Physics of Negative Energy Solitons

-- Frontiers in Gravity and Fundamental Physics --

YU Workshop 2022 @ Yamagata Nov 27, 2022

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References

Qunatum nucleation of topological solitons
M.Eto & MN, *JHEP* 09 (2022) 077 [2207.00211 [hep-th]]

Non-Abelian chiral soliton lattice in QCD
M.Eto, K.Nishimura & MN, *JHEP* 08 (2022) 305 [2112.01381 [hep-ph]]

Domain wall lattice in chiral magnets
C.Ross, N.Sakai, MN, *JHEP* 12 (2021) 163 [2012.08800 [cond-mat]]]

Skyrmion lattice in chiral magnets
C.Ross, N.Sakai, MN,
***JHEP* 02 (2021) 095 [2003.07147 [cond-mat]]**

Topological solitons(or defects) can have negative energy(tension).

What are physical consequences?

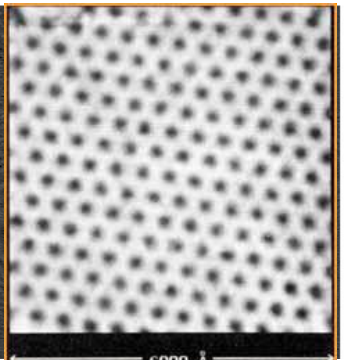
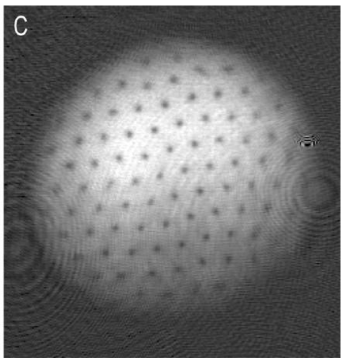
Ground states are solitonic

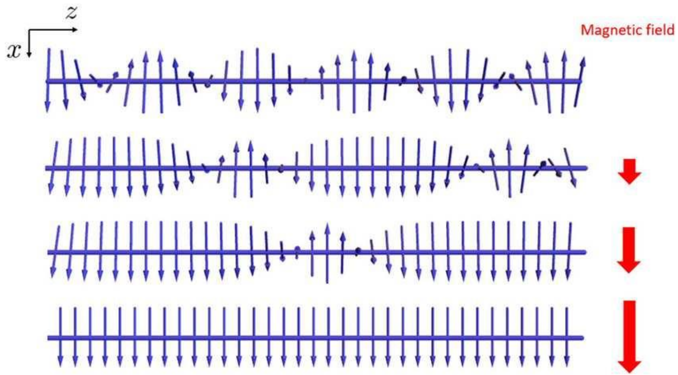
(when they are repulsive)

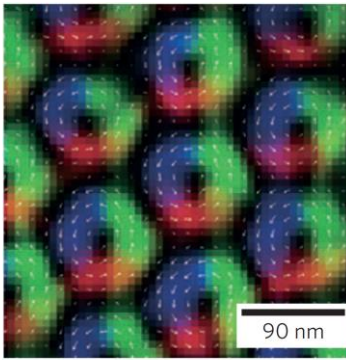
Solitonic ground states

1. Abrikosov lattice in Type-II superconductors under B
2. Abrikosov lattice in superfluids under rotation
3. Chiral soliton lattices in chiral magnets
4. Skyrmion lattices in chiral magnets
5. Chiral soliton lattices in QCD under
   - strong magnetic field Son-Stephanov('07), Brauner-Yamamoto ('16)
   - rapid rotation Huang-Nishimura-Yamamoto('17)

Cond-mat examples
Important for industry





Classification of phase transition

deGennes said that there are
two kinds of continuous (2$^{\text{nd}}$ order) transitions:
instability type & nucleation type.

Instability type: ordinary one described by Landau
Nucleation type:
soliton energy <0, soliton interaction >0(repulsive)

The model at IR (common for chiral magnets & QCD)
= sine-Gordon model + topological term
(chiral sine-Gordon model)

$$\mathcal{H}_{\rm IR} = v^2\left[\dot{\theta}^2 + (\nabla\theta)^2 - 2m^2(\cos\theta - 1) - \underline{c\boldsymbol{B}\cdot\nabla\theta}\right]$$

topological term
Not affecting EOM

The origin of the term
1. Dzyaloshinskii–Moriya interaction for chiral magnets
2. WZW term for QCD @ finite density
under magnetic field or rotation

Chiral sine-Gordon model in QCD [Son-Stephanov('07)]

SU(2) Nambu-Goldstone fields $\Sigma = \exp\left(\frac{i\sigma^a\pi^a}{f_\pi}\right)$

Chiral Lagrangian with Wess-Zumino-Witten term

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[D_\mu\Sigma D^\mu\Sigma^\dagger + m_\pi^2(\Sigma+\Sigma^\dagger)\right] + \mathcal{L}_{\mathrm{WZW}}$$

$$\mathcal{L}_{\mathrm{WZW}} = -\left(A_\mu^{\mathrm{B}} + \frac{1}{2}A_\mu^{\mathrm{EM}}\right) j_{\mathrm{B}}^\mu$$

$L_\mu = \Sigma\partial_\mu\Sigma^\dagger,\ R_\mu = \partial_\mu\Sigma^\dagger\Sigma$

$$j_{\mathrm{B}}^\mu = -\frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\left\{\mathrm{Tr}L_\nu L_\alpha L_\beta - \frac{3}{2}i\partial_\nu\left(A_\alpha\sigma^3(L_\beta+R_\beta)\right)\right\}$$

A constant magnetic field $\boldsymbol{H}$

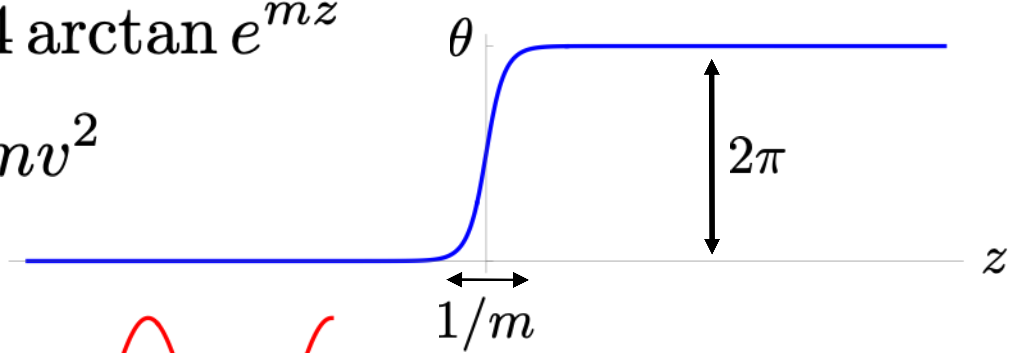
A baryon chemical potential $A_\mu^{\mathrm{B}} = (\mu_{\mathrm{B}}, \vec{0})$

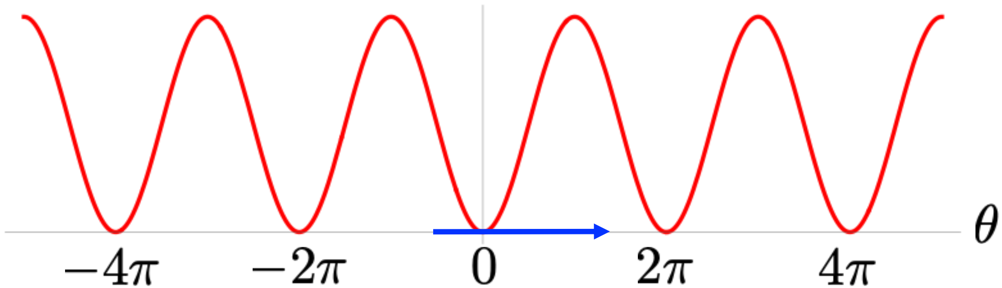
Ignore charged pions $\pi^1 - i\pi^2 = 0$

$c = 0$ no topological term

Single soliton $\theta(z) = 4 \arctan e^{mz}$

tension $\sigma = 16 m v^2$





2 solitons repel (no static sol)

Soliton lattice

$$\theta(z) = 2\mathrm{am}\left(\frac{mz}{k}, k\right) + \pi$$

Jacobi amplitude function

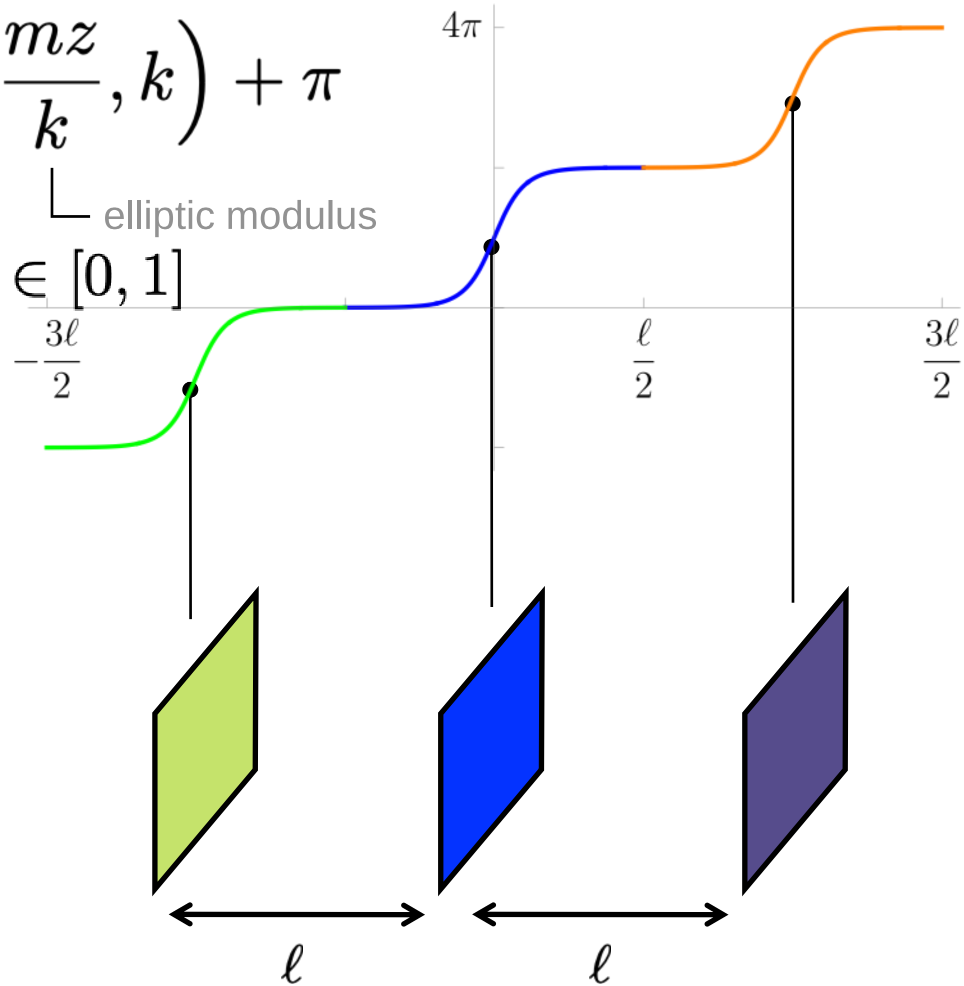
elliptic modulus

$k \in [0, 1]$

lattice spacing

$$\ell(k) = 2kK(k)/m$$

elliptic integral of 1st kind



$c \neq 0$ no topological term

tension $\sigma = 16mv^2 - 2\pi v^2 cB$

Soliton #
Negative contribution

When $B > 8m/\pi c$, tension is negative $\sigma < 0$

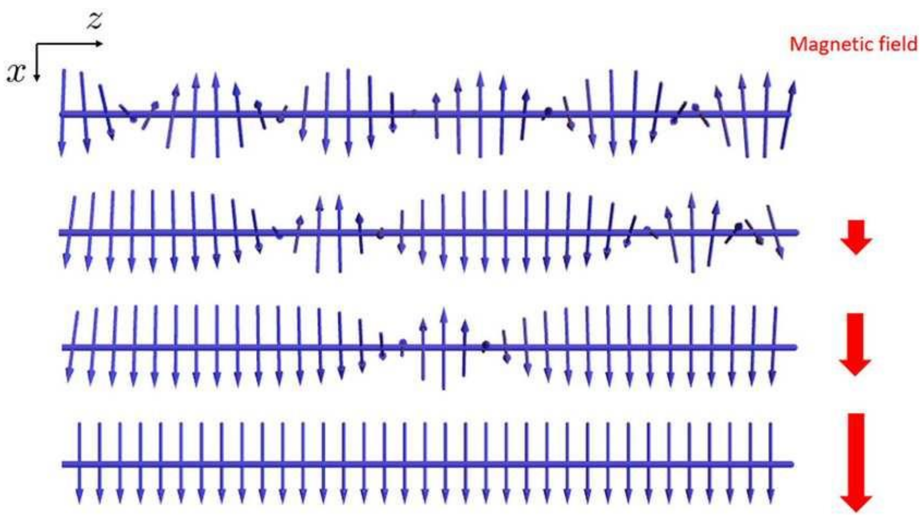
Solitons are created (come in) in the ground state, but not infinitely because of repulsion.

$\rightarrow$ The ground state is a chiral soliton lattice.

How are they created?

In condensed matter physics

--- usually defect, or at boundary of a finite system.



How about infinite systems?

Topological soliton creation Ours ('22)
= Quantum nucleation of a soliton

Is it possible?

$$E = +\pi R^2 T_{wall} + 2\pi R T_{string}$$

YES!!

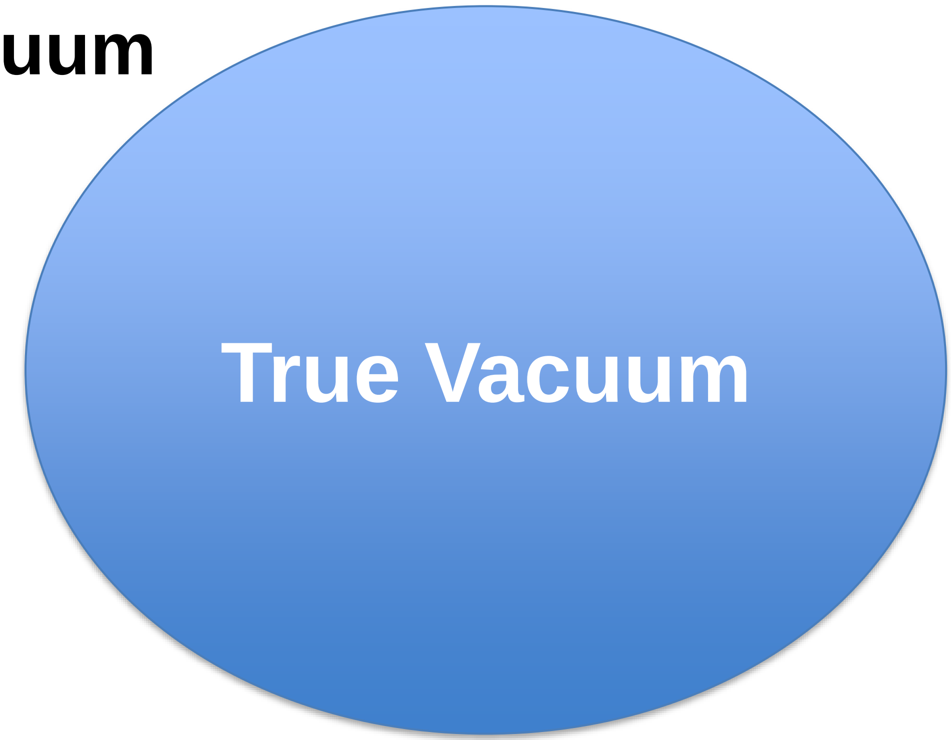
Possible if $T_{wall} < 0$

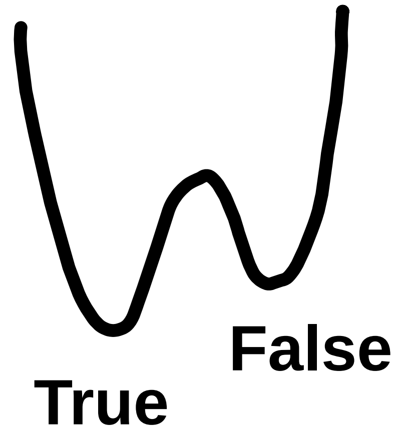
Quantum nucleation of topological solitons
M.Eto & MN, *JHEP* 09 (2022) 077 [2207.00211 [hep-th]]
***Also* Higaki, Kamada, Nishimura, 2207.00212 [hep-th]**

Nucleation of topological defects in de Sitter space,
(string loop, spherical domain wall, monopole-anti-monopole)
Basu-Guth-Vilenkin('91), Basu-Vilenkin('92),
Garriga-Vilenkin('93), Garriga('94)

False vacuum decay **Coleman('77)**
= Quantum nucleation of a bubble

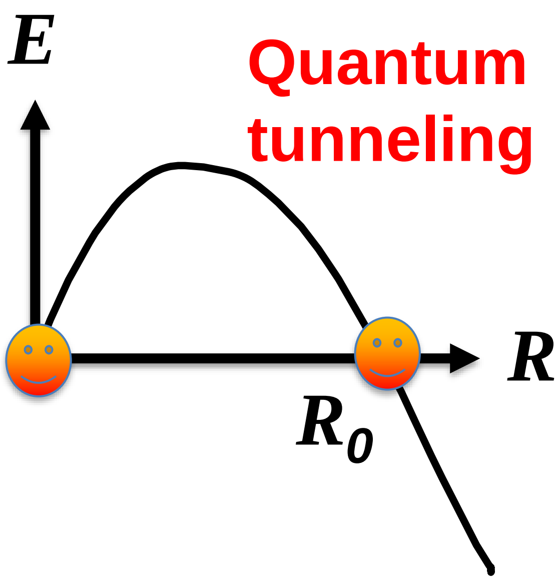
False Vacuum

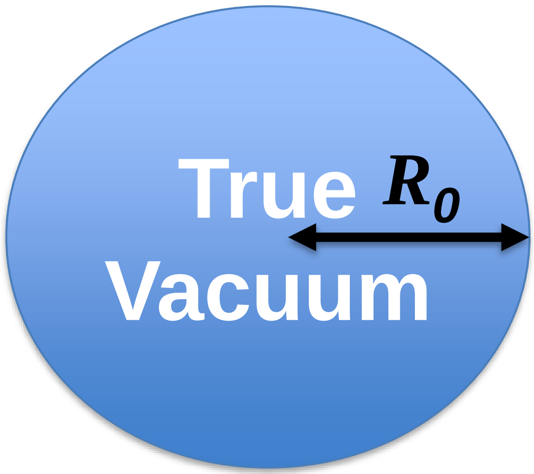


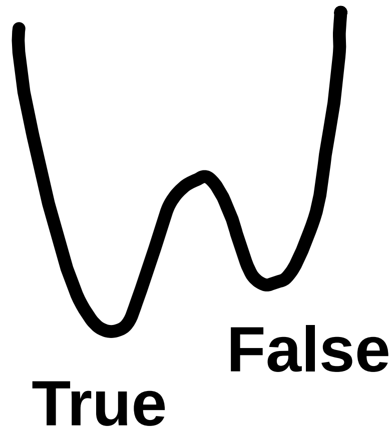


False vacuum decay **Coleman('77)**
= Quantum nucleation of a bubble

False Vacuum





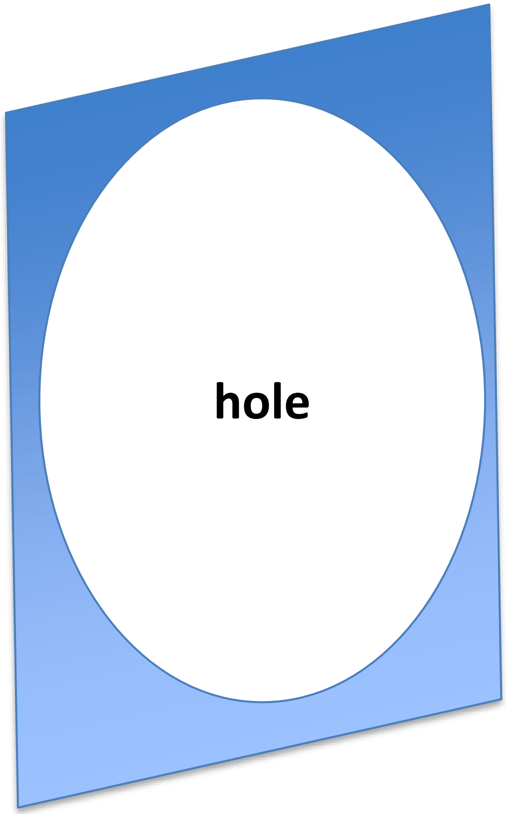


Domain wall tension @surface

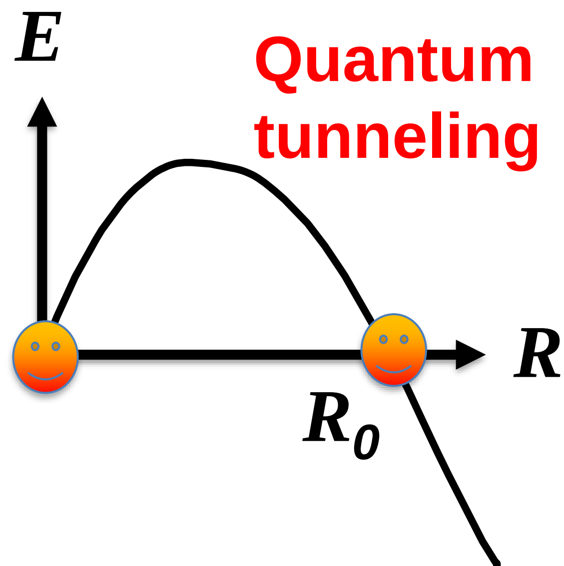
Topological soliton decay Preskill & Vilenkin('93) = Quantum nucleation of a hole

Domain wall

A hole bound by a string loop



$$E = -\pi R^2 T_{wall} + 2\pi R T_{string}$$



Topological soliton creation Ours ('22)
= Quantum nucleation of a soliton

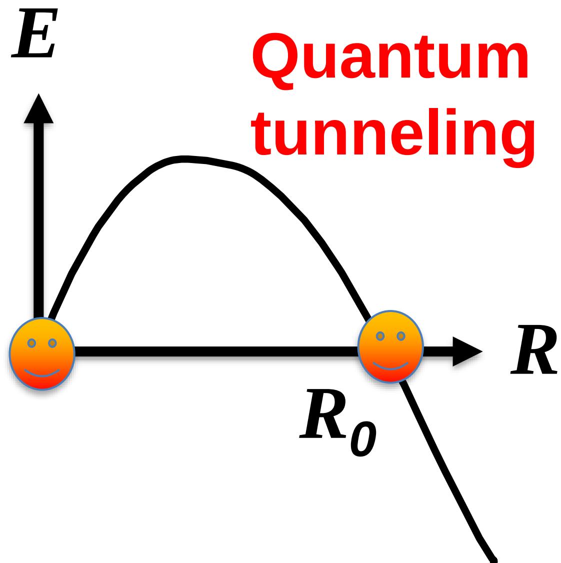
Vacuum

A soliton disk bound by a string loop

"Pancake soliton"

$$E = +\pi R^2 T_{wall} + 2\pi R T_{string}$$

Possible if $T_{wall} < 0$



The model at IR (common for chiral magnets & QCD) = sine-Gordon model + topological term

$$\mathcal{L}_{\mathrm{IR}} = v^2 \left[(\partial_\mu \theta)^2 + 2m^2(\cos\theta - 1) + \underline{c\boldsymbol{B} \cdot \nabla\theta} \right]$$

$$\mathcal{H}_{\mathrm{IR}} = v^2 \left[\dot{\theta}^2 + (\nabla\theta)^2 - 2m^2(\cos\theta - 1) - c\boldsymbol{B} \cdot \nabla\theta \right]$$

The UV theory = axion (Goldstone) model + topo

$$\mathcal{L}_{\mathrm{UV}} = |\partial_\mu \phi|^2 - \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 + vm^2(\phi + \phi^*) + c\boldsymbol{j} \cdot \boldsymbol{B}$$

$$j^\mu = -\frac{i}{2}(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) = |\phi|^2 \partial^\mu \theta\,, \qquad \phi = |\phi| e^{i\theta}$$

For $m = 0$ (Goldstone model)

Nambu-Goldstone (NG) mode + Higgs mode $m_h = v\sqrt{\lambda}$

Global string $\delta_{\rm st} \sim m_h^{-1}$, $\mu\big|_{m\to 0} \sim \pi v^2 \log(m_h L)$

thickness **tension**

L**: system size (IR cutoff)**

For $m \neq 0$, pseudo NG mode

We consider $m_h \gg m \quad \Leftrightarrow \quad m^2 \ll \lambda v^2$

For simplicity $m_h \to \infty$

Sine-Gordon(SG) soliton

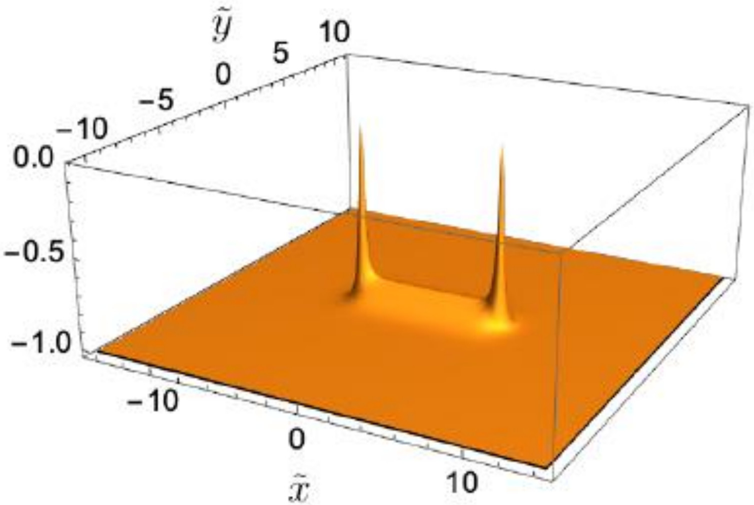
$$\theta = 4\tan^{-1} e^{mz} \qquad \delta_{\mathrm{dw}} = m^{-1}, \qquad \sigma\big|_{\lambda\to\infty} = 16mv^2$$

thickness **tension**

For finite m_h,

1) SG soliton is metastable
2) SG-soliton can be bound by a string
3) String has a *finite* tension $\mu\big|_{m>0} = \text{const.}$

$\tilde{R} = 4.83$



Missing in Preskill-Vilenkin

Without the topological term, the decay probability is (Preskill-Vilenkin)

$$P_{\rm decay} = Ae^{-S}, \qquad S = \frac{16\pi\mu^3}{3\sigma^2}, \qquad R = \frac{2\mu}{\sigma}$$

Nucleation probability with the topological term

$$\tilde{x}^\mu = mx, \qquad \tilde{\phi} = v^{-1}\phi, \qquad \tilde{\lambda} = \frac{m_h^2}{m^2}, \qquad \tilde{\boldsymbol{B}} = m^{-1}c\boldsymbol{B}.$$

$$\mathcal{L}_{\rm UV} = m^2v^2\left[|\tilde{\partial}_\mu\tilde{\phi}|^2 - \frac{\tilde{\lambda}}{4}\left(|\tilde{\phi}|^2 - 1\right)^2 + \tilde{\phi} + \tilde{\phi}^* + \tilde{\boldsymbol{j}}\cdot\tilde{\boldsymbol{B}}\right]$$

$$\tilde{\boldsymbol{j}}\cdot\tilde{\boldsymbol{B}} = \tilde{B}\tilde{j}_z\cos\alpha$$

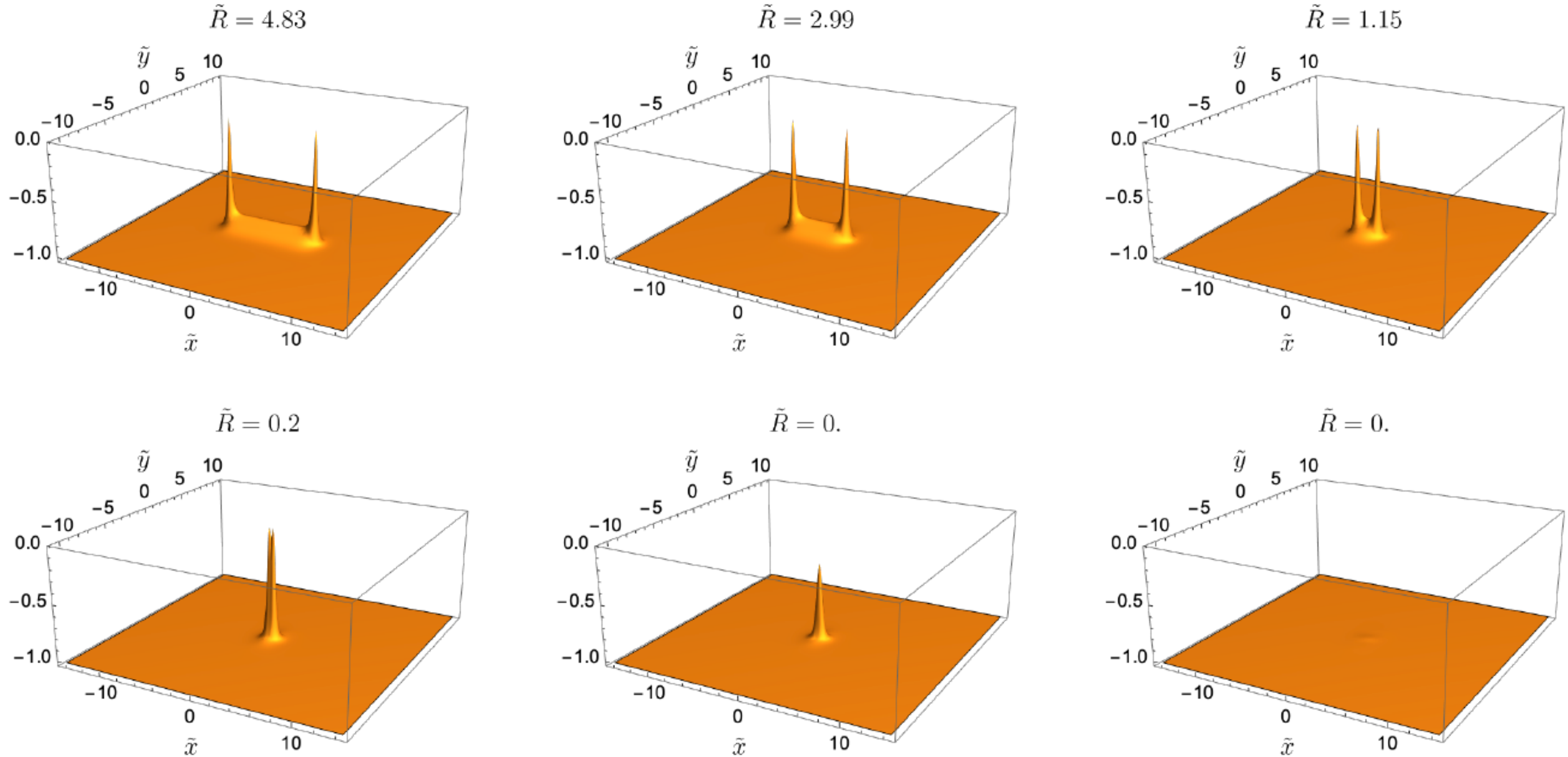
α: Angle between soliton and *B*

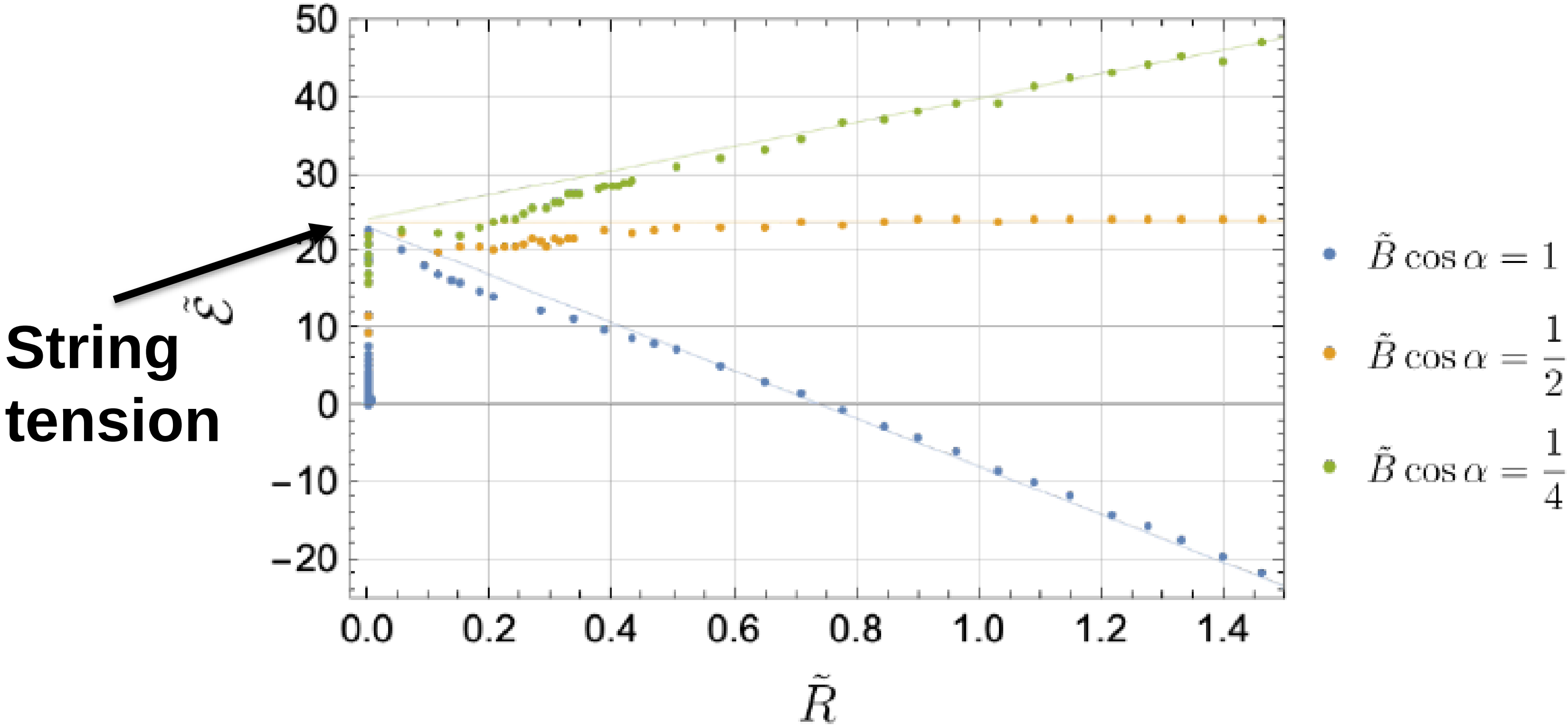
2+1 dim

Thin-defect approx

$$S = 2\pi R\mu + \pi R^2\sigma. \qquad R_0 = \frac{\mu}{-\sigma}, \qquad S_0 = \frac{\pi\mu^2}{-\sigma}$$

Numerical simulation in 2+1 dim: relaxation





Decay prob

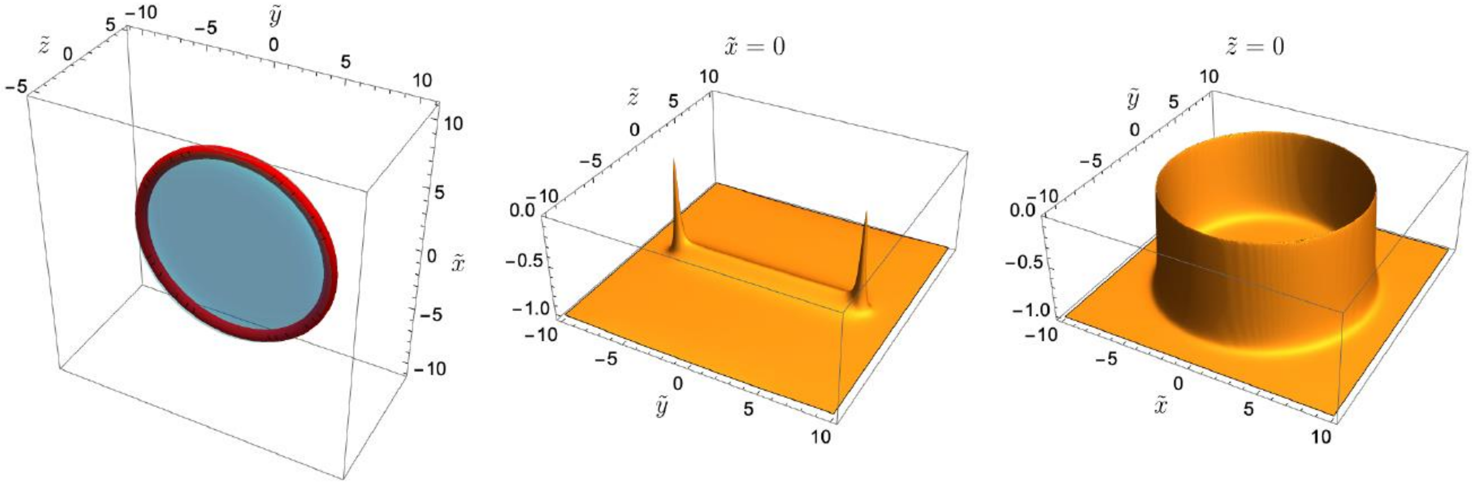
Consistent with thin-defect approx

$$P_{\text{nucleation}} = A \exp\left(-\alpha_1 \frac{v^2}{m} \times 9.0\right)$$

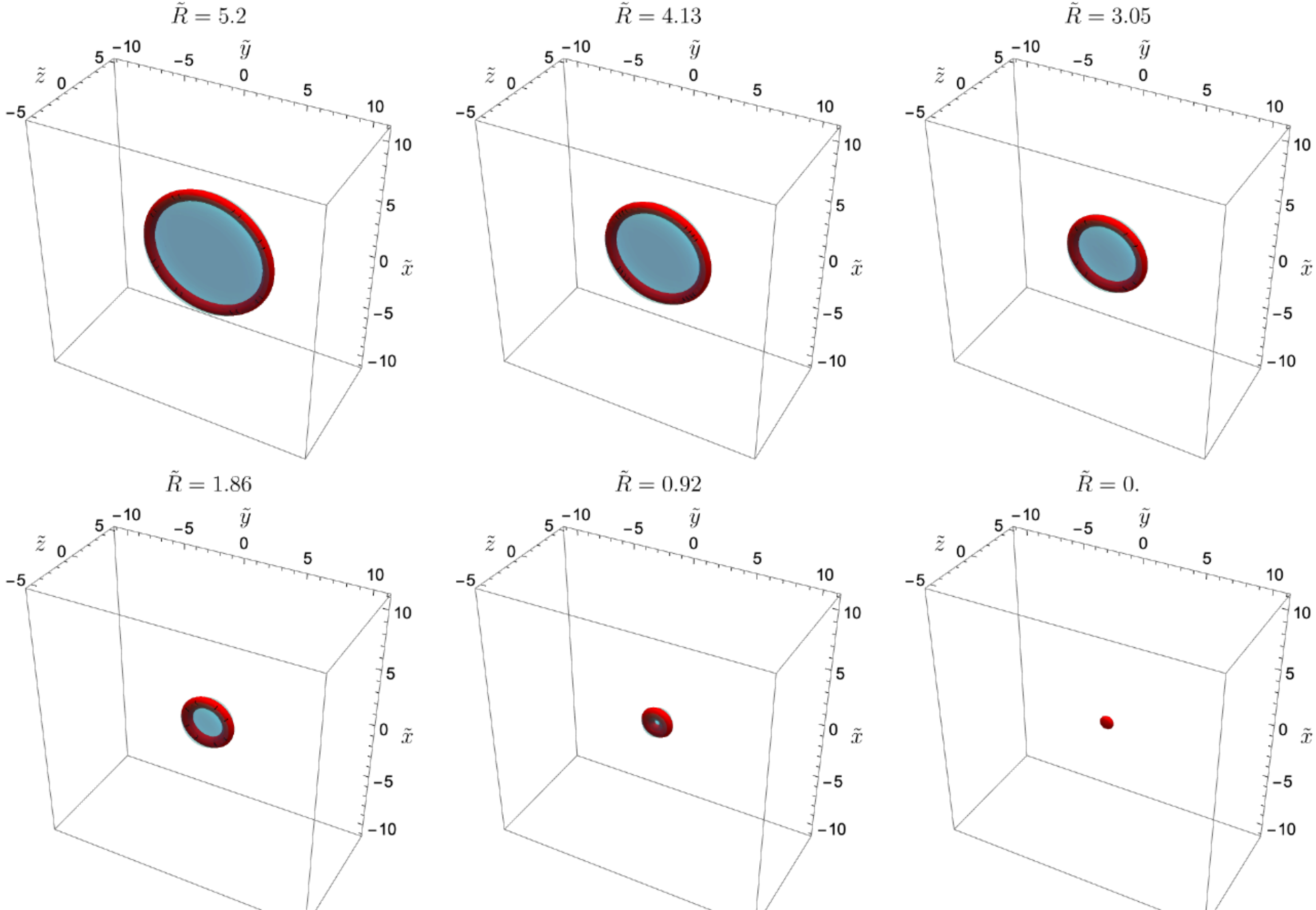
3+1 dim

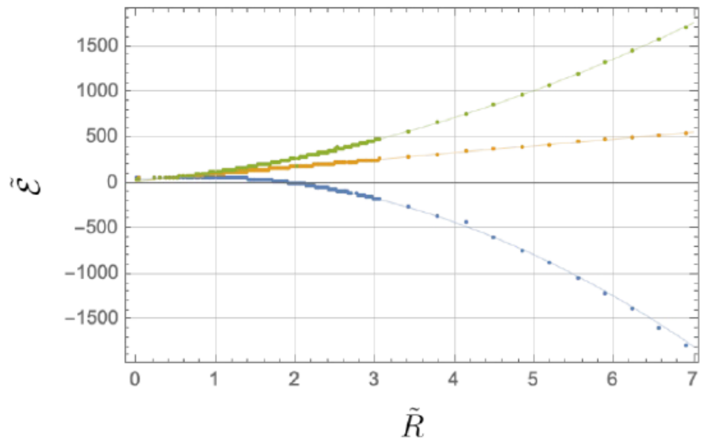
Thin-defect approx

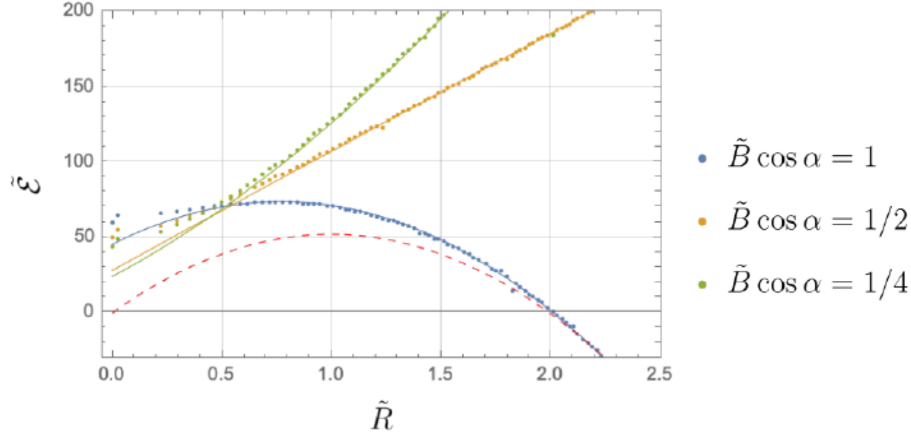
$$S = \pi R^2 \mu + \frac{4\pi}{3} R^3 \sigma \qquad R_0 = \frac{2\mu}{-\sigma}, \qquad S_0 = \frac{16\pi\mu^3}{3\sigma^2}$$



Numerical simulation in 3+1 dim: relaxation







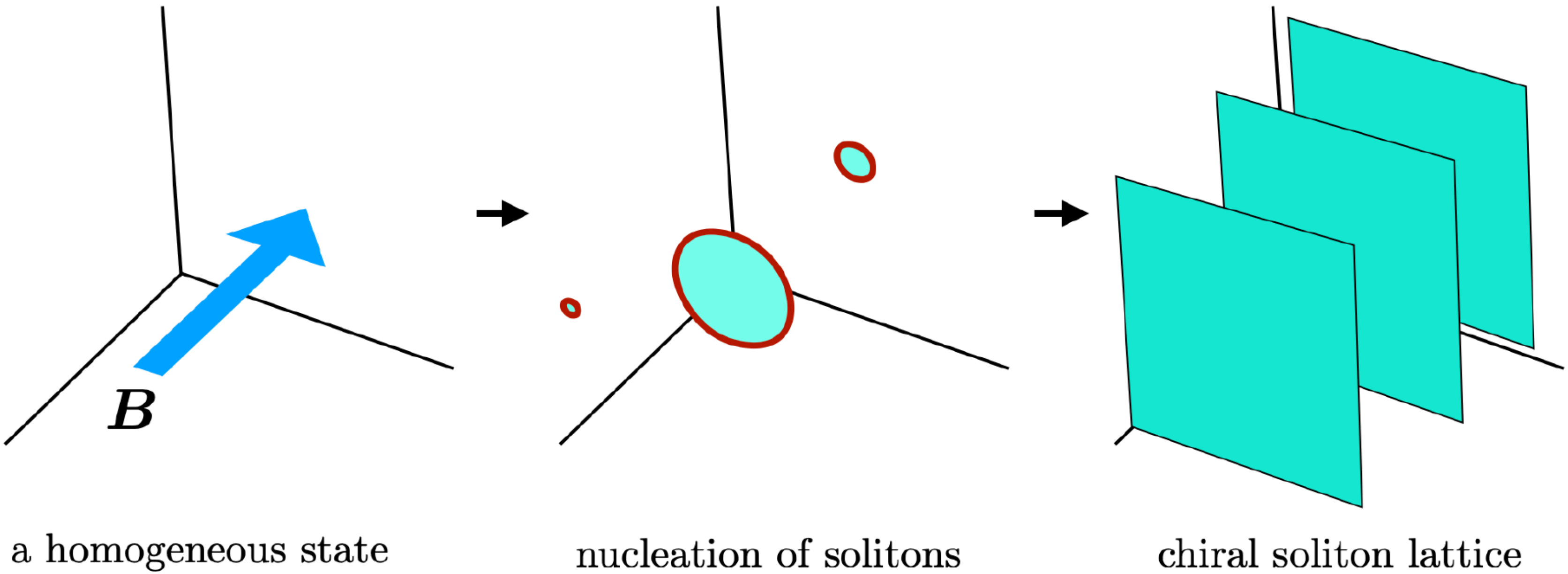
Nucleation probability

$$P_{\text{nucleation}} = A \exp\left(-111\alpha_2 \frac{v^2}{m^2}\right)$$

$$\tilde{\mathcal{E}} = \pi \tilde{R}^2 a + 2\pi \tilde{R} b + c.$$

We found a remnant energy *c*
giving a correction to the thin-defect approx

Formation of chiral soliton lattice

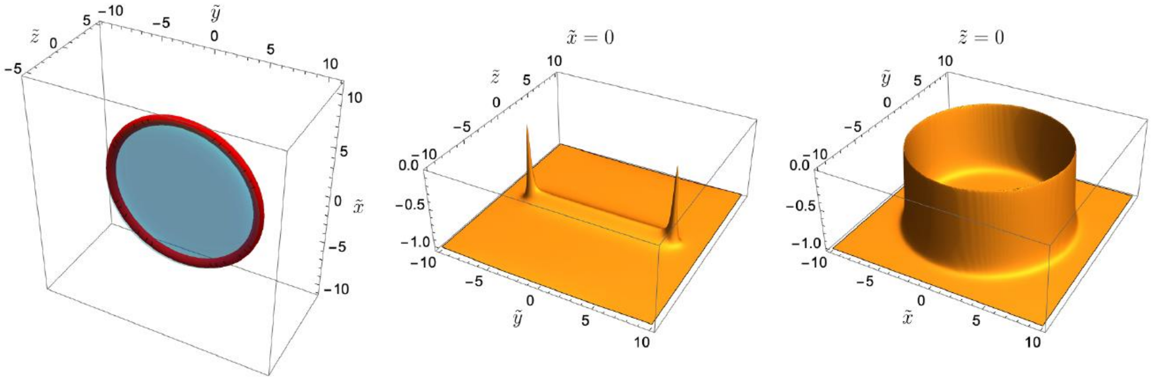


Summary

Negative energy(tension) soliton appears in various cond-mat, QCD

Solitonic ground state

We have proposed
Quantum nucleation of such topological solitons



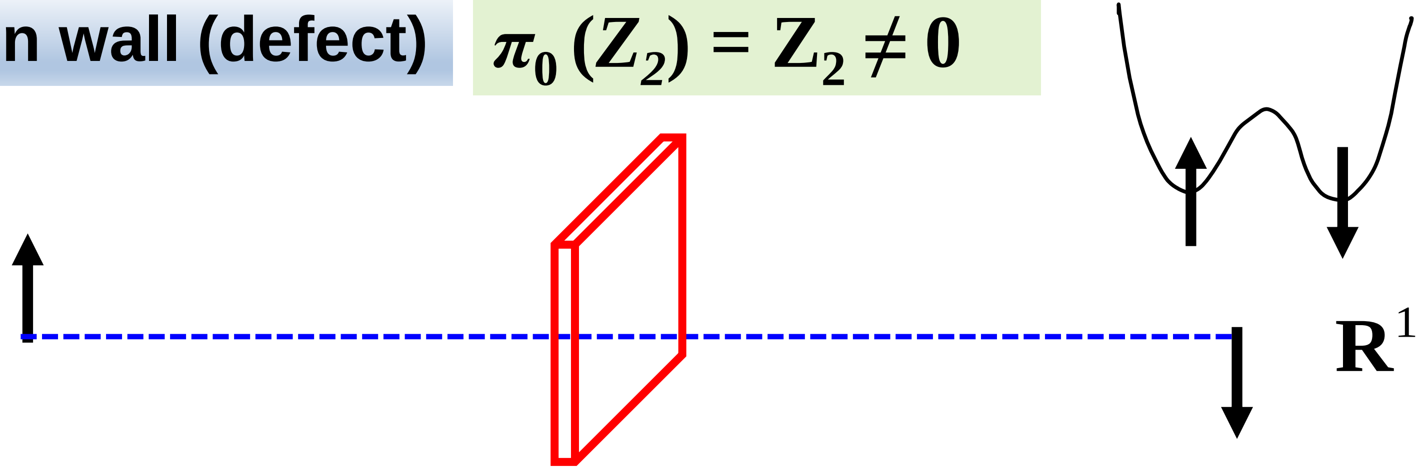
Classification of topological solitons: 3 types

d	Defects		Textures		Gauge Structure	
1	Domain wall, Kink	π_0	Sine-Gordon soliton	π_1		
2	Vortex, Cosmic string	π_1	Lumps, Baby Skyrmion	π_2		
3	Monopole	π_2	Skyrmion, Hopfion	π_3		
4					YM instanton	π_3
	$\partial R^d \cong S^{d-1} \to G/H$		$R^d + \{\infty\} = S^d \to G/H$		$\partial R^d \cong S^{d-1} \to G$	
	$\pi_{d\text{-}1}(G/H) \neq 0$		$\pi_d(G/H) \neq 0$		$\pi_{d\text{-}1}(G) \neq 0$	

d: codimensions (in which solitons are particles, or on which solitons depend)

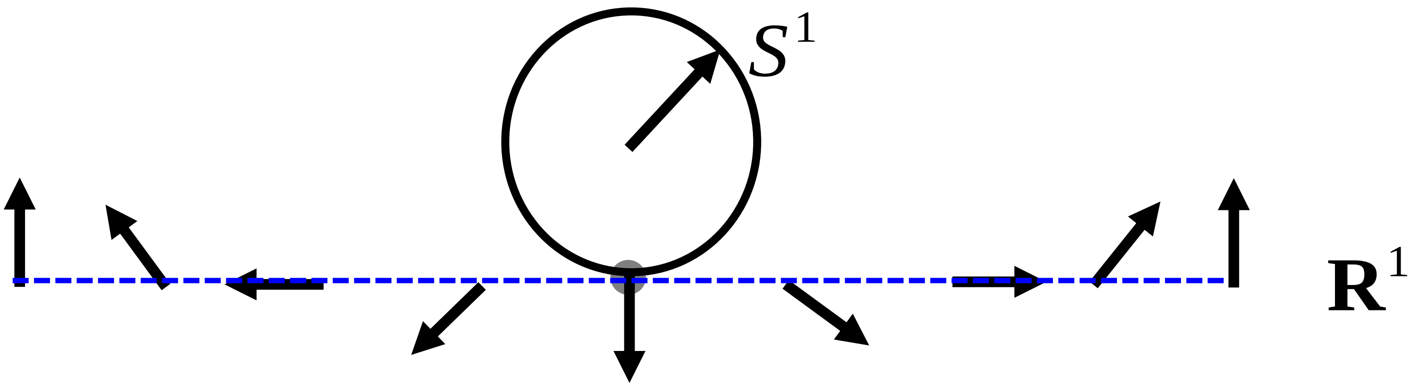
Domain wall (defect)

$\pi_0(Z_2) = Z_2 \neq 0$



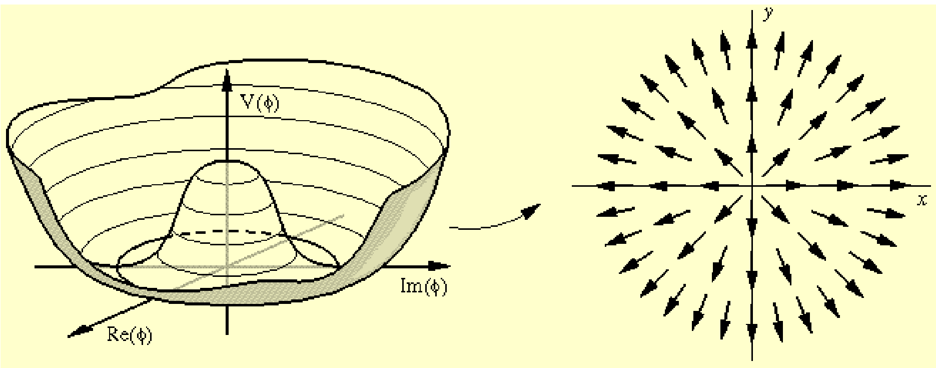
Sine-Gordon soliton (texture)

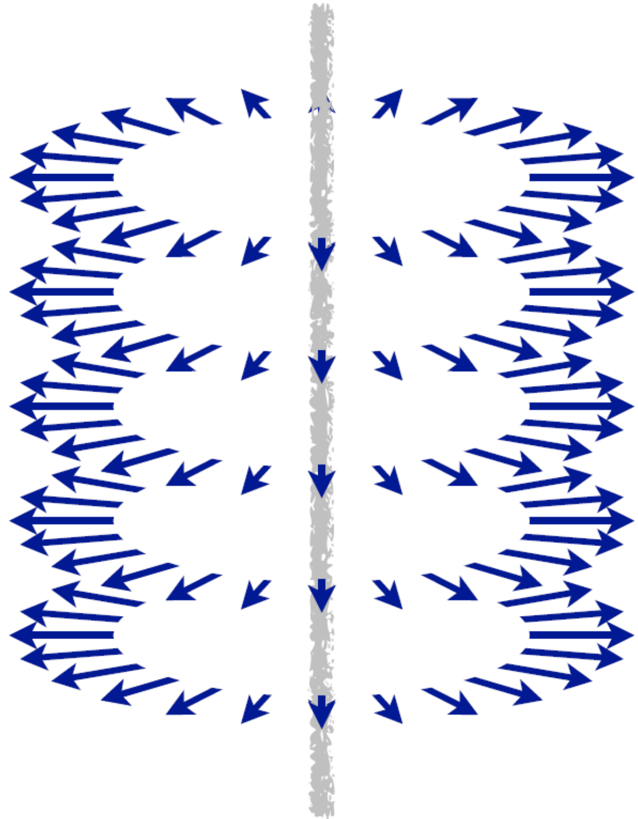
$\pi_1(S^1) = Z \neq 0$



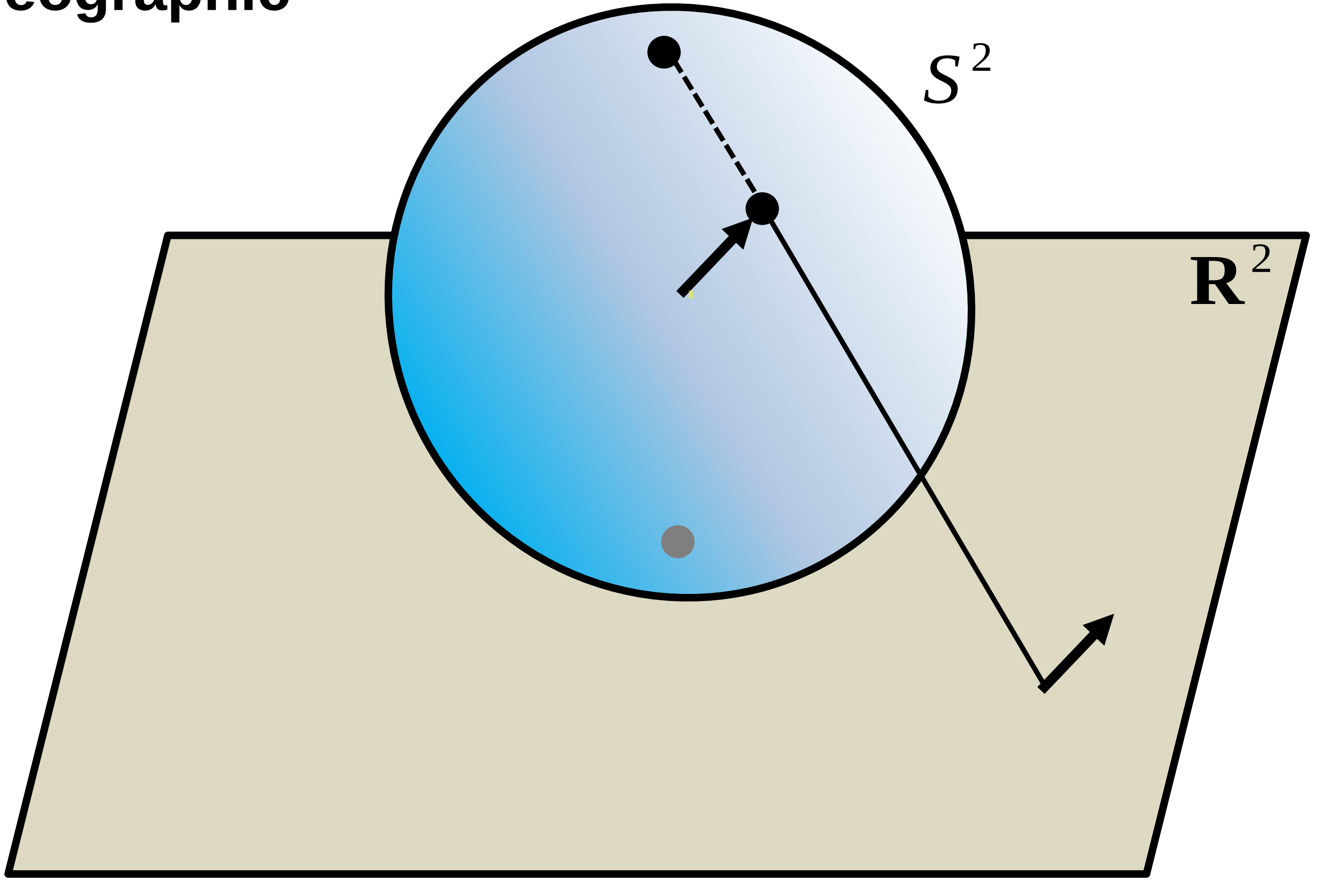
Vortex, cosmic string (defect)

$$\pi_1(S^1) = Z \neq 0$$



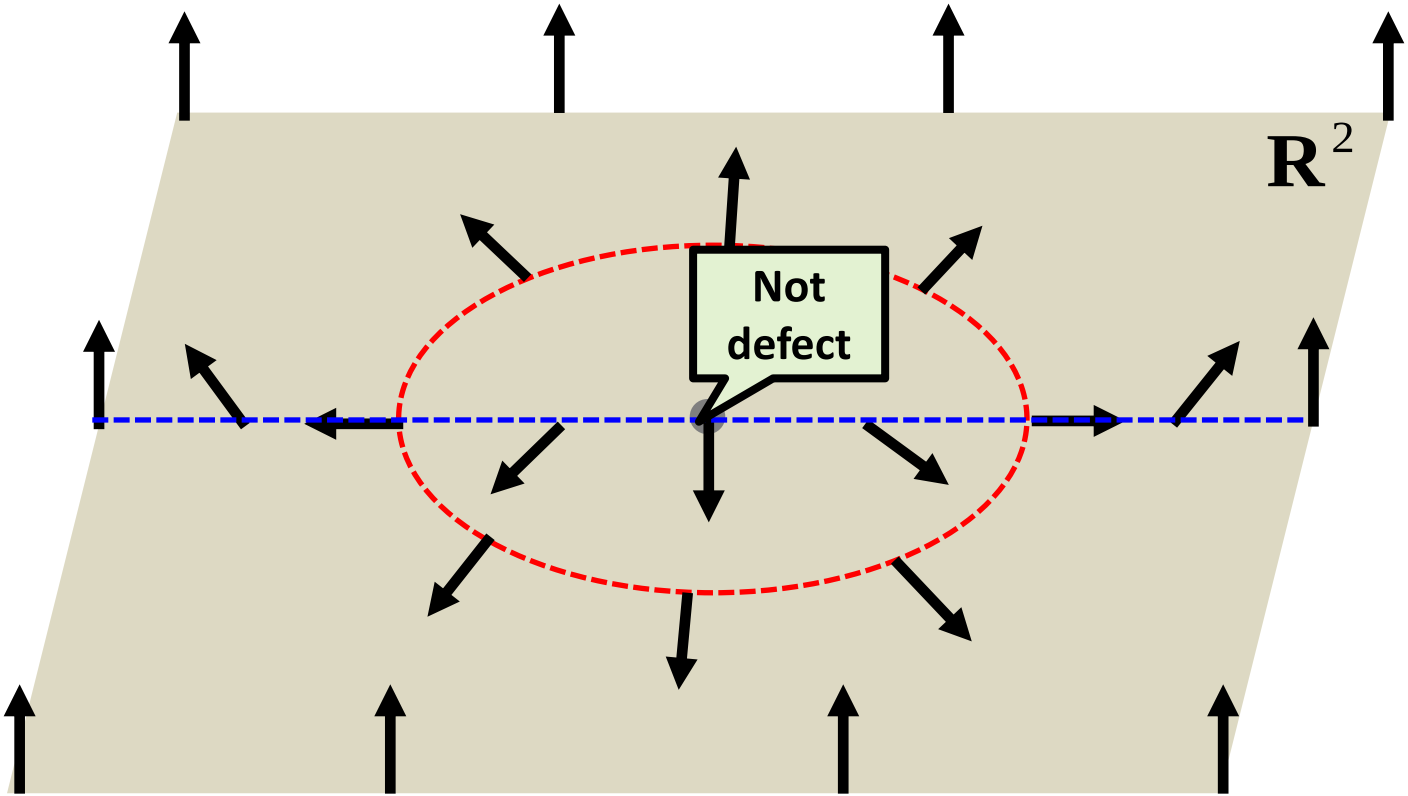


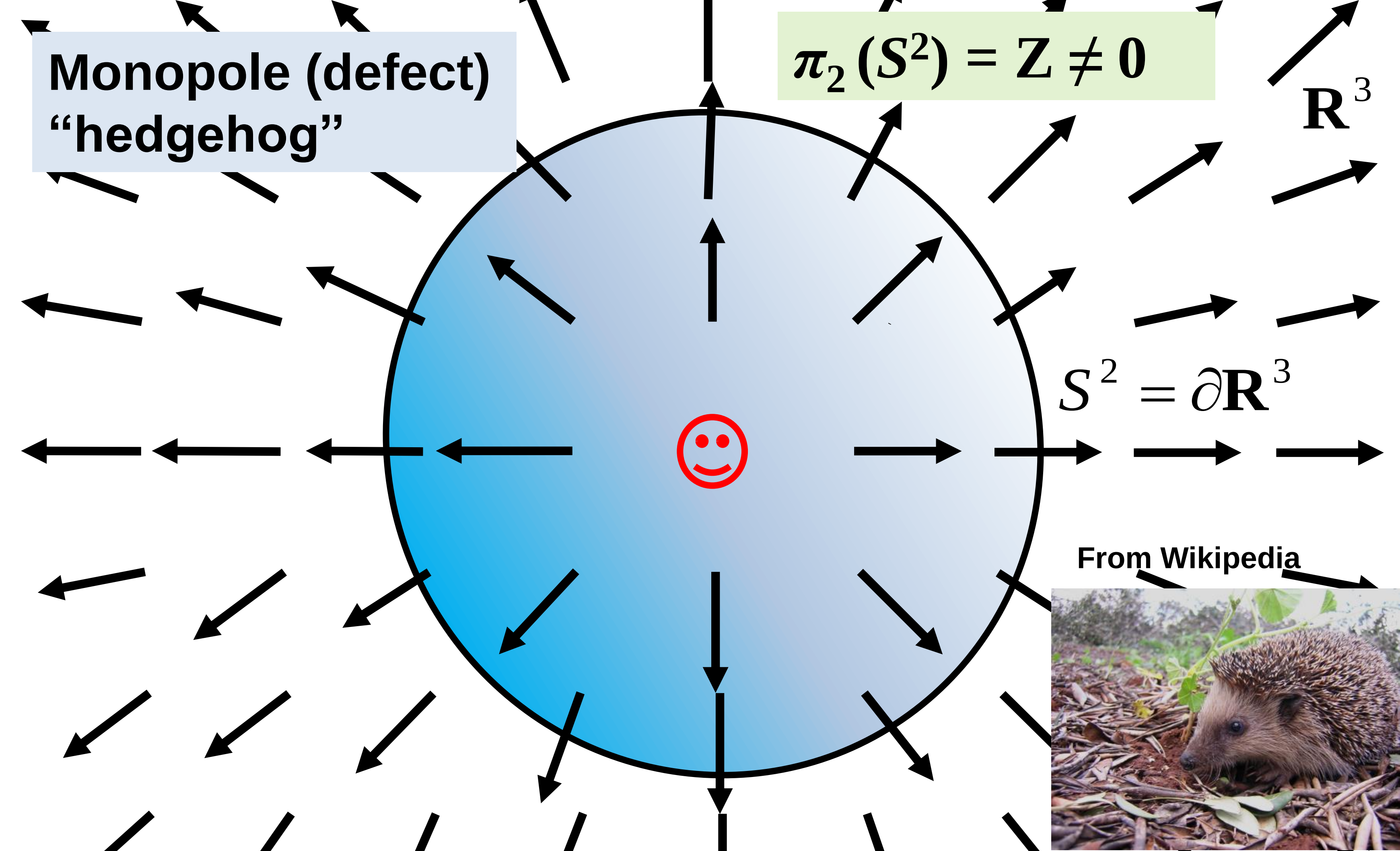
stereographic



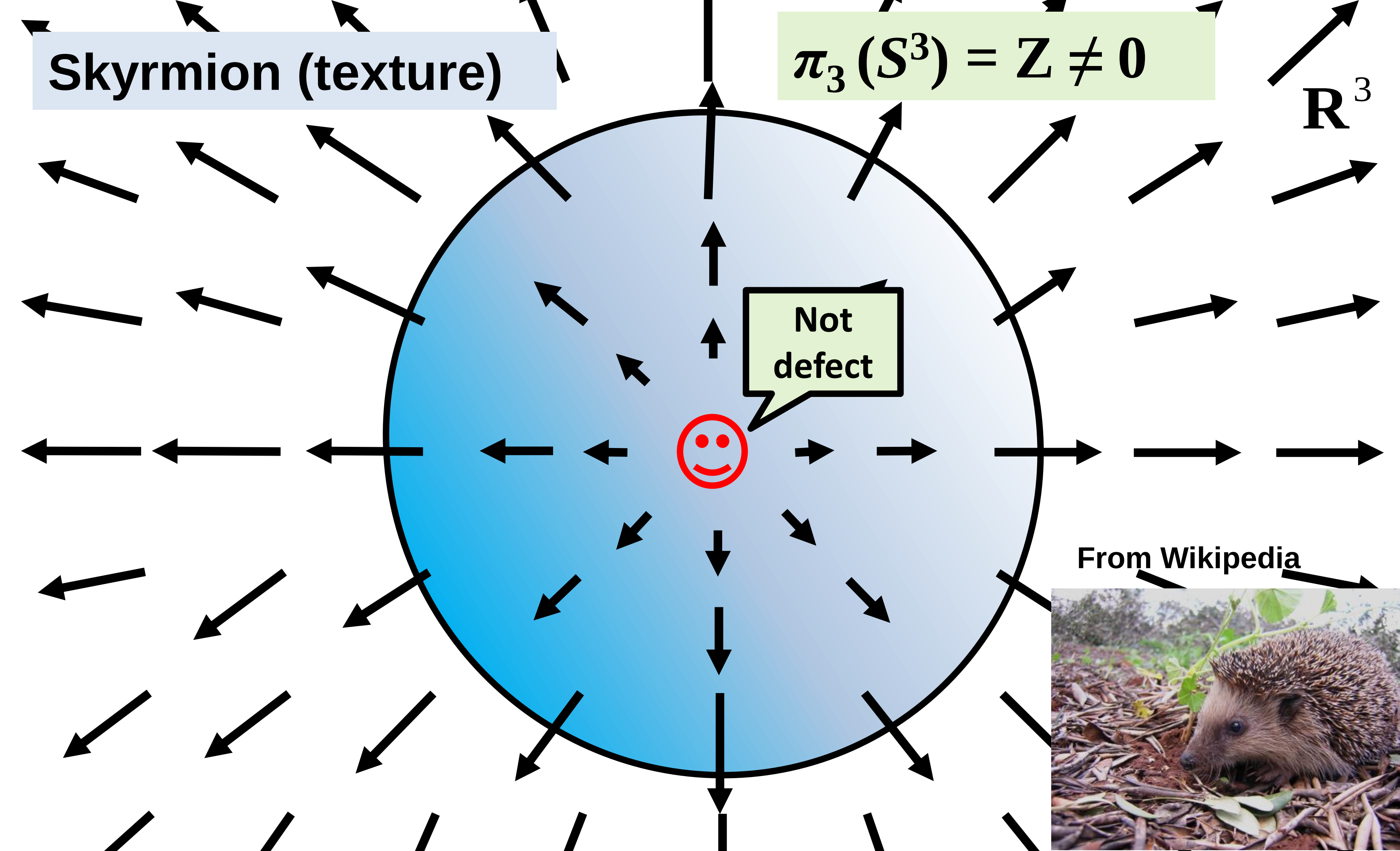
Lump, baby Skyrmion (texture)

$\pi_2(S^2) = \mathbf{Z} \neq 0$





Monopole (defect)
“hedgehog”
$\pi_2(S^2) = Z \neq 0$
R^3
$S^2 = \partial R^3$
From Wikipedia



From Wikipedia

Symmetry breaking: $G \to H$
Either gauge or global symmetries

↓

Nambu-Goldstone modes
Vacuum manifold or Order parameter space(OPS): G/H

↓

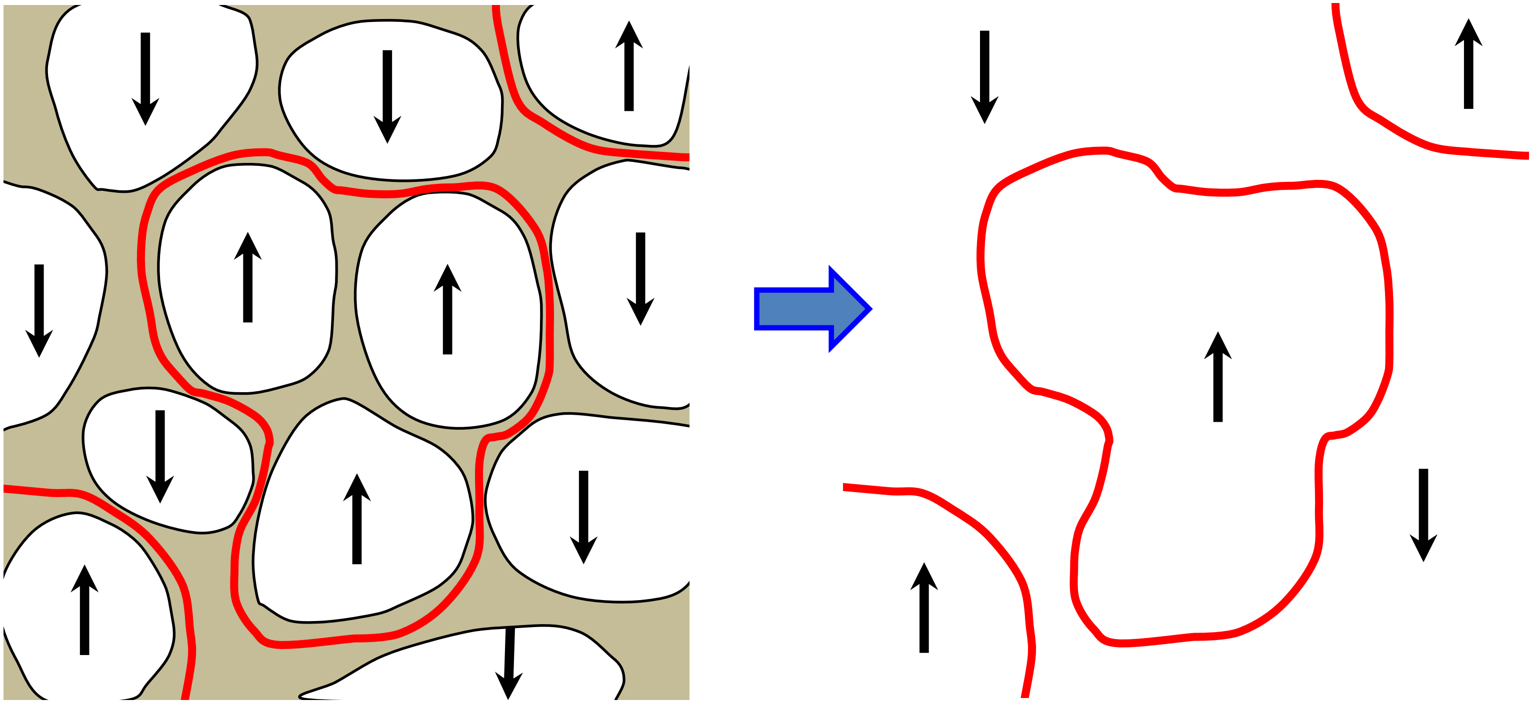
Topology of OPS: $\pi_n\,(G/H)$

↓

Topological solitons, defects/textures

T.Kibble,
N.D.Mermin
Rev.Mod.Phys.('79),
G.E.Volovik
Universe in a helium droplet

How are they created?
e.g. Kibble-Zurek mechanism @ phase transition



Domain walls