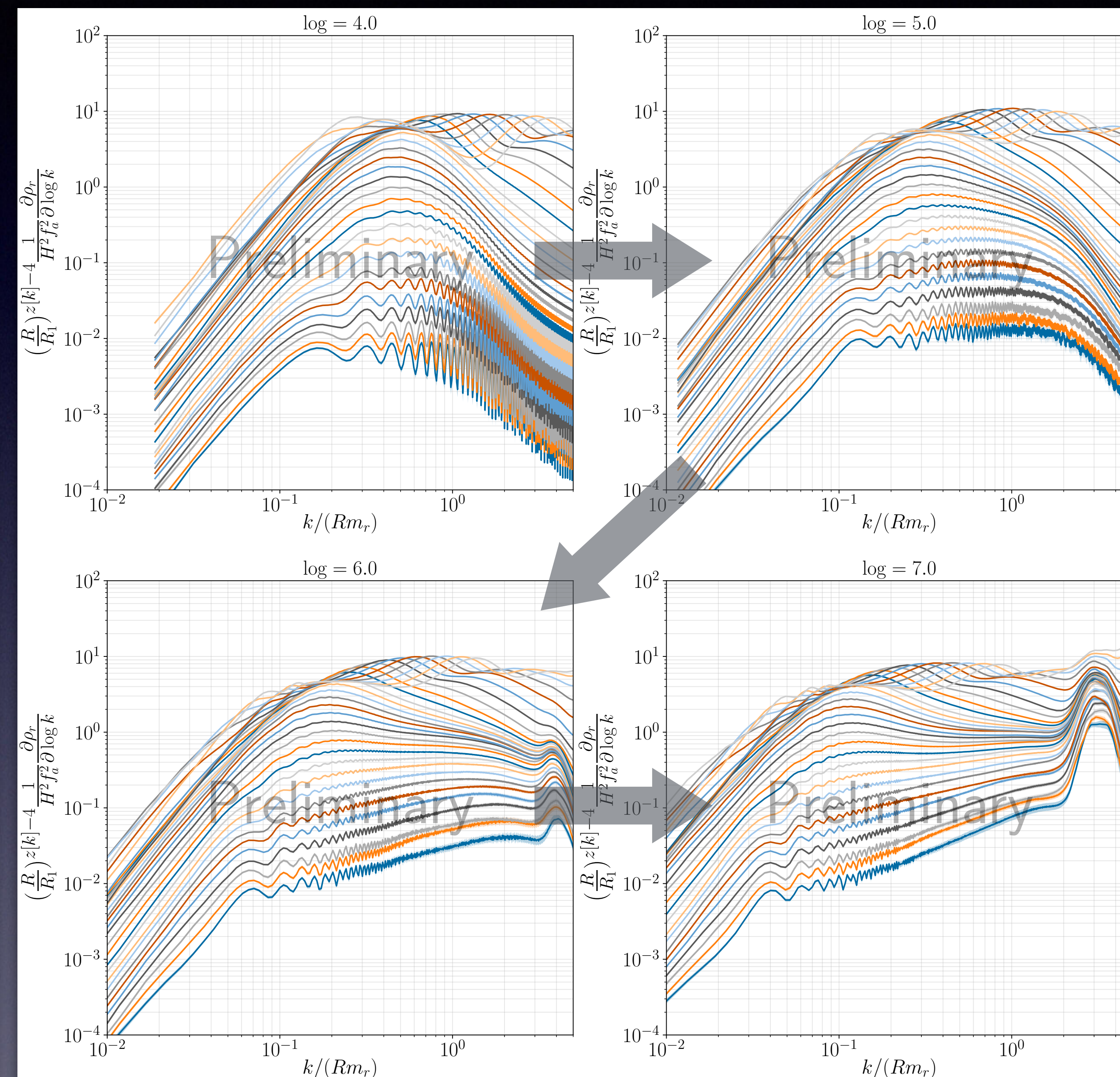
Spectra also show the attractor behavior

Axion spectrum



$$\frac{1}{(f_a H)^2}\frac{\partial \rho_a}{\partial \log k}$$

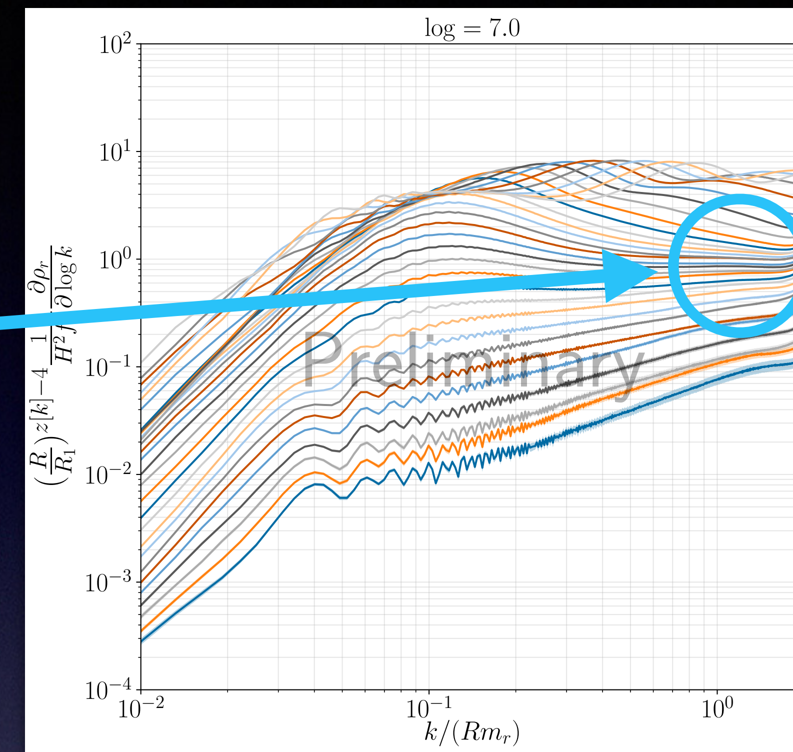
Radial field ("saxion") spectrum



$$R^{z[k]-4}\frac{1}{(f_a H)^2}\frac{\partial \rho_r}{\partial \log k} \qquad z[k] = 3 + \frac{\left(\frac{k}{Rm_r}\right)^2}{1+\left(\frac{k}{Rm_r}\right)^2}$$

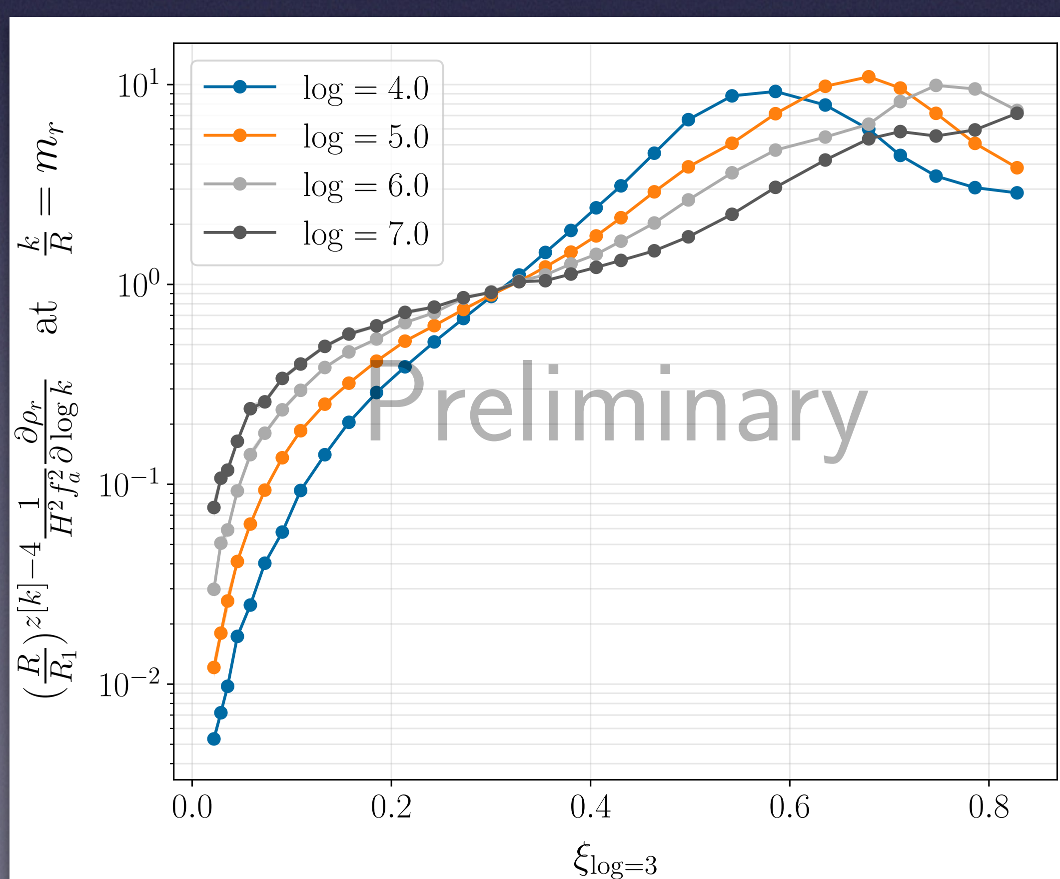
Finding the attractor

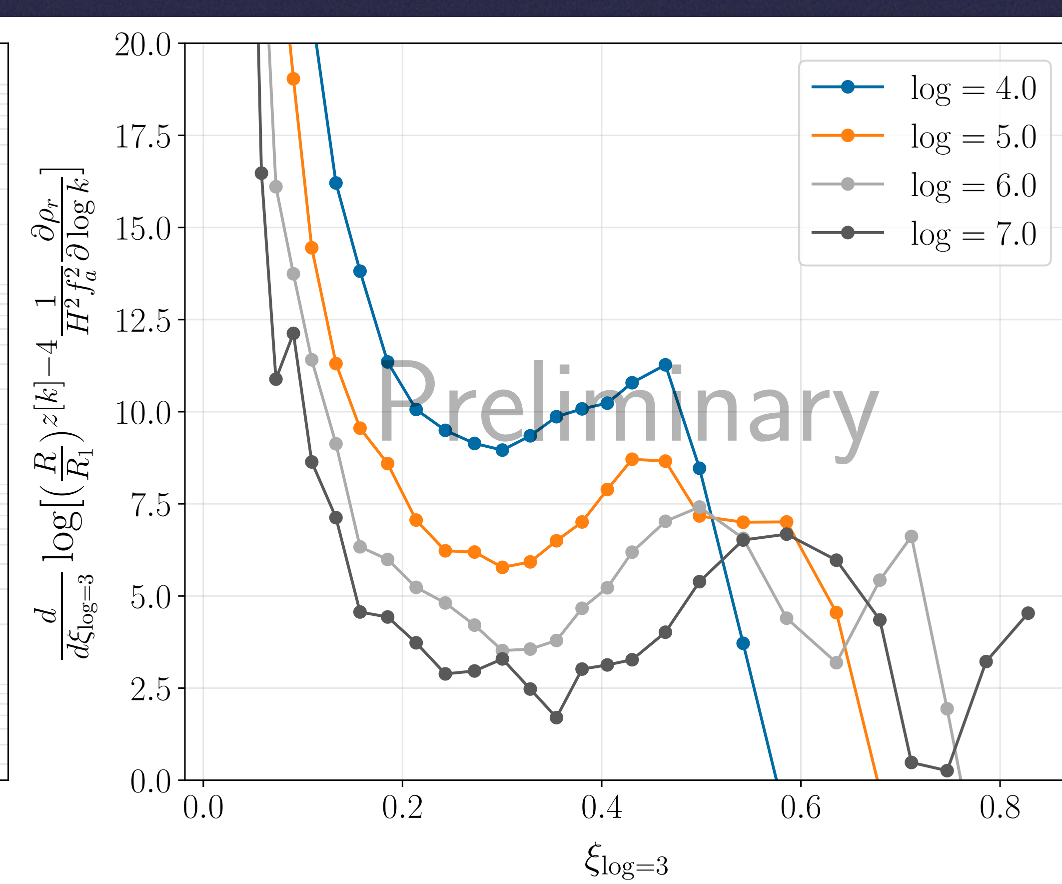
- The convergence behavior is most visible in the radial field spectra: Saxions are only produced at $k/R \sim m_r$.



- Existence of the point that is least sensitive to the initial condition.

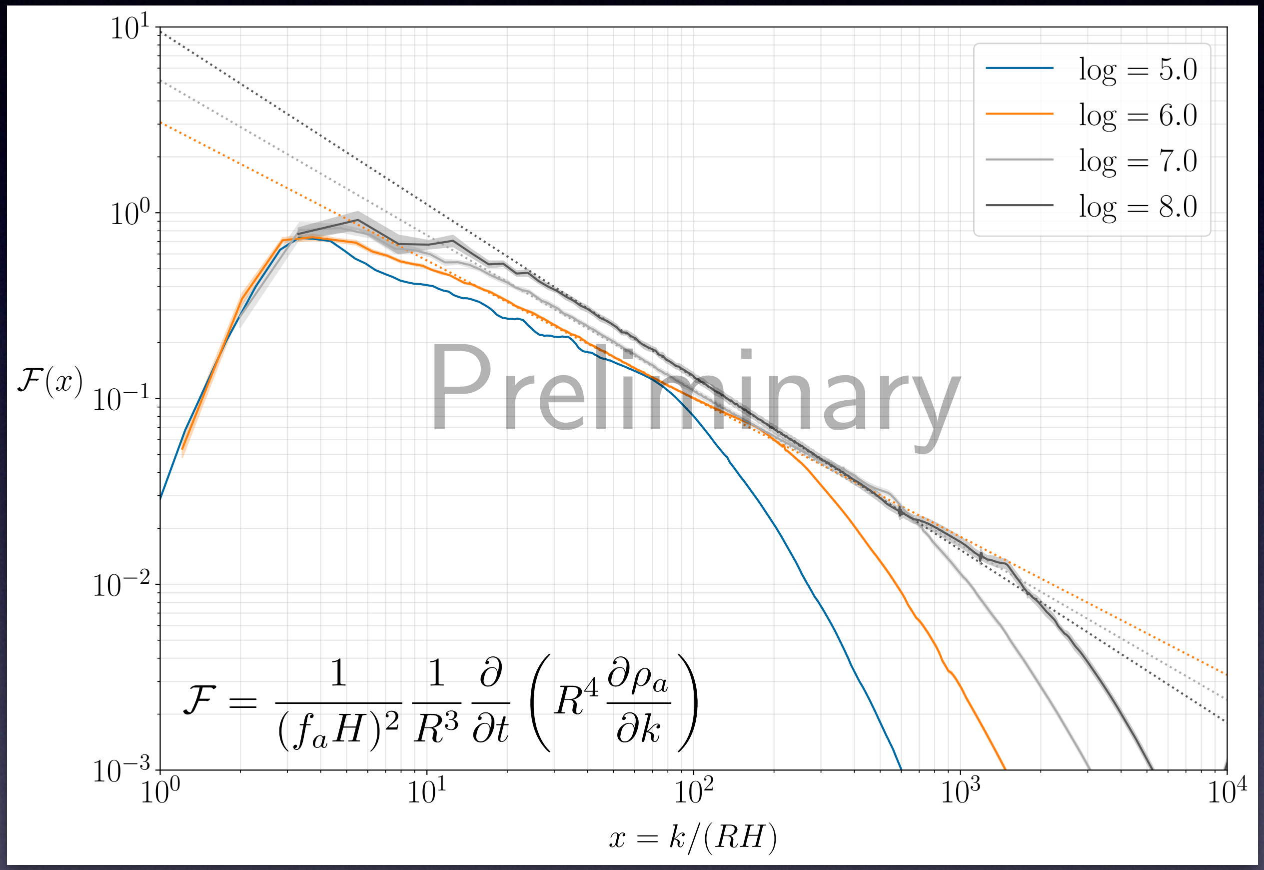
➡ Identified as "attractor".





Differential spectrum and spectral index

Fit a power law $\mathcal{F}(x) \propto x^{-q}$

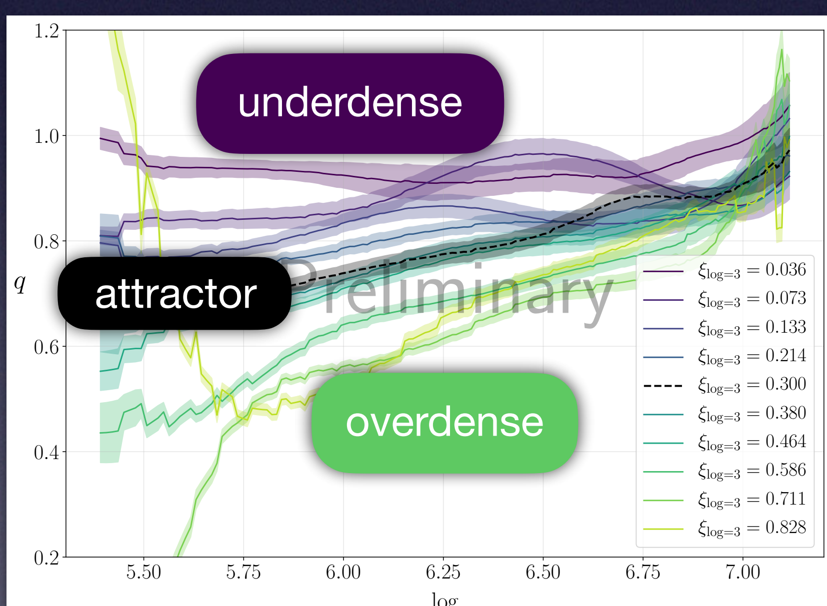


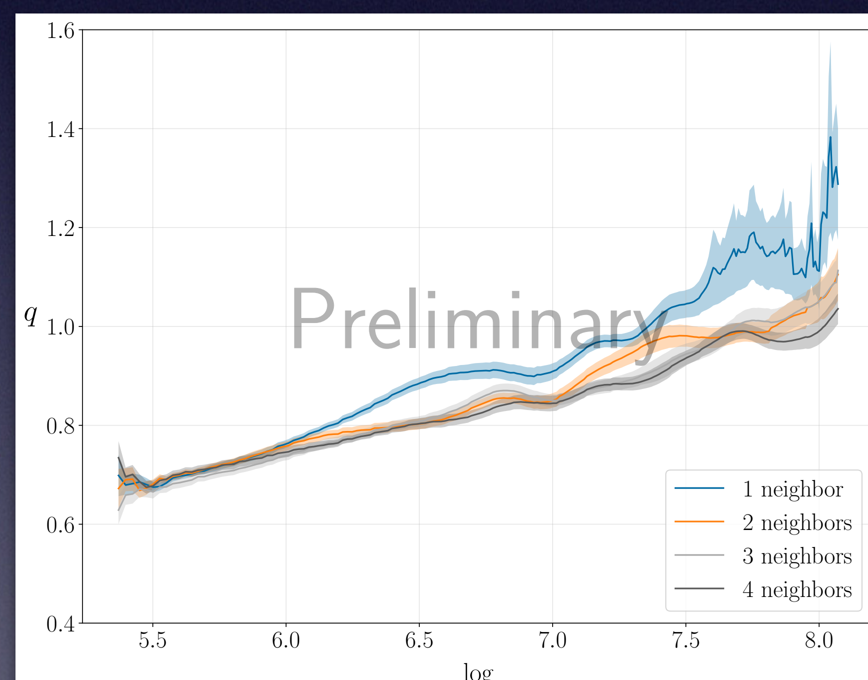
Results indicate the logarithmic growth of q and do not agree with the scenario of q = 1 (= constant) suggested by the recent AMR simulation.

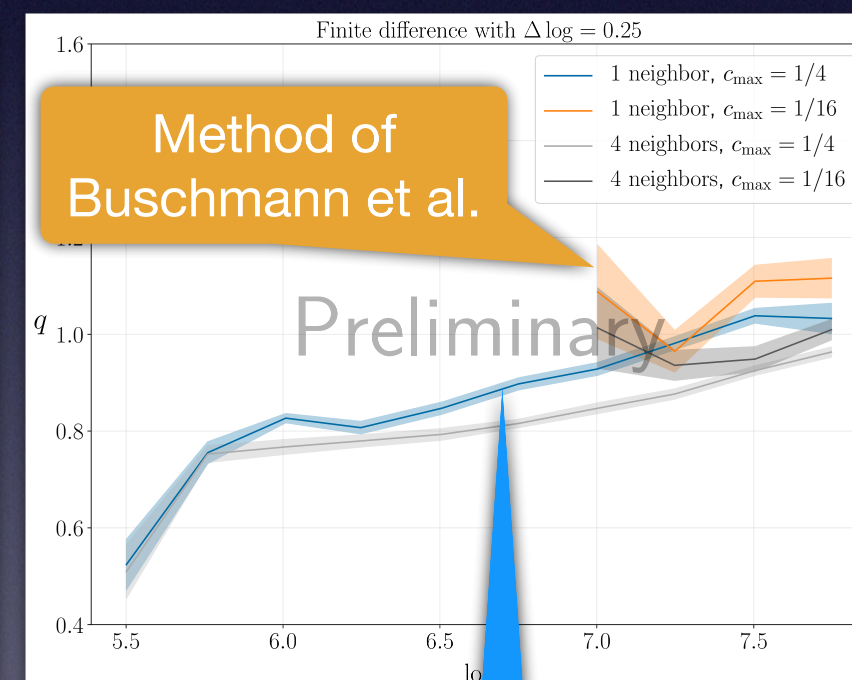
⇨ IR-dominated spectrum at large log

Possible sources of discrepancy

- Difference in the initial string density.
- Discretization scheme of laplacian.
- Time derivative of the spectrum.



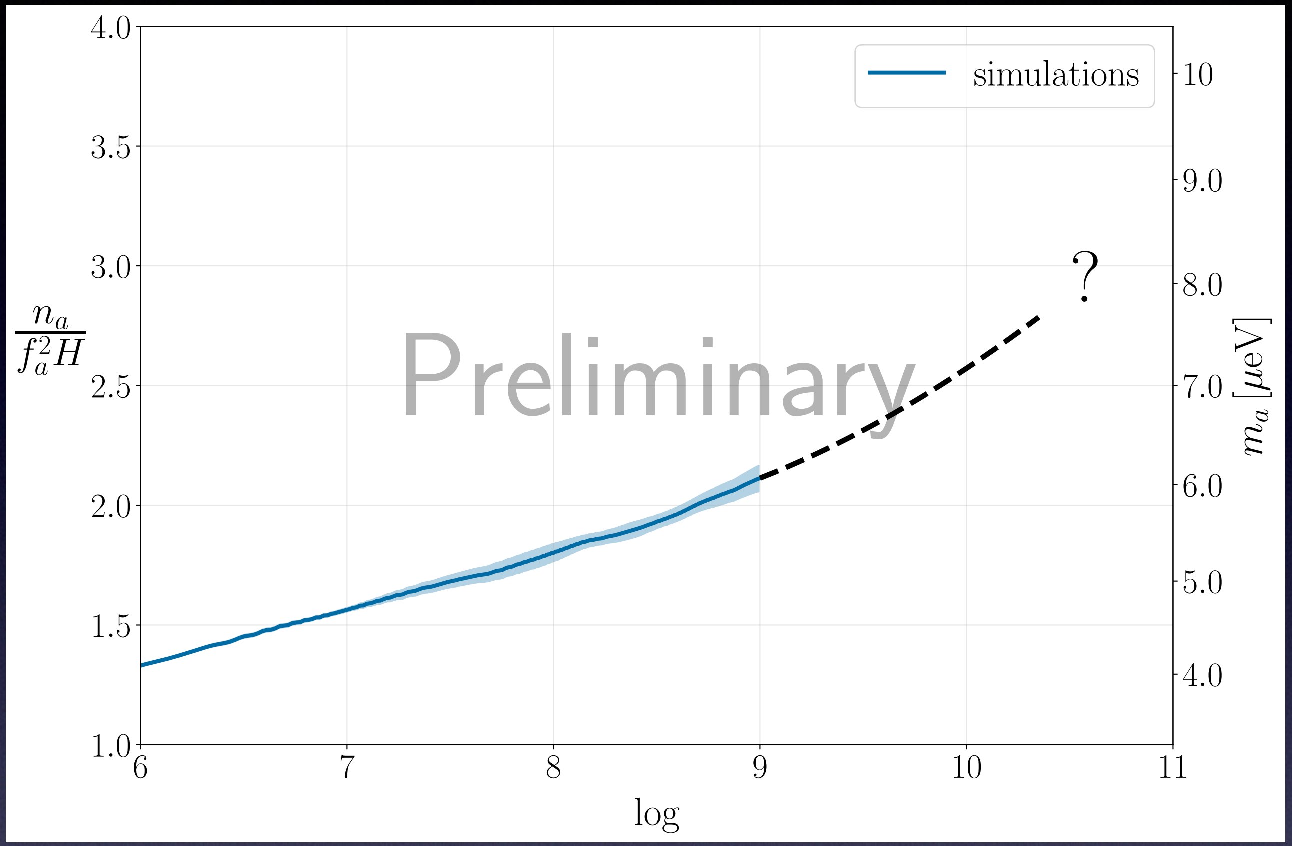




Broader fit range, less affected by IR oscillations

Axion abundance

- IR-dominated spectrum implies larger axion number (and hence larger axion DM mass) at large log.
- Error bar? ... work in progress
- Need further improvements in dynamical range to extract the trend more accurately.



$$\frac{n_a}{f_a^2 H} \approx \frac{\Gamma}{f_a^2 H^3} \left\langle \left(\frac{k}{RH}\right)^{-1} \right\rangle$$

$$\sim 8\pi\xi \log \left\langle \left(\frac{k}{RH}\right)^{-1} \right\rangle$$

$$\left\langle \left(\frac{k}{RH}\right)^{-1} \right\rangle \equiv \frac{\int \frac{dx}{x} \mathcal{F}(x)}{\int dx \mathcal{F}(x)}$$

Summary

- Understanding of the global string dynamics is indispensable for a sharp prediction for "typical" axion dark matter mass, which will serve as a guide for forthcoming axion searches.

- Fast developments in recent simulations allow us to have a better understanding, albeit serious discrepancies.

- The latest simulation clearly shows a trend of growing spectral index, experiencing a transition into the IR-dominated spectrum.

- Further improvement in the dynamical range would be helpful to be sure of extrapolation.