



# Axion domain walls, and cosmic birefringence

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— Frontiers in Gravity and Fundamental Physics —

Fumi Takahashi (Tohoku)

Based on [2012.11576](#) with Wen Yin, [2205.05083](#) with Naoya Kitajima, Fumiaki Kozai, and Wen Yin  
[2211.06849](#) with Diego Gonzalez, Naoya Kitajima, and Wen Yin

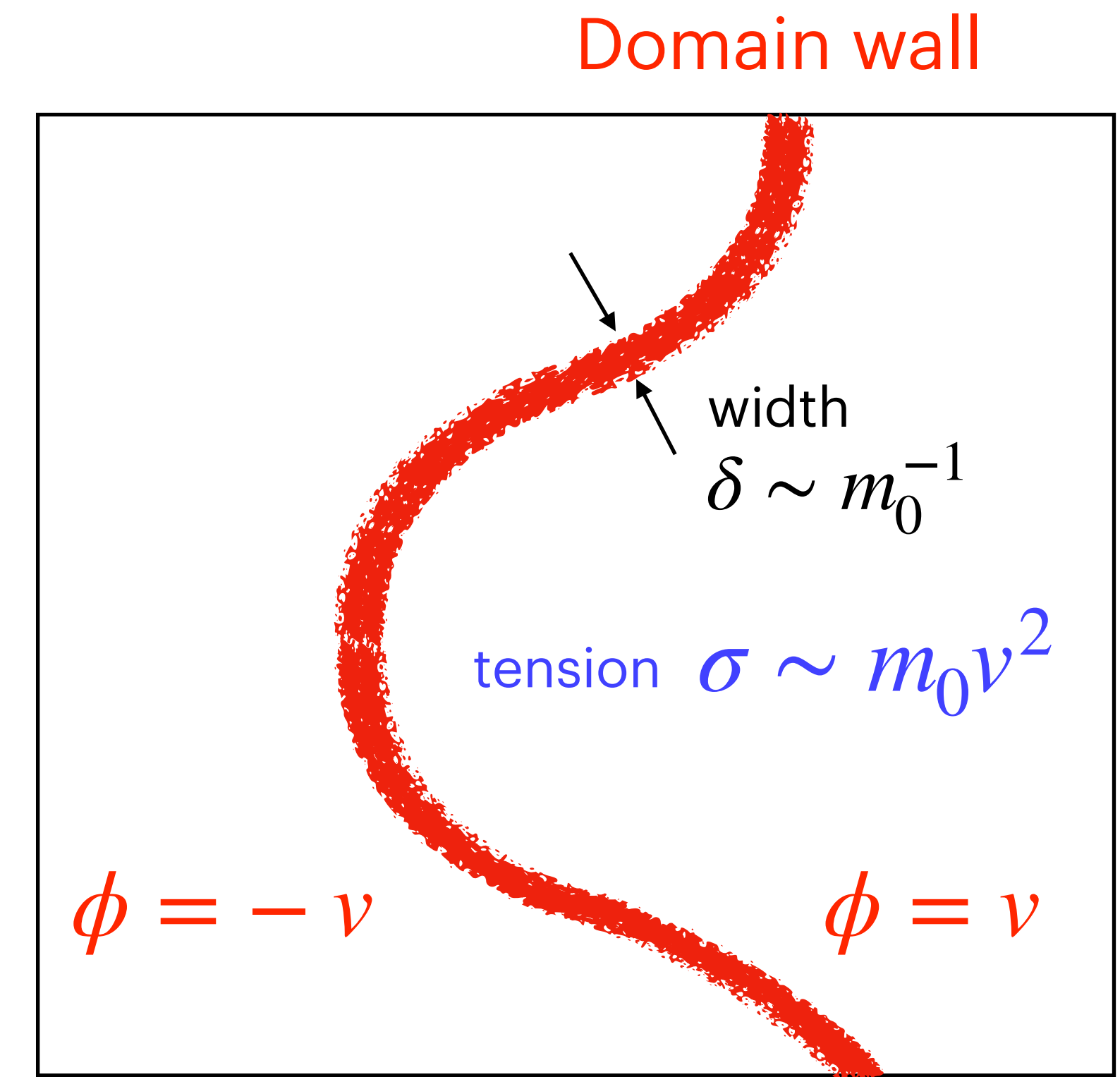
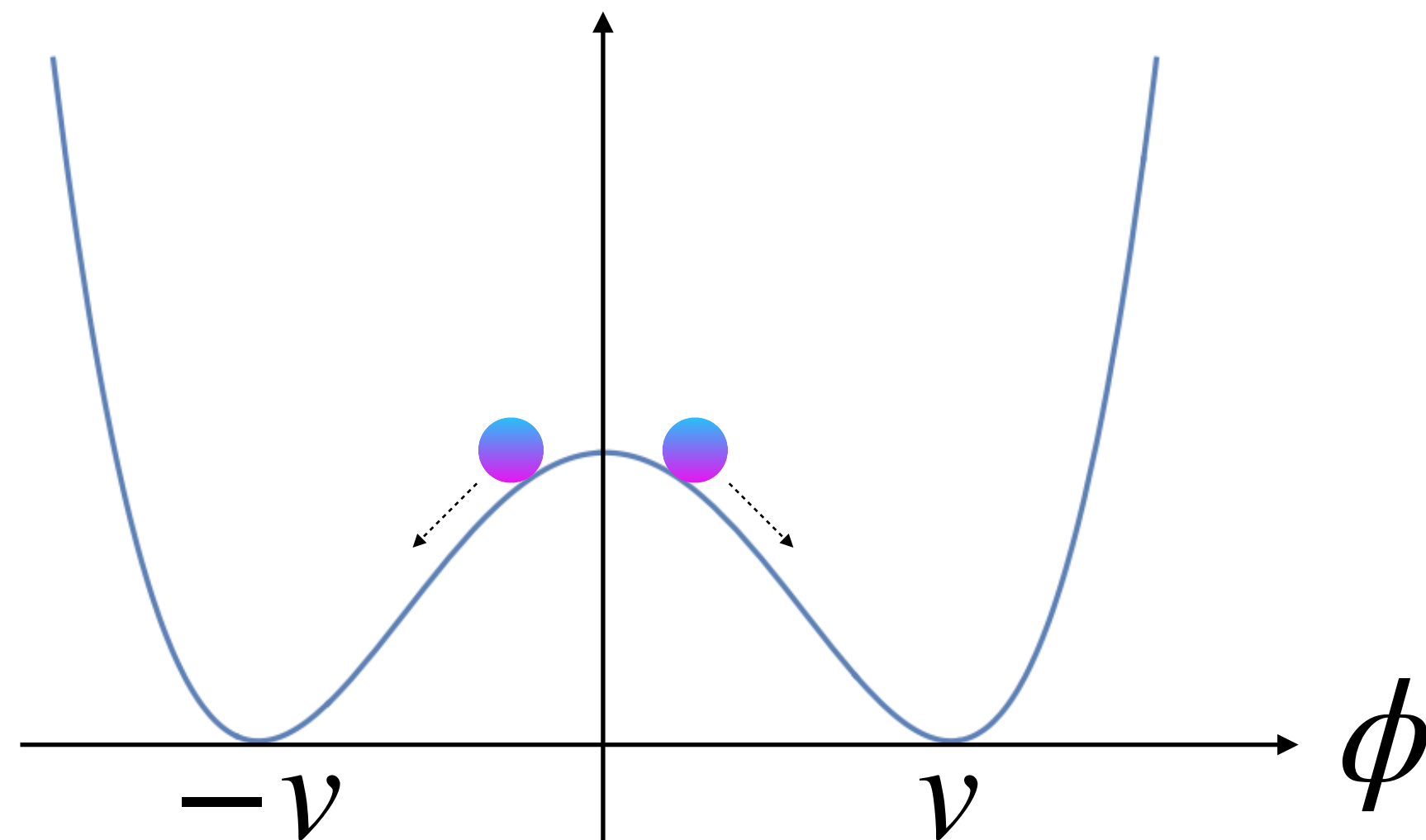
# 1. Introduction

Domain walls (DWs) are produced when discrete symmetry is spontaneously broken.

Zeldovich, Kobzarev, and Okun '74, Kobzarev, Oku, Voloshin '74, Kibble '76  
see also Bogolyubov '66

e.g.)  $Z_2$  symmetry

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 = \frac{\lambda}{4}v^4 - \frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4}\phi^4$$



# Scaling regime

Press, Ryden, and Spergel '89, Hindmarsh '96, Garagounis and Hindmarsh '03, + many

Once formed, the DW network soon converges to the so-called scaling solution, a situation where there is **on average about one DW per the Hubble horizon**.

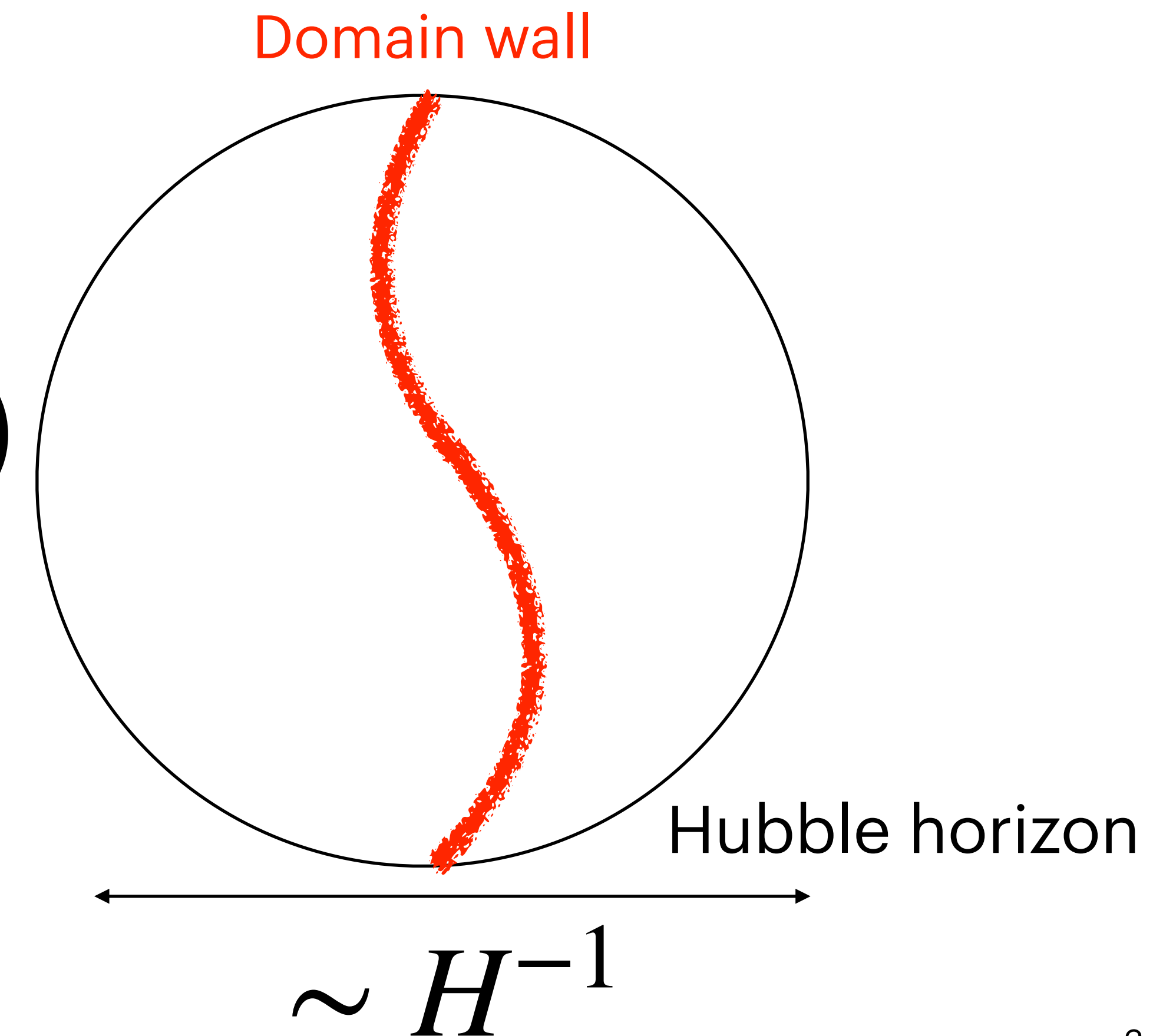
Scaling solution:

$$\rho_{\text{DW}} \sim \sigma H$$

$$(\propto a^{-2} \text{ for RD})$$

The scaling solution has been usually regarded as an attractor.

This is true for at least the cases that have been studied numerically so far.



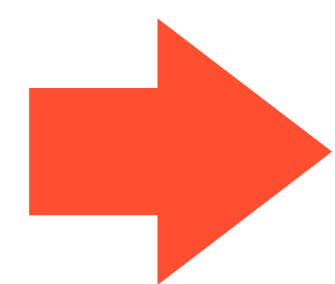
# Cosmological DW problem

Zeldovich, Kobzarev, and Okun '74

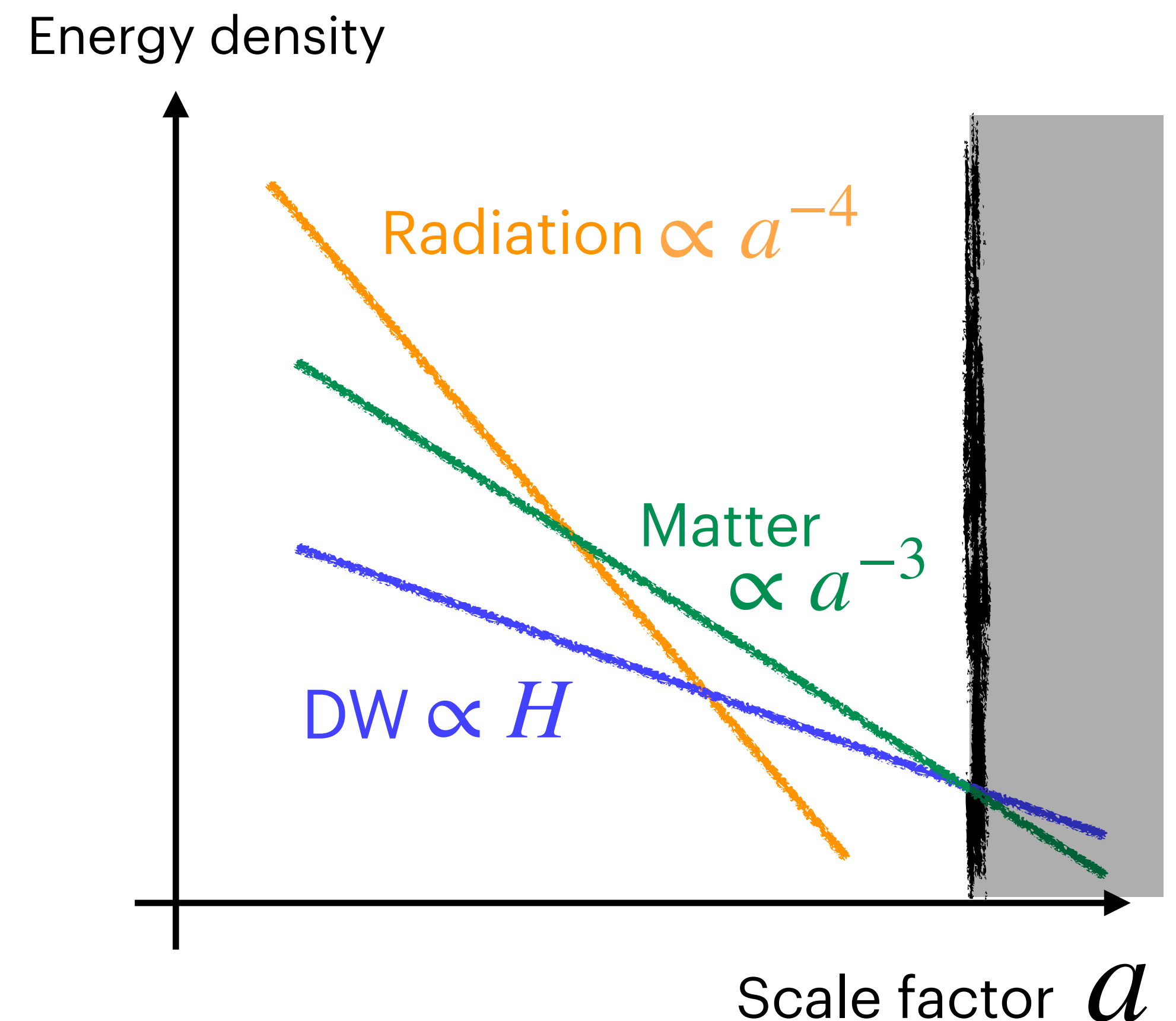
DWs can easily dominate the universe, which makes it anisotropic and inconsistent with observations.

Even if DW is subdominant, its energy density is bounded above due to the CMB observations.

$$\rho_{\text{DW}} \underset{\substack{\uparrow \\ \text{Scaling solution}}}{\sim} \sigma H < 10^{-5} H_0^2 M_p^2$$



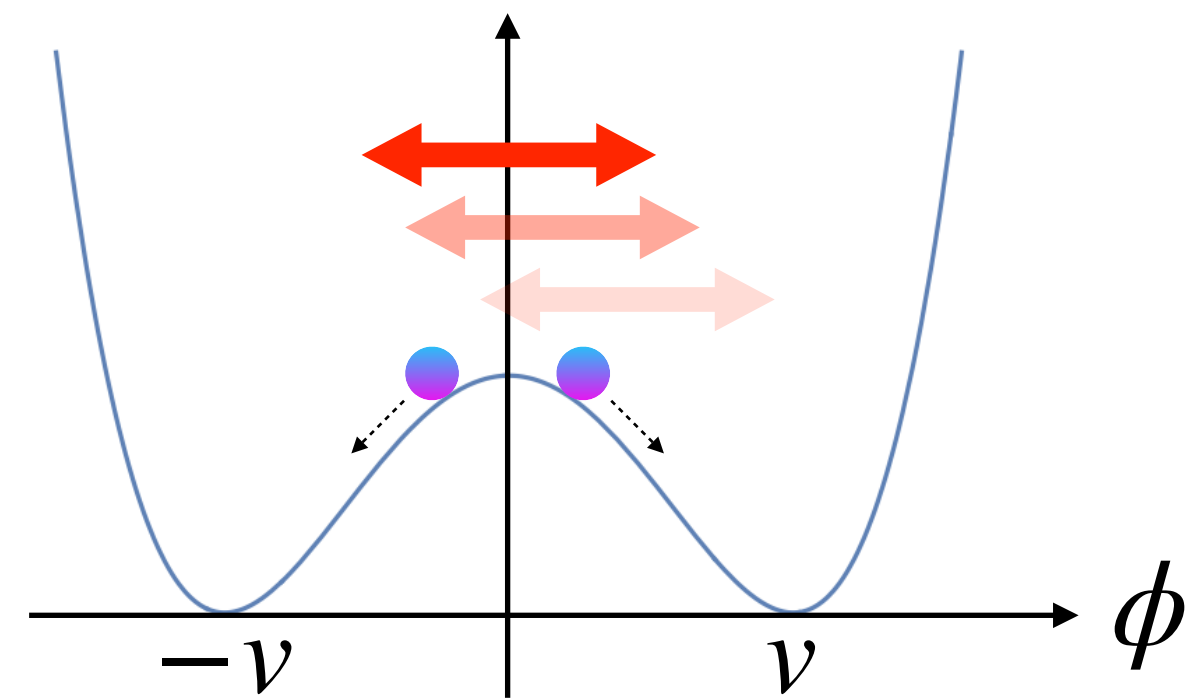
$$\sigma < (\text{MeV})^3$$



I will talk about the two recent topics related to DWs;

- Stability of the DW network with inflationary fluctuations

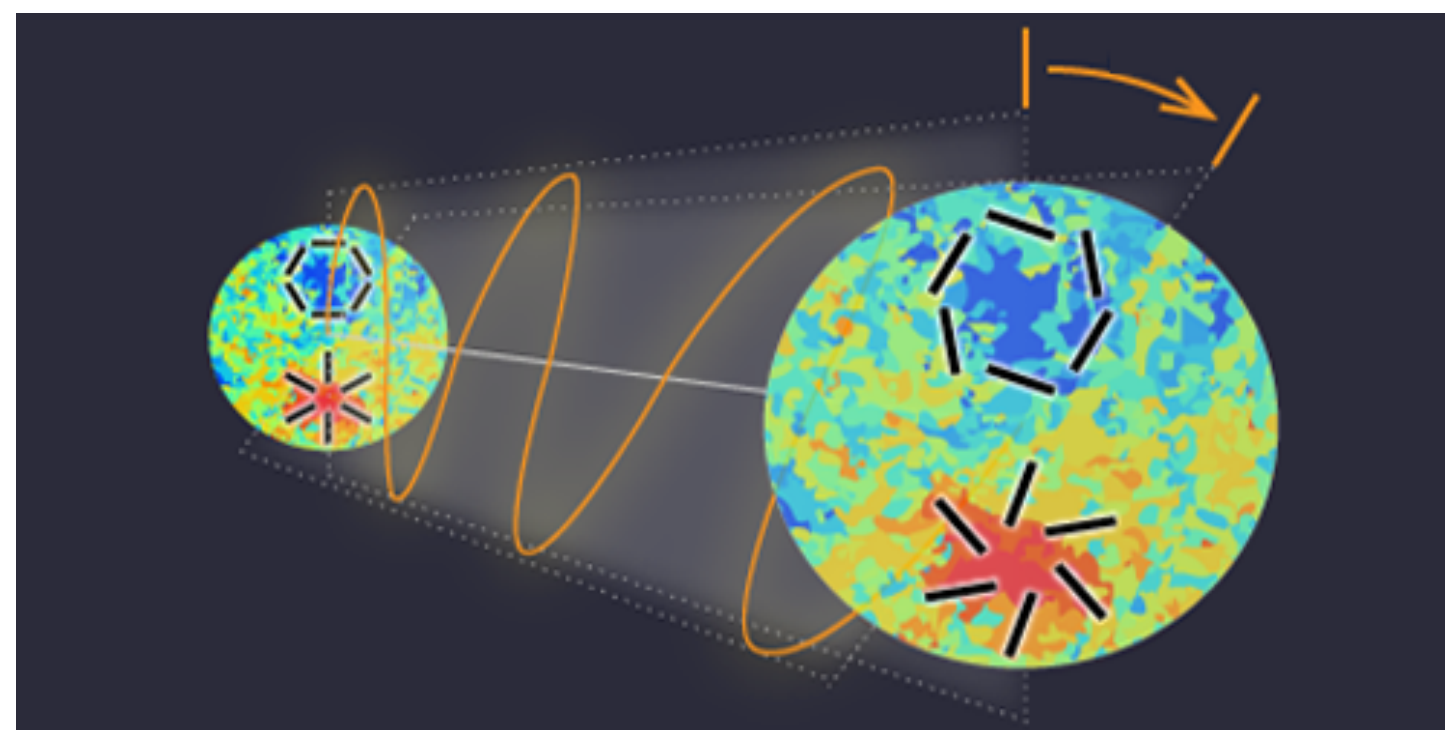
Gonzalez, Kitajima, FT and Yin 2211.06849



- Thermal or non-thermal fluctuations?
- Decay fast in the presence of bias?

- Cosmic birefringence caused by axion DWs

FT and Yin 2012.11576, Kitajima, Kozai, FT and Yin 2205.05083,  
Gonzalez, Kitajima, FT and Yin 2211.06849



- The field value, not the energy, of the scalar field that constitutes DW can be probed by cosmic birefringence.

<https://physics.aps.org/articles/v13/s149>

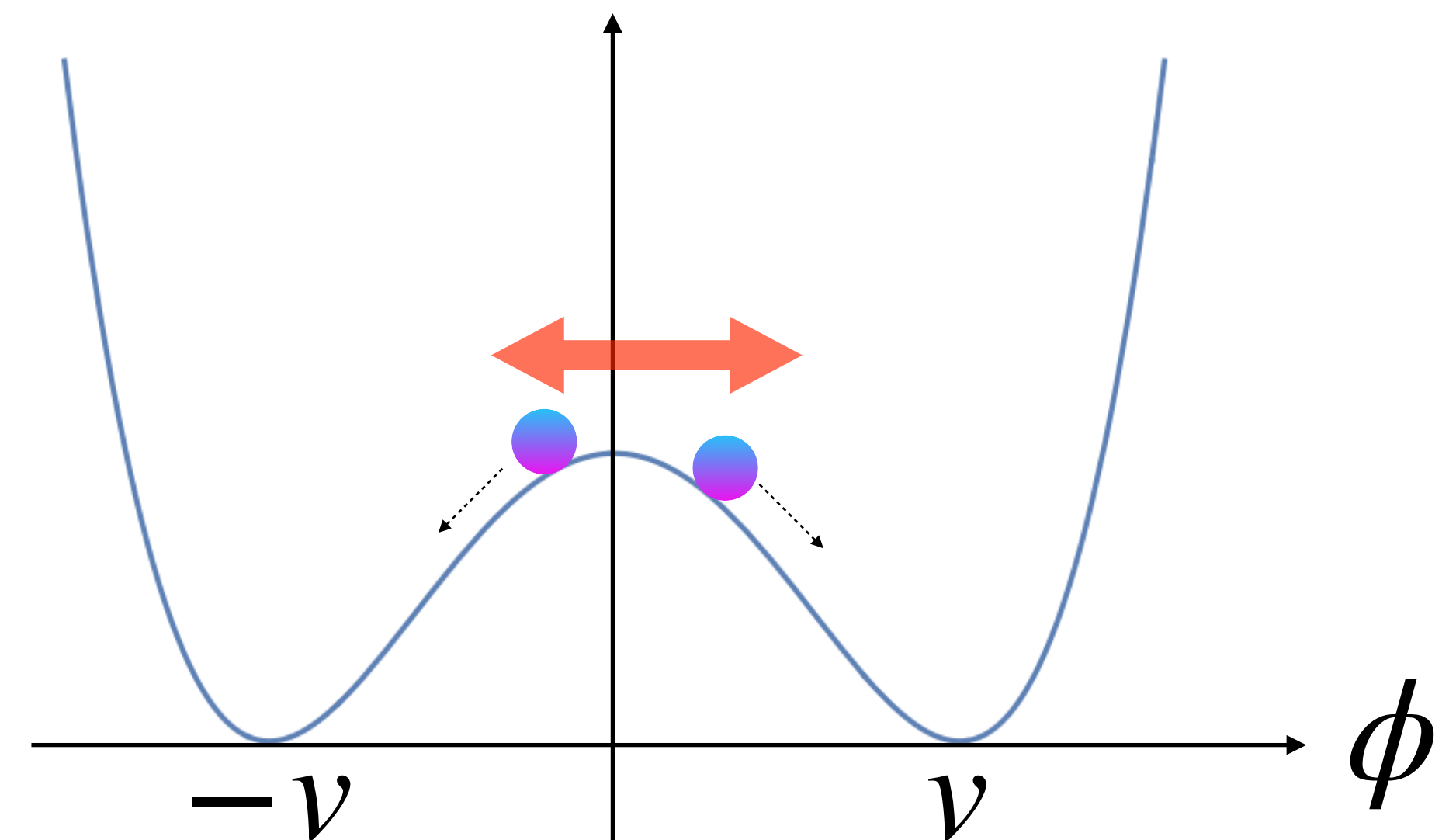
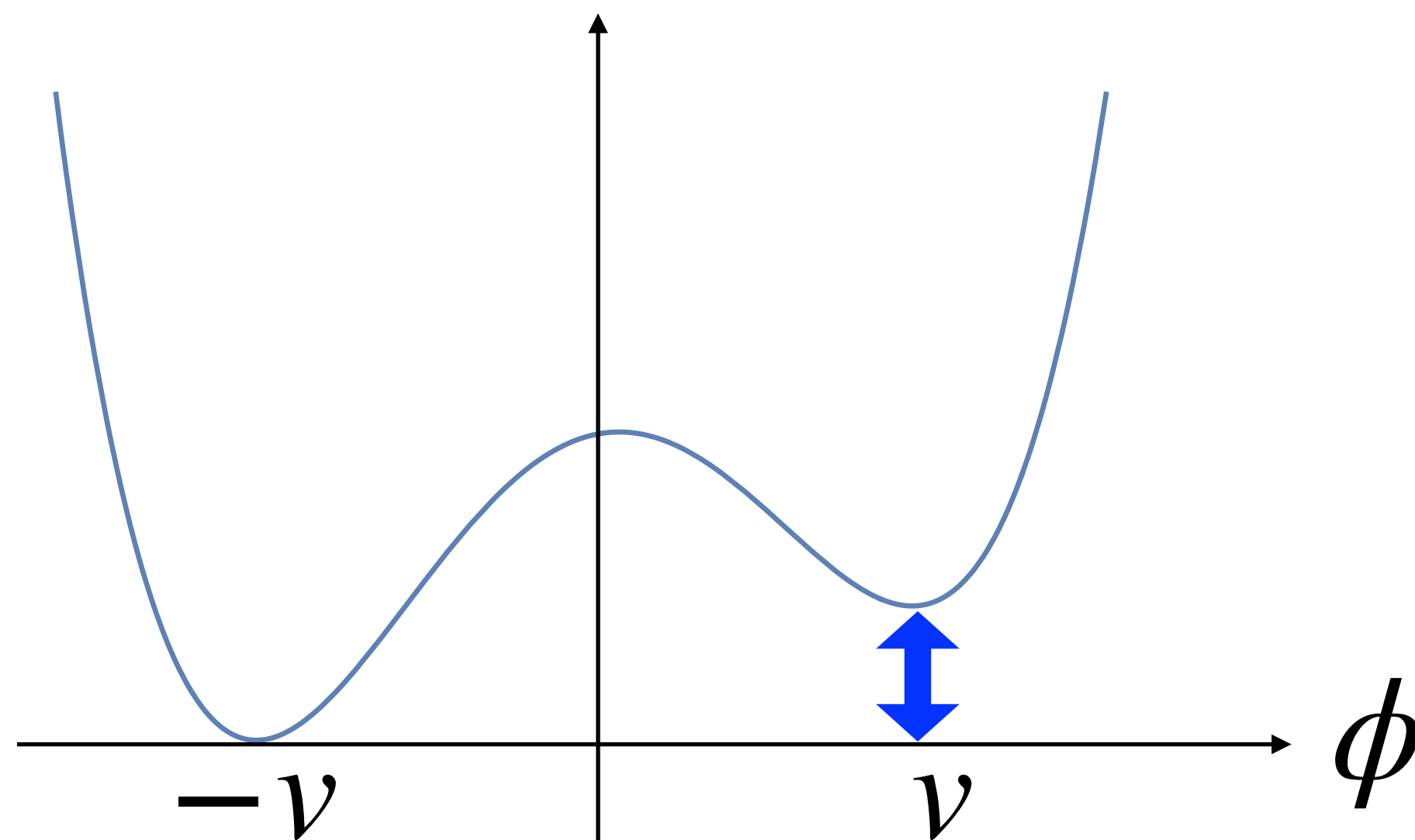
## **2. Stability of the DW network**

# Destabilization of DW network

One way to avoid the cosmological DW problem is to destabilize the DW network and make it disappear in the early Universe.

To this end, we need to introduce a small bias either **in the potential** or **in the initial distribution**.

Vilenkin '81, Sikivie '82, Mohanty and Stecker '84, Gelmini, Gleiser and Kolb '89, Lalak and Thomas '93, + many

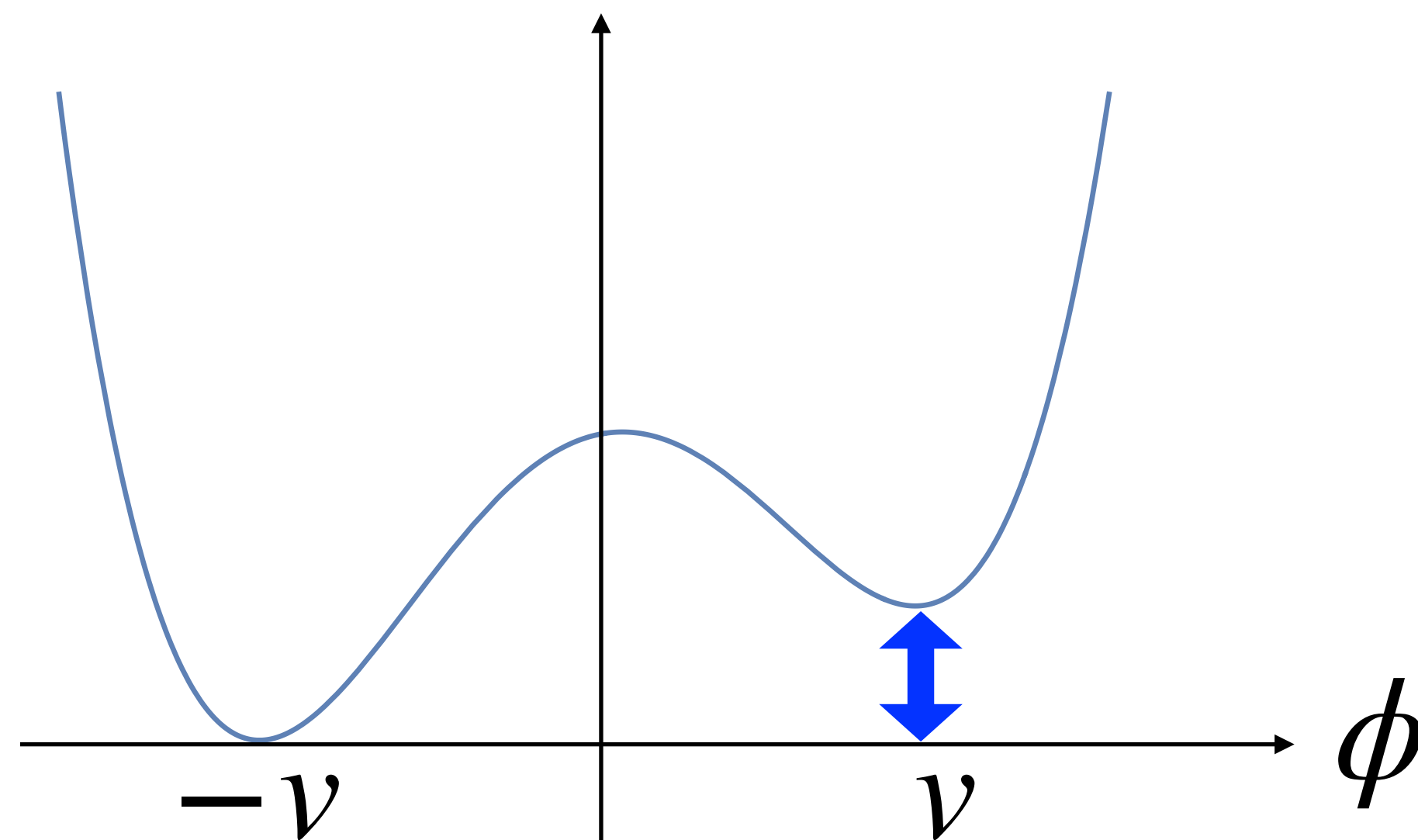


# Destabilization of DW network

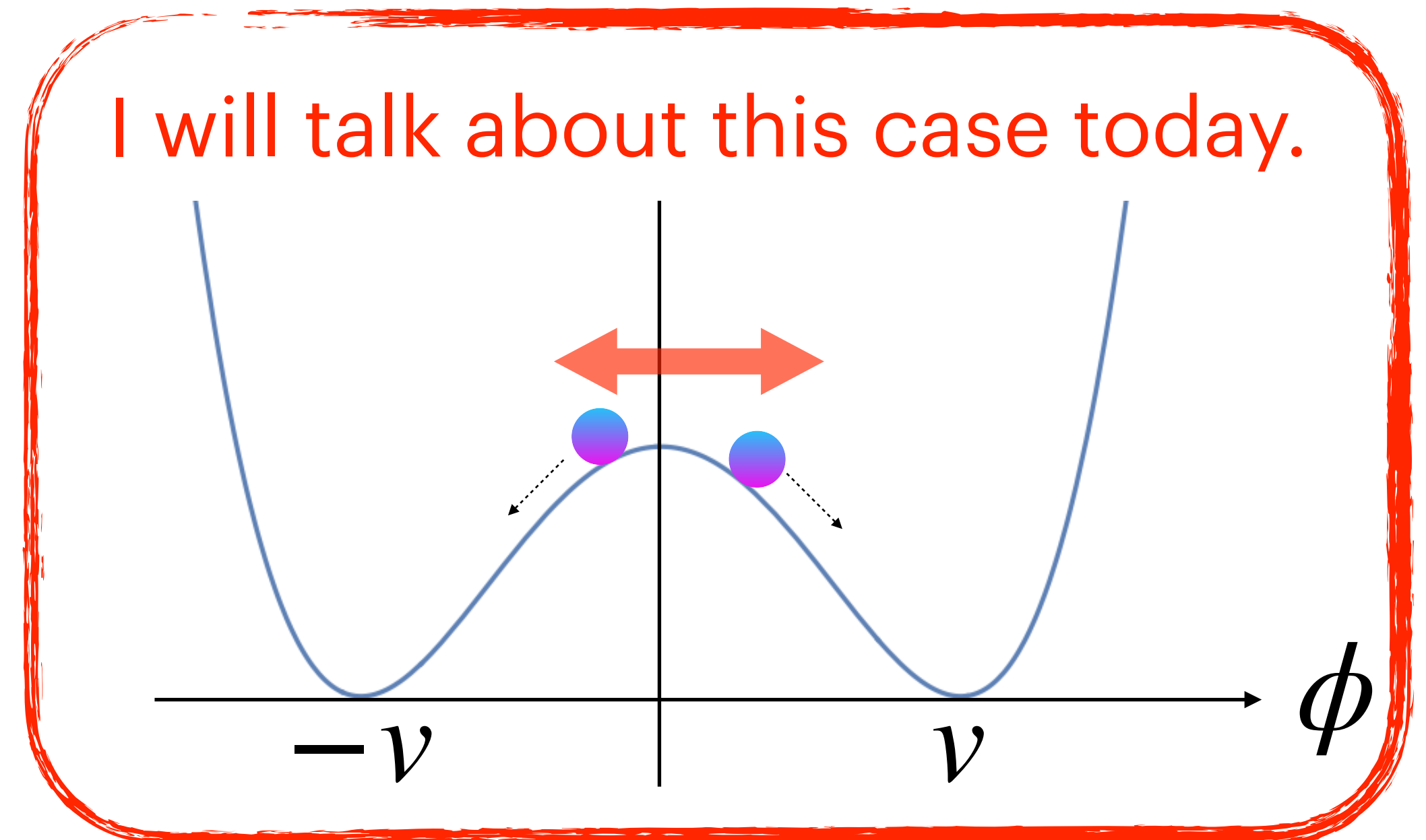
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Kitajima, Lee, FT, and Yin, in preparation

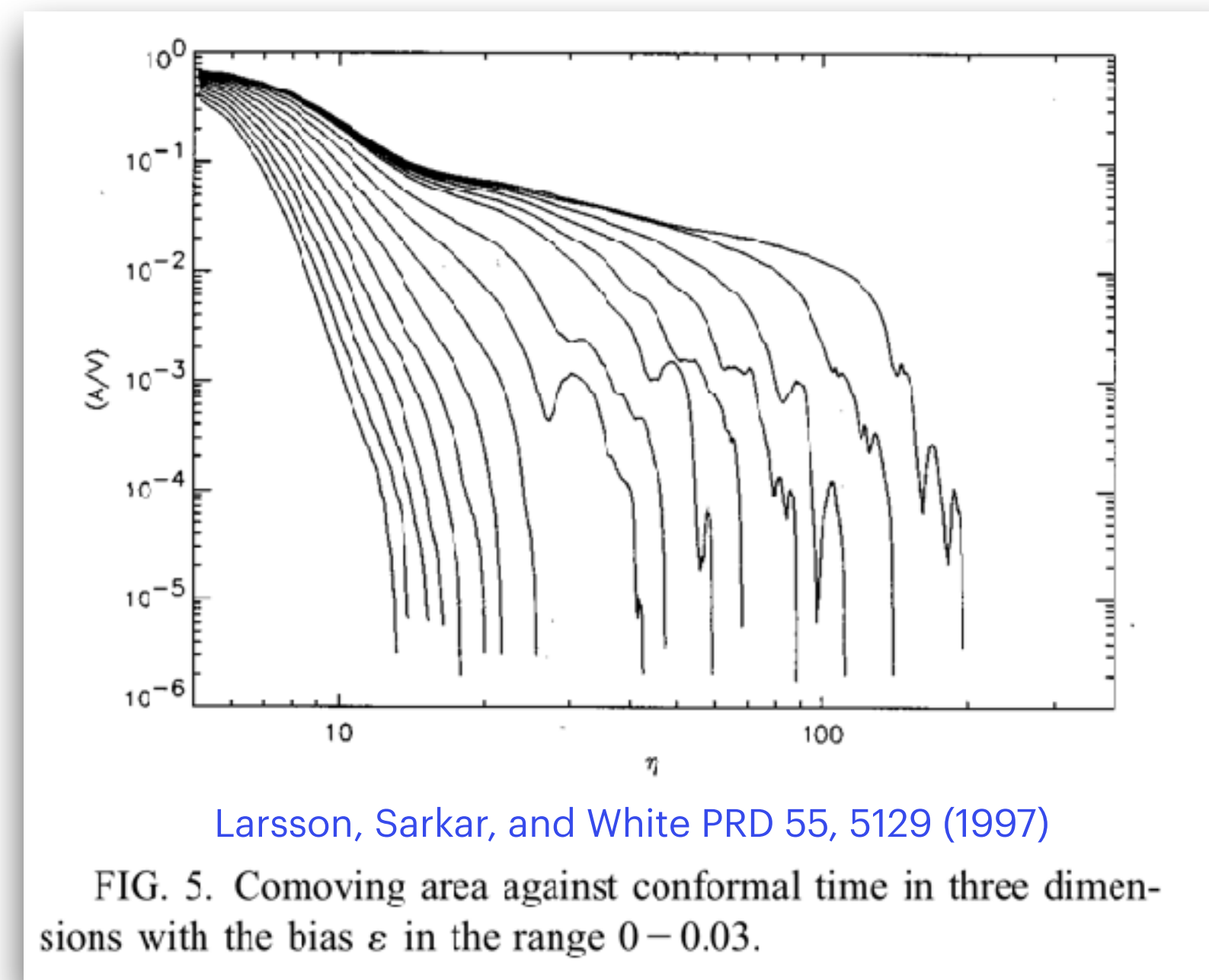
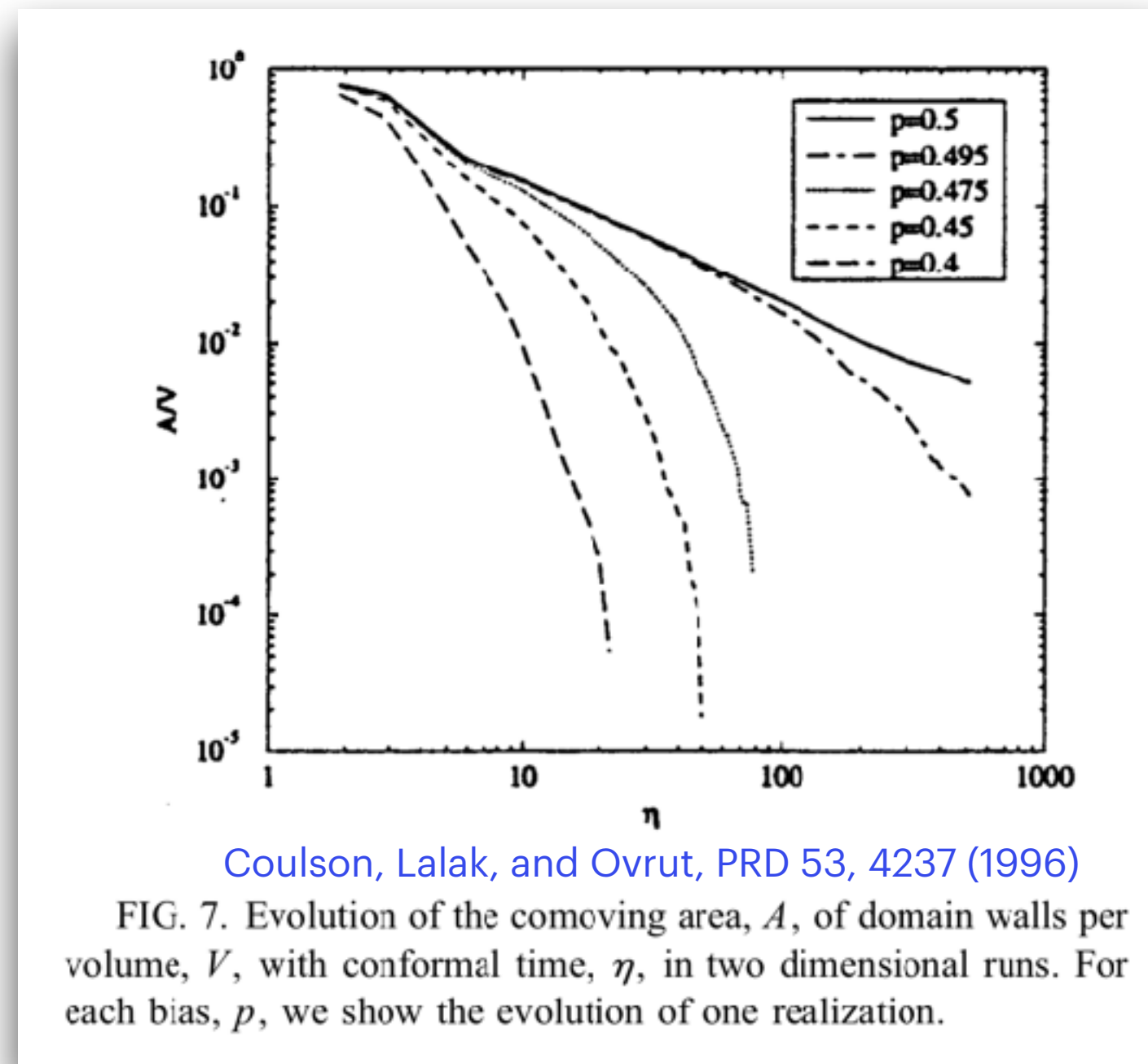




# DW network vulnerable to bias in the initial distribution?

According to numerical simulations in the literature, the DW network quickly disappears when the initial distribution is only slightly biased.

Coulson, Lalak, and Ovrut, PRD 53, 4237 (1996), Larsson, Sarkar, and White PRD 55, 5129 (1997), Correia, Leite, and Martins, PRD 90, 023521 (2014), Correia, Leite, and Martins, PRD 97, 083521 (2018).



White noise (random) initial distribution was assumed.

It was claimed that a random distribution with correlation length of  $O(H^{-1})$  can mimick the inflationary fluctuations. 🤔

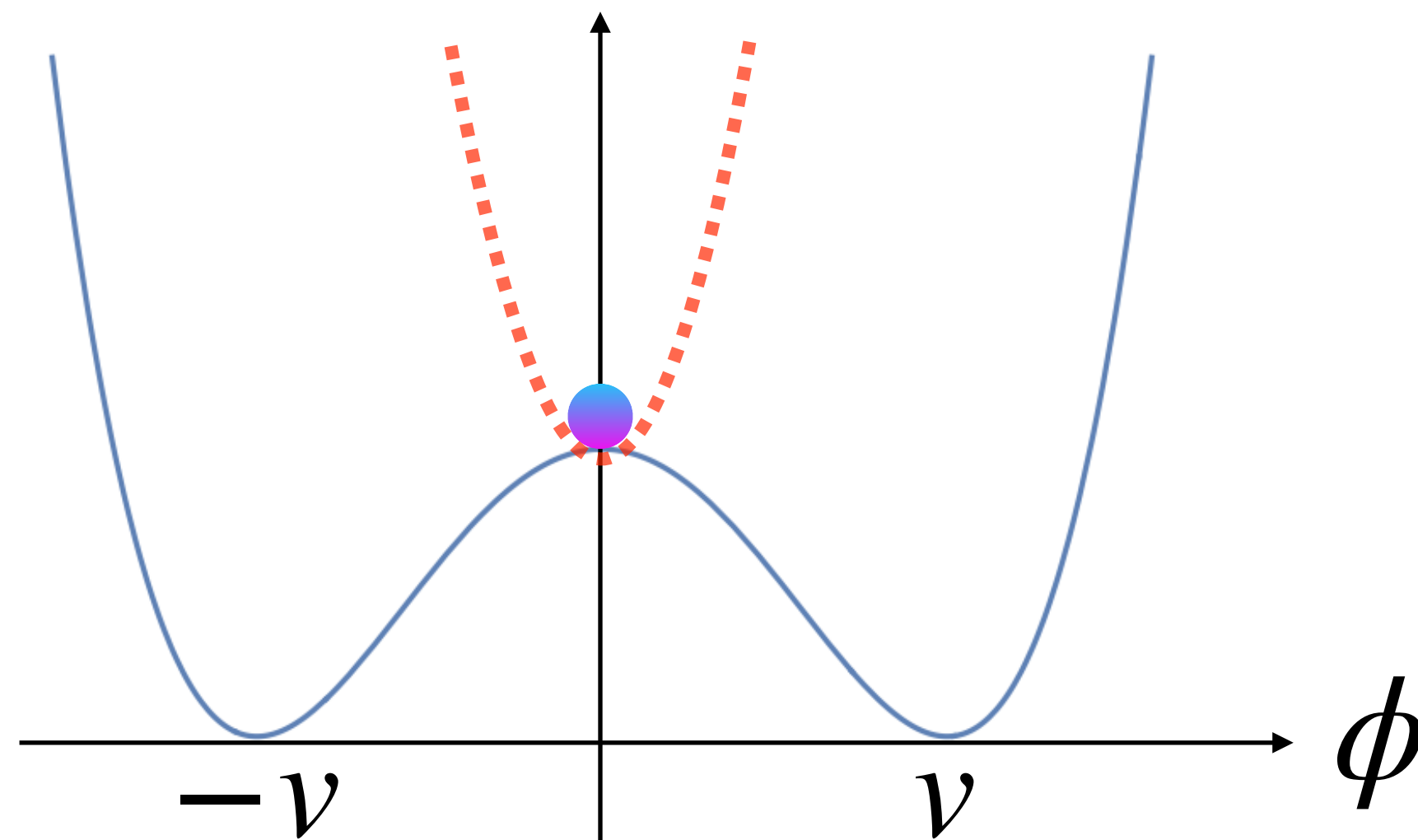
e.g. Lalak and Thomas, Phys. Lett. B 306, 10 (1993), Lalak, Ovrut, and Thomas, Phys. Rev. D 51, 5456 (1995), Larsson, Sarkar, and White Phys. Rev. D 55, 5129 (1997), Correia, Leite, and Martins, PRD 90, 023521 (2014)

# Initial distributions

Initial distributions are broadly classified into **thermal** and **non-thermal** fluctuations.

Thermal fluctuations/white noise

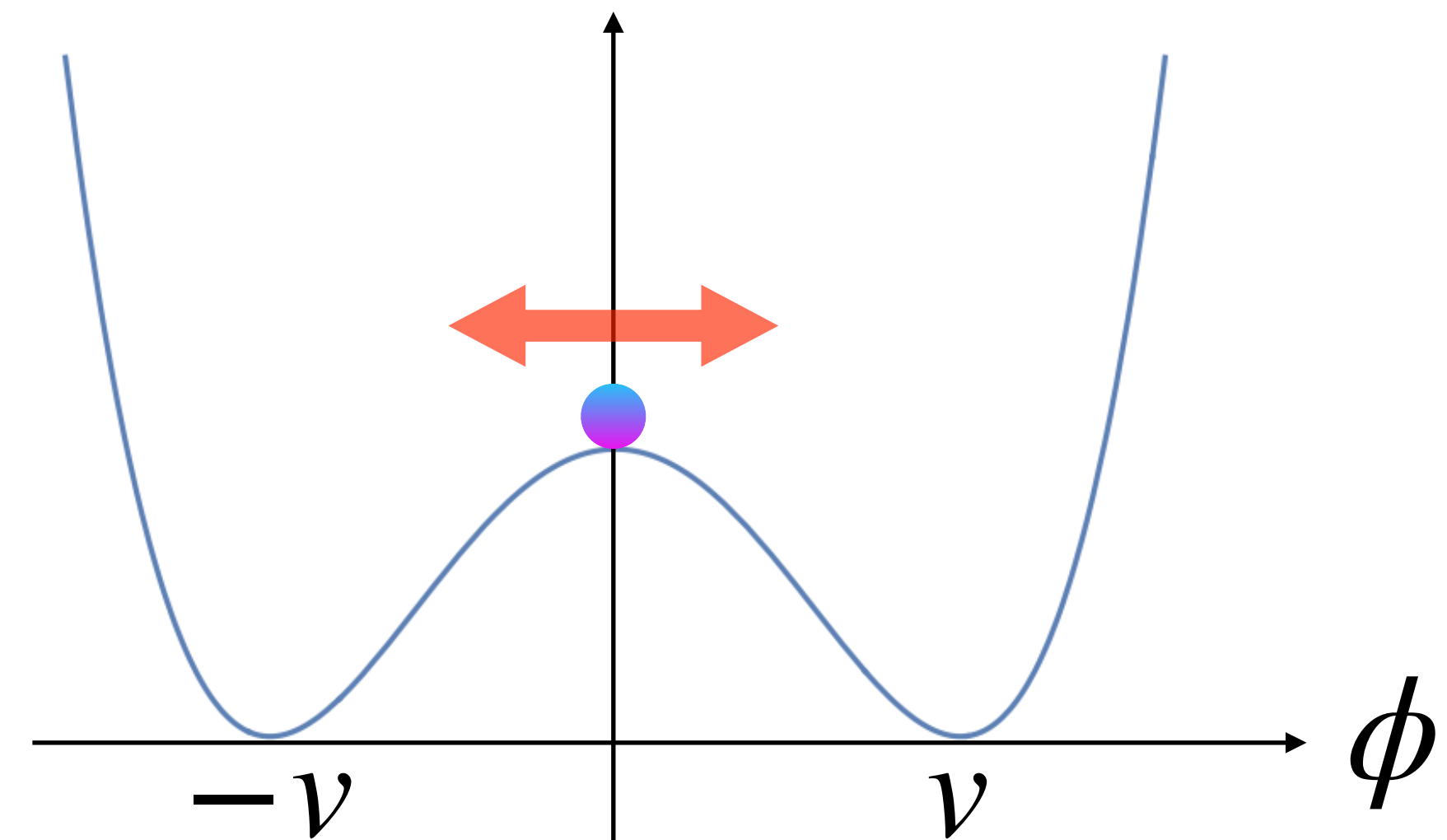
**No large-scale correlations**



$$\delta V = T^2 \phi^2 \text{ or } H^2 \phi^2$$

Inflationary fluctuations

**Correlation on superhorizon scales**



$$\delta\phi = \frac{H_{\text{inf}}}{2\pi}$$

Linde and Lyth '90,  
Nagasawa and Yokoyama '92

# Initial distributions

During inflation, a massless (or light) scalar acquires almost scale-invariant fluctuations;

(Here the bias is set to be zero)

$$\langle \phi(\mathbf{k})\phi(\mathbf{k}') \rangle = (2\pi)^d \delta^{(d)}(\mathbf{k} + \mathbf{k}') P(k) \quad d = 2 \text{ or } 3$$

$$\mathcal{P}(k) \simeq \left( \frac{H_{\text{inf}}}{2\pi} \right)^2 \quad \text{with} \quad \mathcal{P}(k) = \frac{k^d}{2\pi^{d-1}} P(k)$$

$$\langle \phi(\mathbf{x})^2 \rangle = \int \frac{d^d \mathbf{k}}{(2\pi)^d} P(k) = \int d \ln k \mathcal{P}(k),$$

Correlation on large scales

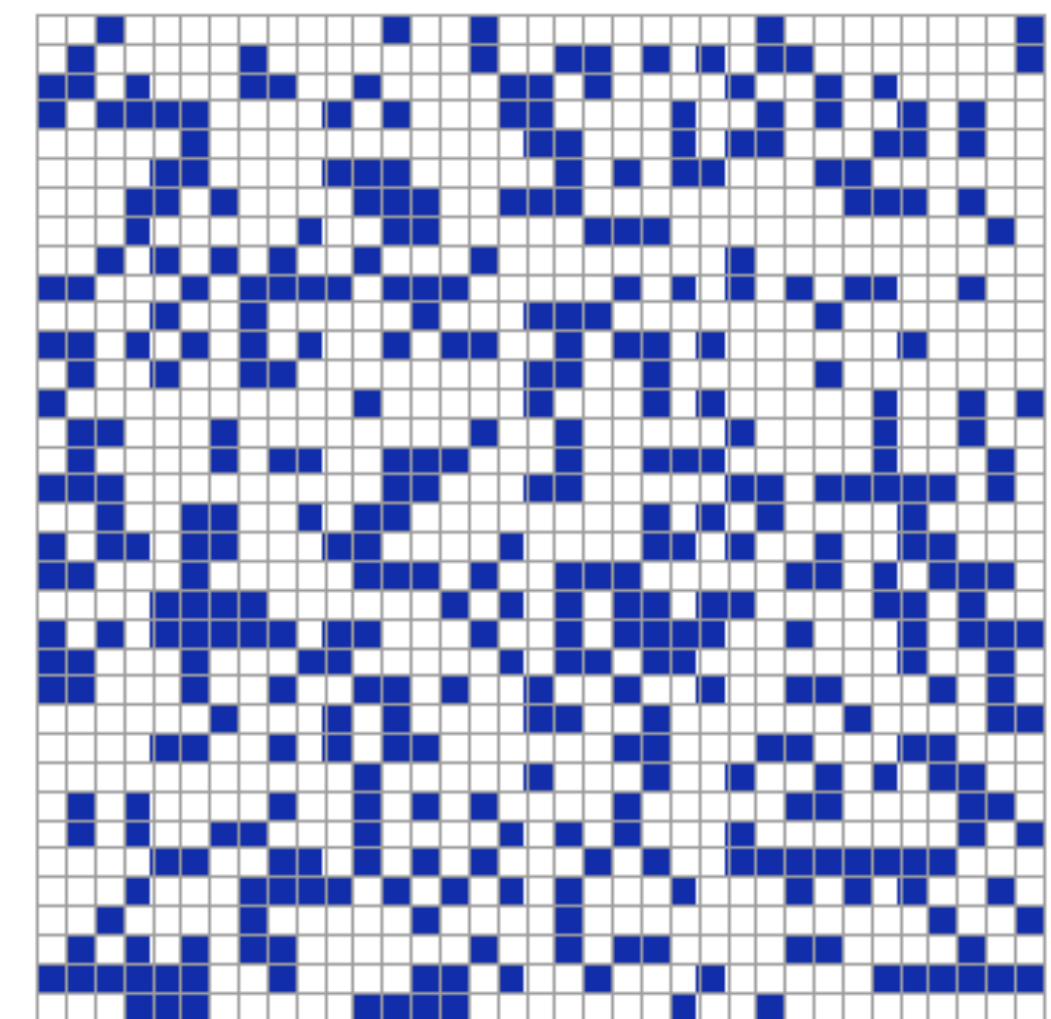
# Standard lore about DWs?

- *The scaling solution is an attractor.*
- *DW is a nonlinear object, and so, the dynamics is determined locally.*
  - ➔ *Thus, the DW network forgets the initial condition soon.*
- *One can use the percolation theory to see if infinitely large DWs are formed or not.*

however that this situation will change if we take into account the effect of the cosine potential on the values of  $\chi$ . We will then find that the gaussian distribution will evolve into one which is non-gaussian, displaying peaks near the two minima  $\chi = \pm\pi$  of the potential. In this way we have a situation which is much closer to the one relevant to percolation theory.

from Lalak and Thomas, Phys. Lett. B 306, 10 (1993)

For the inflationary fluctuations, based on percolation theory, it was assumed that for each Hubble horizon, the scalar field randomly falls into one of the two vacua, which is the same as the case of white noise, without proper numerical simulations.



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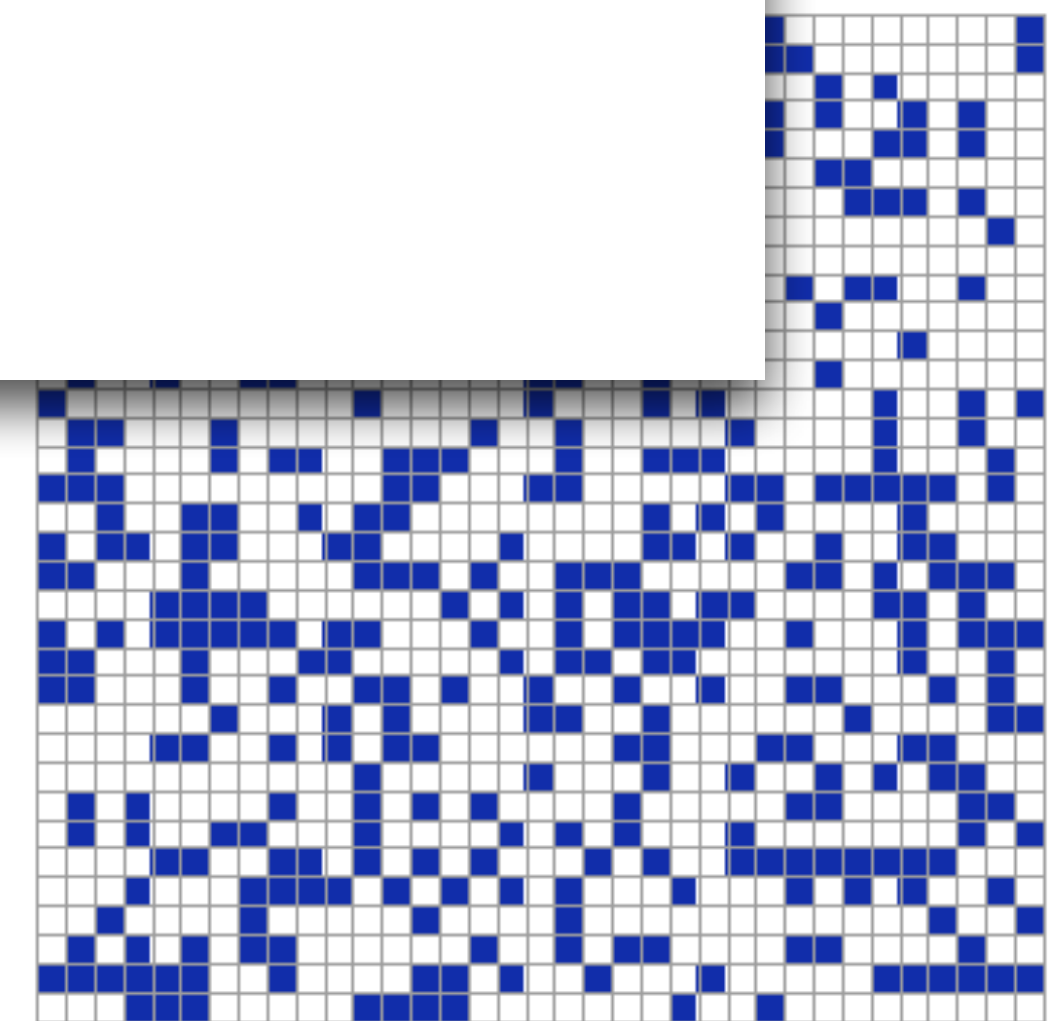
- *One ca*  
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**These do not apply to DWs with  
inflationary fluctuations!**

*Ws are*

For the inflationary fluctuations, based on percolation theory, it was assumed that for each Hubble horizon, the scalar field randomly falls into one of the two vacua, which is the same as the case of white noise, without proper numerical simulations.



# Numerical lattice simulations

Gonzalez, Kitajima, FT and Yin 2211.06849

- EOM: 
$$\frac{\partial^2 \phi(\mathbf{x}, t)}{\partial t^2} + 3H \frac{\partial \phi(\mathbf{x}, t)}{\partial t} - \frac{\partial_i^2 \phi(\mathbf{x}, t)}{a^2} + \frac{\partial V}{\partial \phi} = 0,$$

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 = \frac{\lambda}{4}v^4 - \frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4}\phi^4$$

- Conformal time:  $\tau = \sqrt{2t/m_0}$

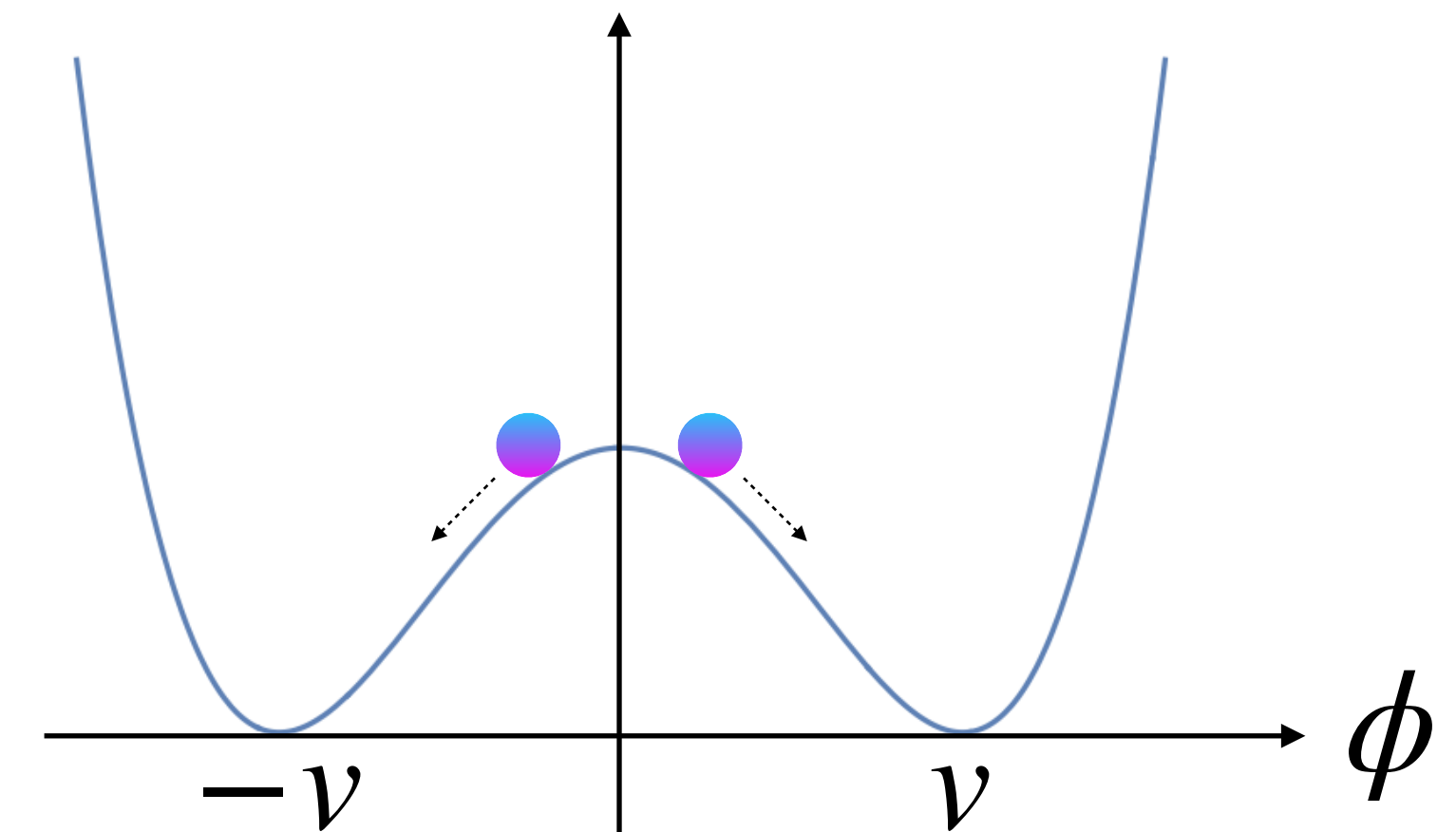
- Set-up

- 2D lattice with a grid size of 4096 x 4096 (or 16384 x 16384)

- Initial fluctuations are set with white noise and inflationary fluctuations.

- The bias parameter:

$$b_d \equiv \frac{\langle \phi(\mathbf{x}) \rangle}{\sqrt{(\langle \phi(\mathbf{x} - \langle \phi(\mathbf{x}) \rangle)^2)}}$$



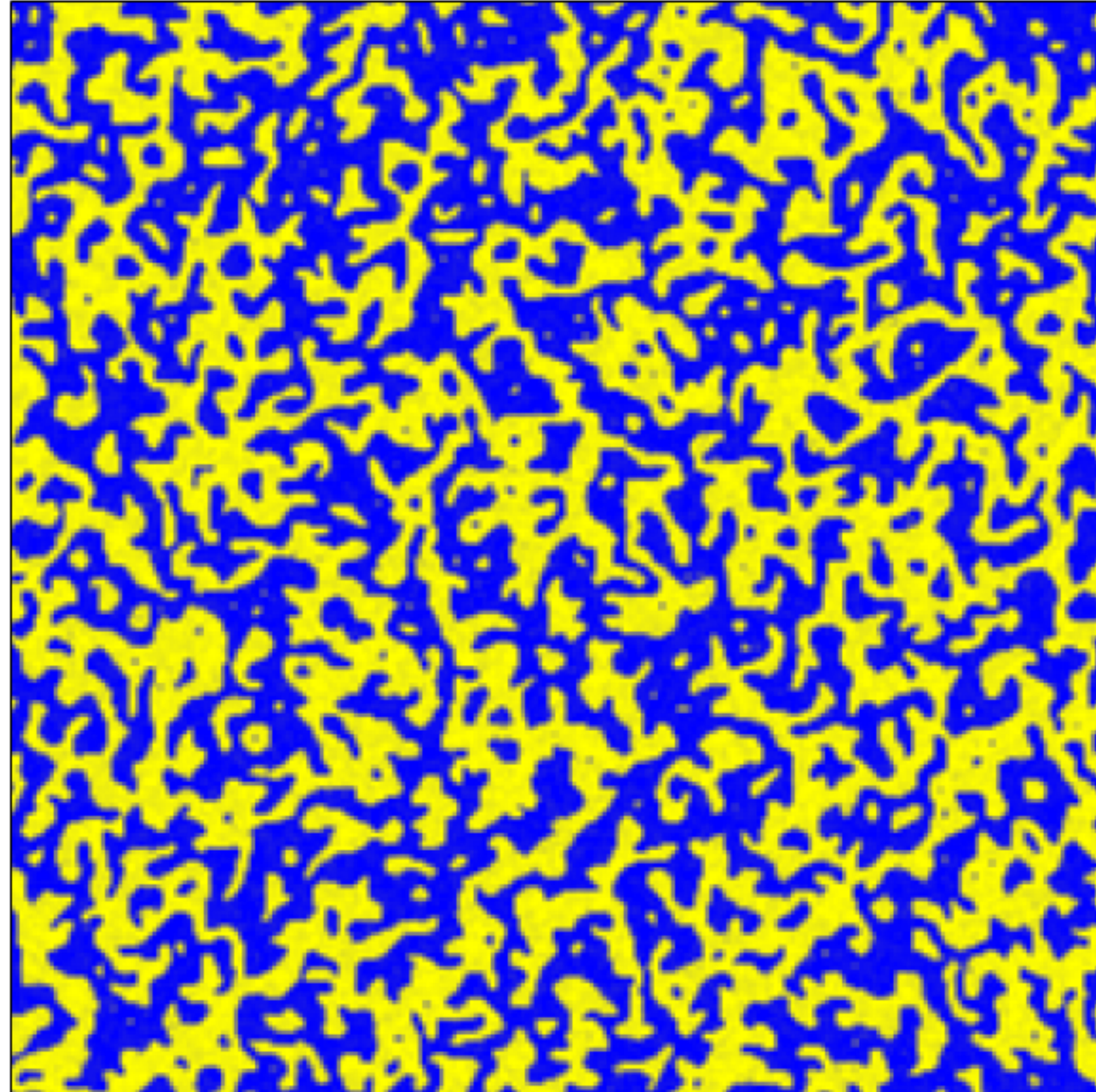
Note: for the inflationary fluctuations, we use  $\sqrt{\mathcal{P}(k)}$  for the denominator to avoid the logarithmic dependence on the box size.

# Snapshots of the DW network

$$b_d = 0 \quad \tau = 10/m_0$$

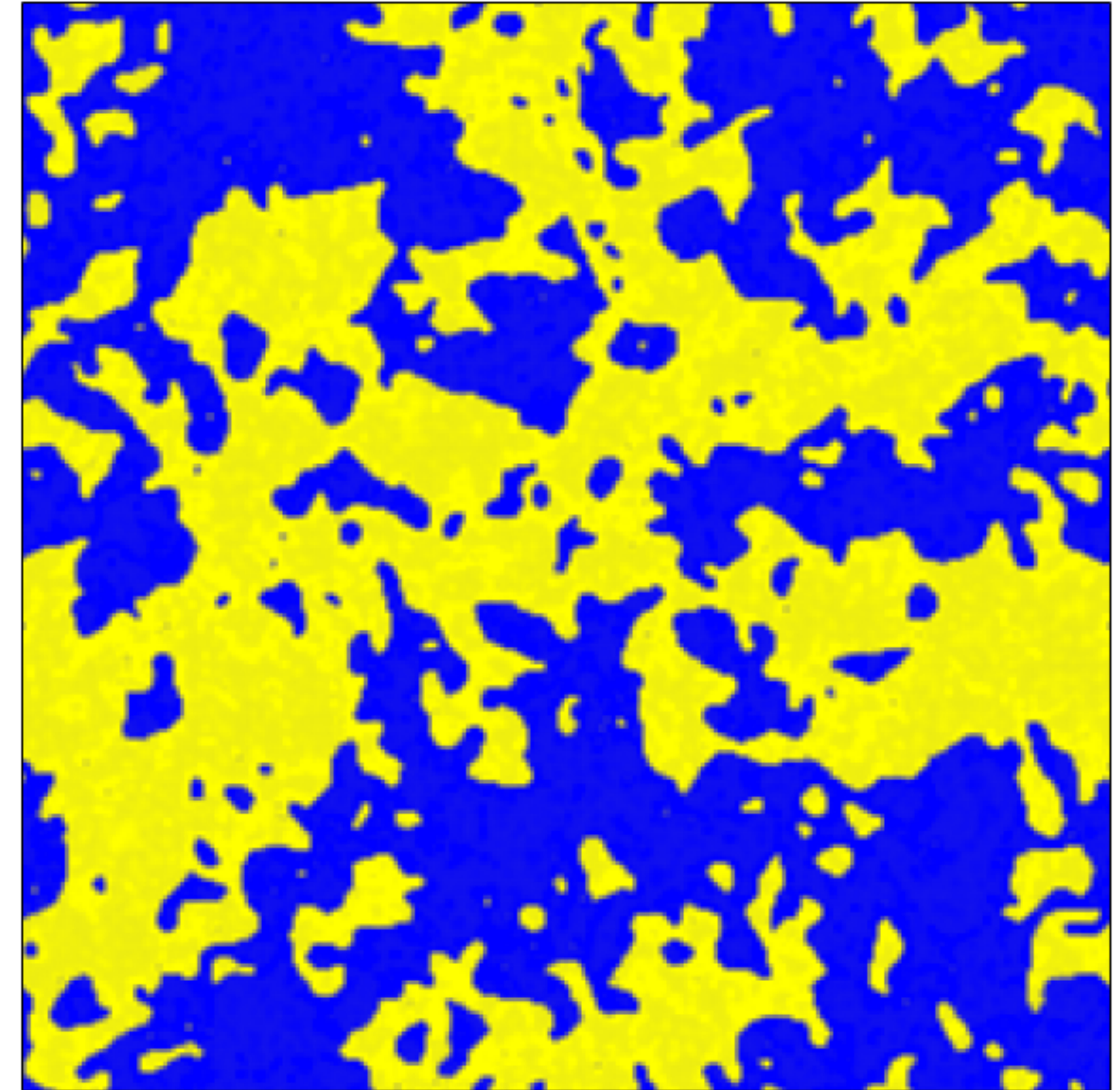
$\sim 40^2$  horizons

White noise



Typical structure of  $\mathcal{O}(H^{-1})$

Inflationary fluctuations



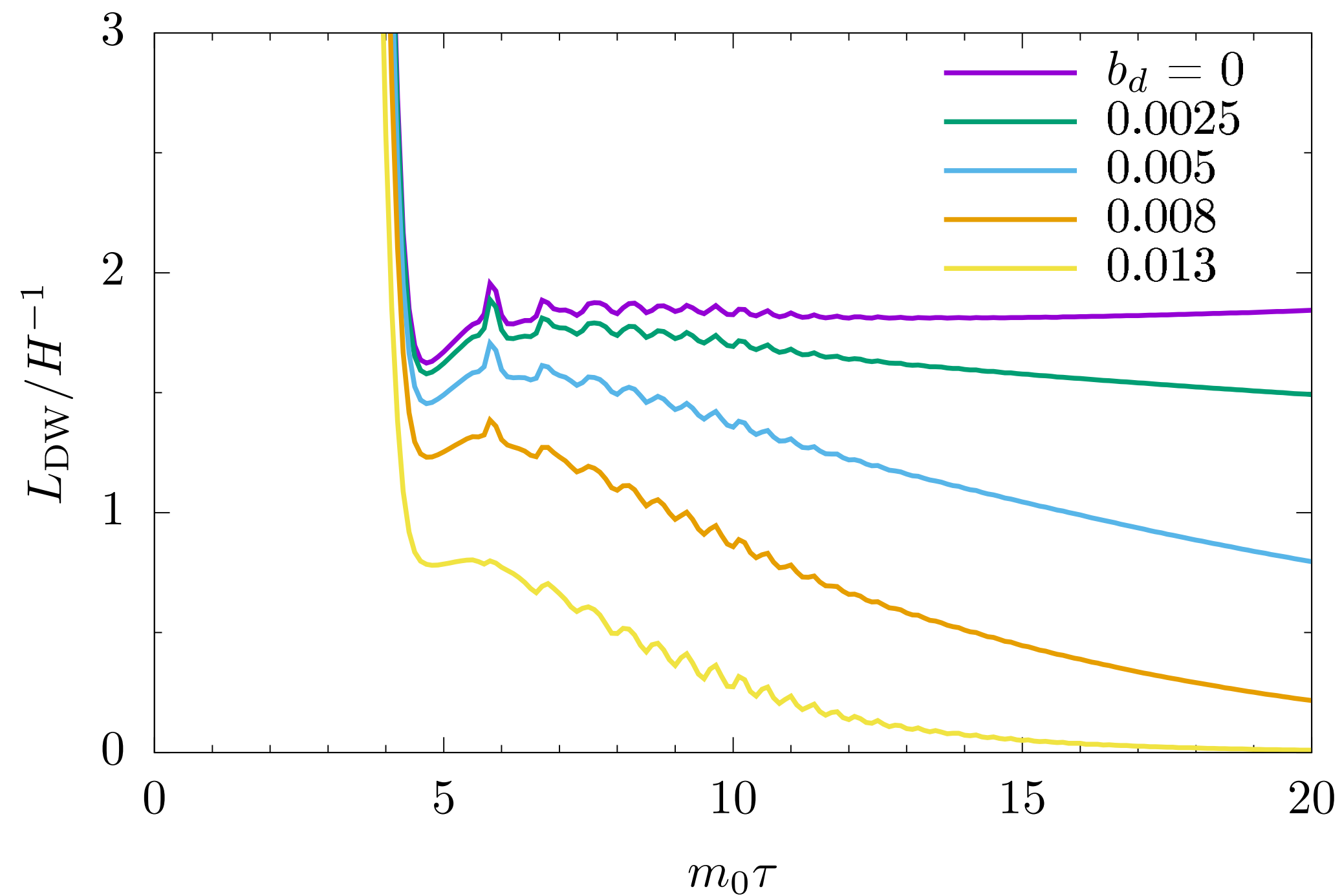
Structure of various scales

# DW length per horizon

Scaling solution:  $L_{\text{DW}}/H^{-1} = \mathcal{O}(1)$

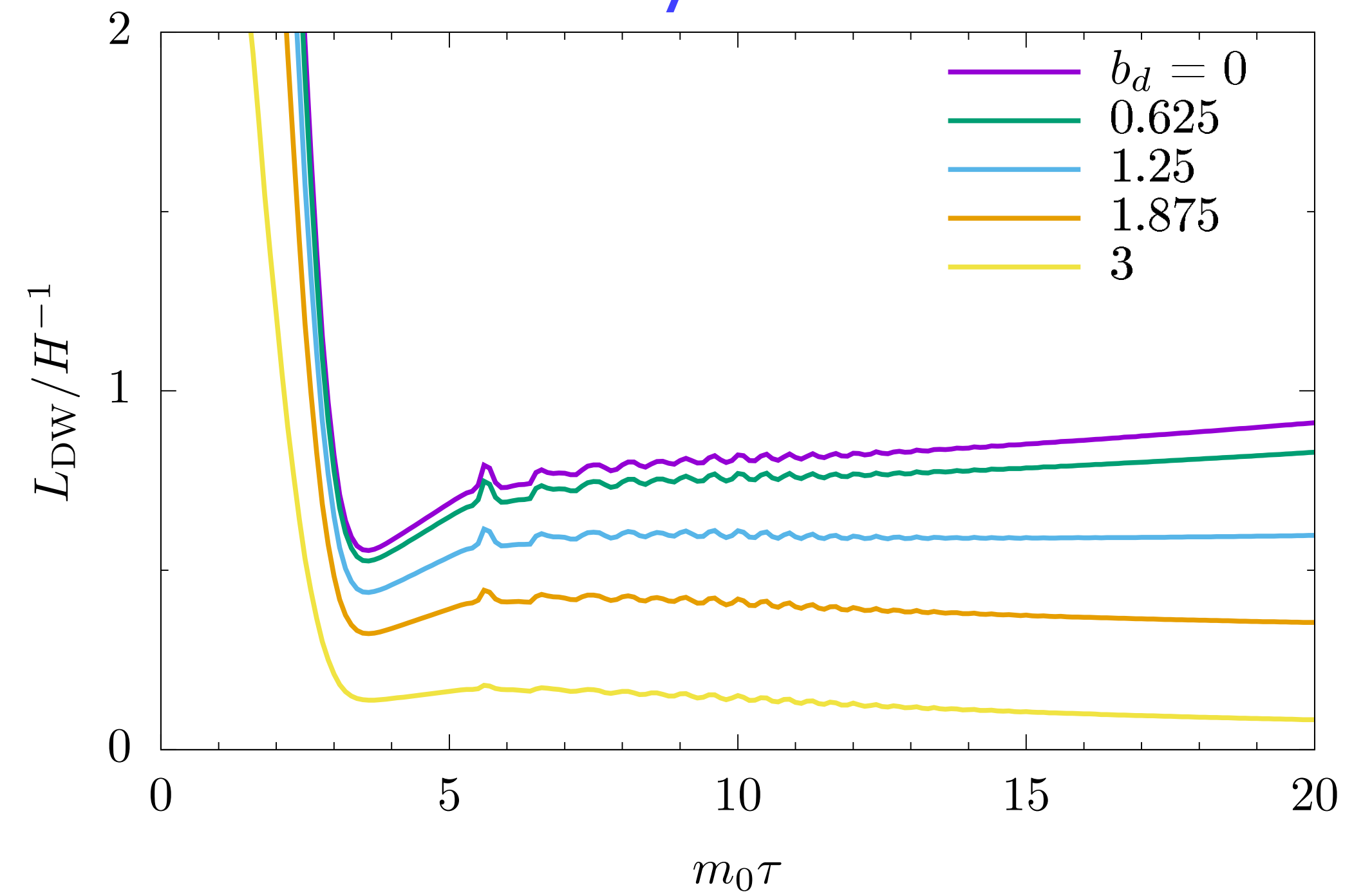
$L_{\text{DW}}$  : the average DW length per horizon

White noise



**DW decays fast for small bias**

Inflationary fluctuations



**DW is rather stable for large bias**



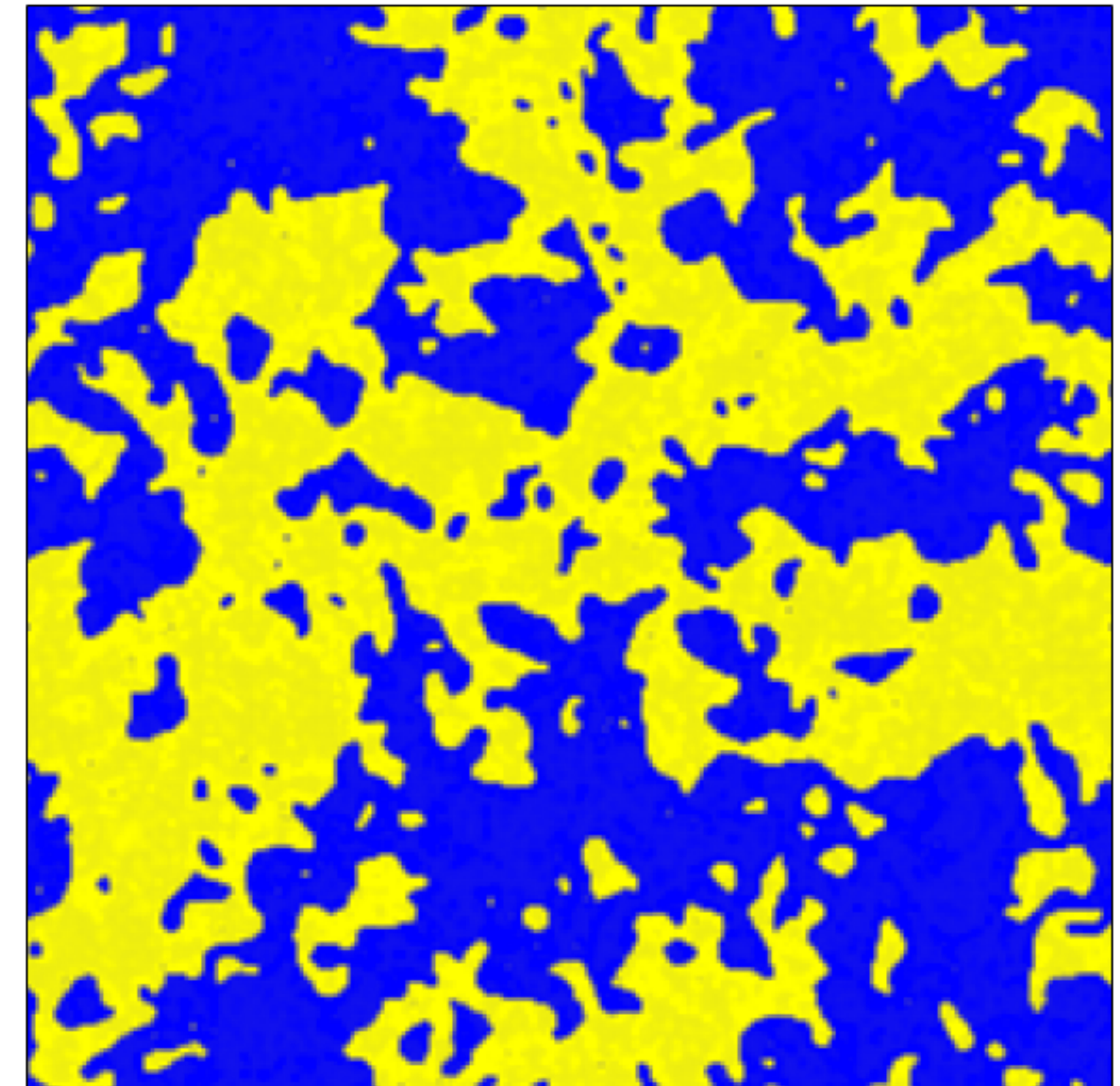
# Why is the DW with inflationary fluctuations so stable?

The DW network retains information on the initial distributions, and in this sense, the scaling solution is not a local attractor.

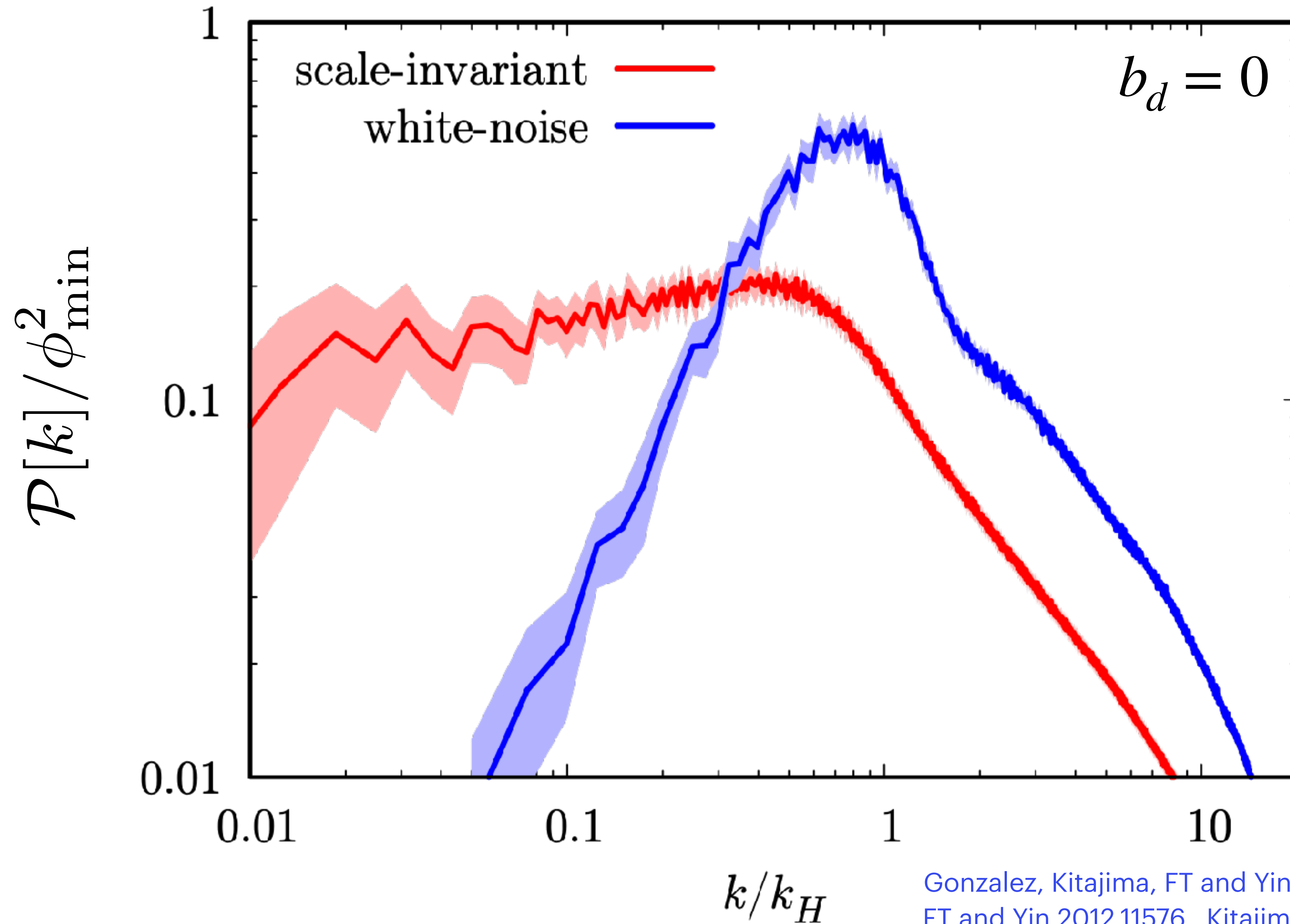
Clearly, the percolation theory is not applicable.

There are large voids without DWs, which remain until the Hubble horizon grows to their sizes.

**It is the large-scale correlation that makes the DW network so stable!**



# Power spectrum



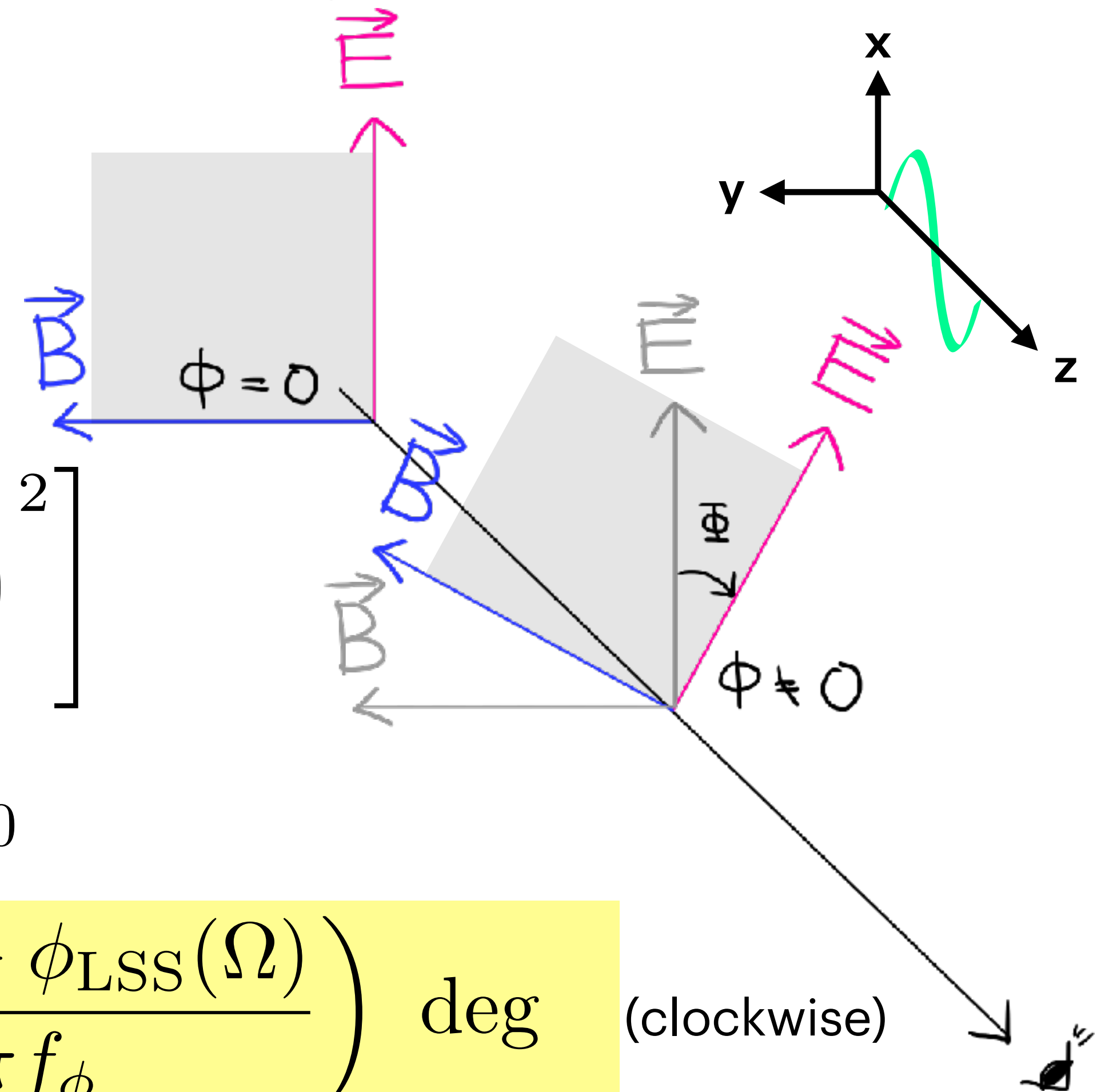
# **3. Axion DW and cosmic birefringence**

# Cosmic birefringence

$$\mathcal{L}_{\phi\gamma} = -c_\gamma \frac{\alpha}{4\pi} \frac{\phi}{f_\phi} F_{\mu\nu} \tilde{F}^{\mu\nu} = -\frac{1}{4} g_{\phi\gamma\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu},$$

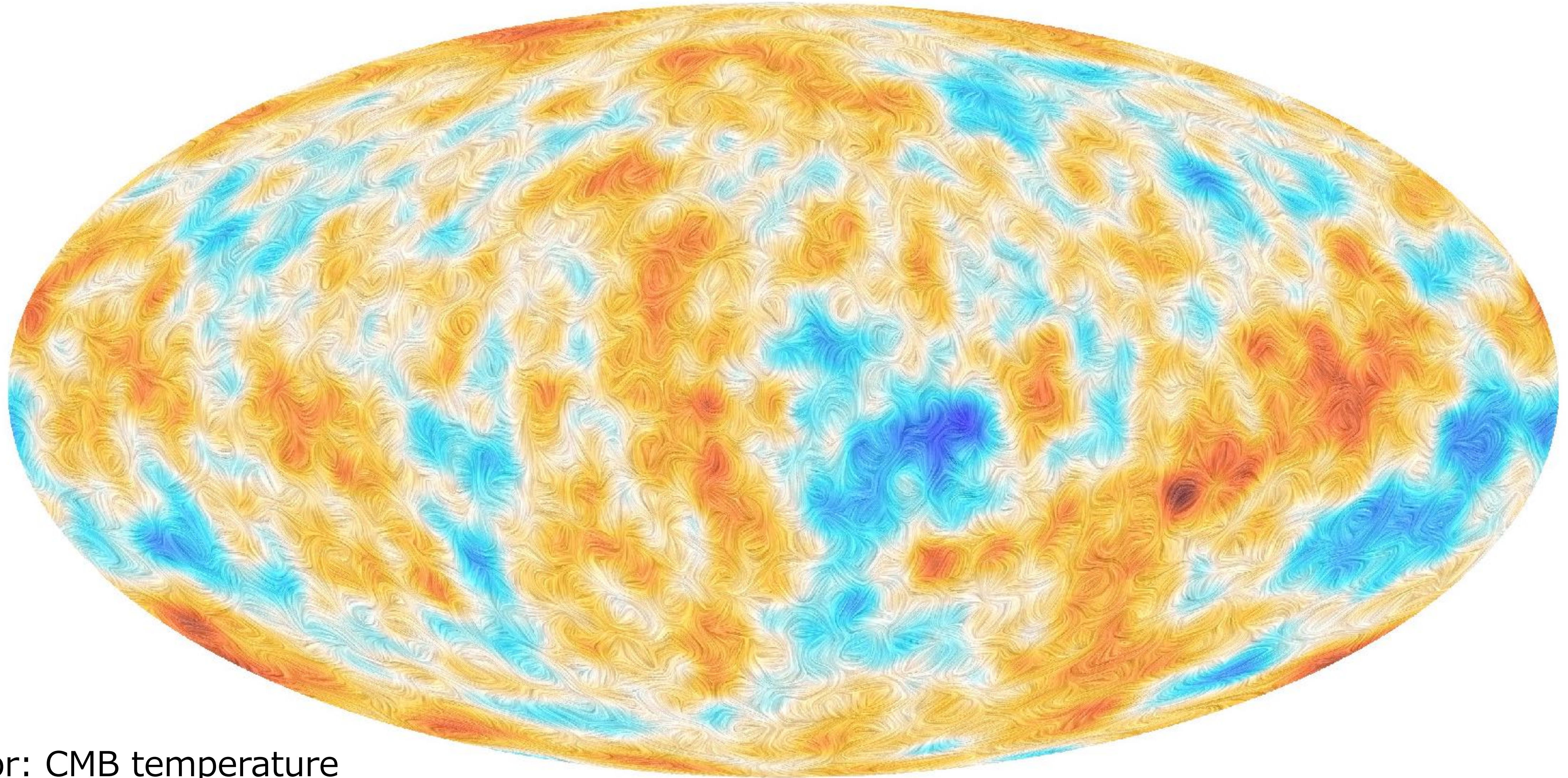
The axion dynamics rotates the polarization plane of linearly polarized light through the axion-photon coupling.

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{\phi\gamma\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}, \\ &= \frac{1}{2} \left( \vec{E}^2 - \vec{B}^2 \right) + g_{\phi\gamma\gamma} \phi \vec{E} \cdot \vec{B} \\ &\simeq \frac{1}{2} \left[ \underbrace{\left( \vec{E} + \frac{g_{\phi\gamma\gamma} \phi}{2} \vec{B} \right)^2}_{\vec{E} \text{ when } \phi = 0} - \underbrace{\left( \vec{B} - \frac{g_{\phi\gamma\gamma} \phi}{2} \vec{E} \right)^2}_{\vec{B} \text{ when } \phi = 0} \right] \end{aligned}$$



Thus, 
$$\Phi = \frac{g_{\phi\gamma\gamma} \Delta\phi}{2} \simeq 0.42 c_\gamma \left( \frac{\phi_{\text{today}} - \phi_{\text{LSS}}(\Omega)}{2\pi f_\phi} \right) \text{ deg} \quad (\text{clockwise})$$

# CMB photons are polarized (dominated by E-mode)

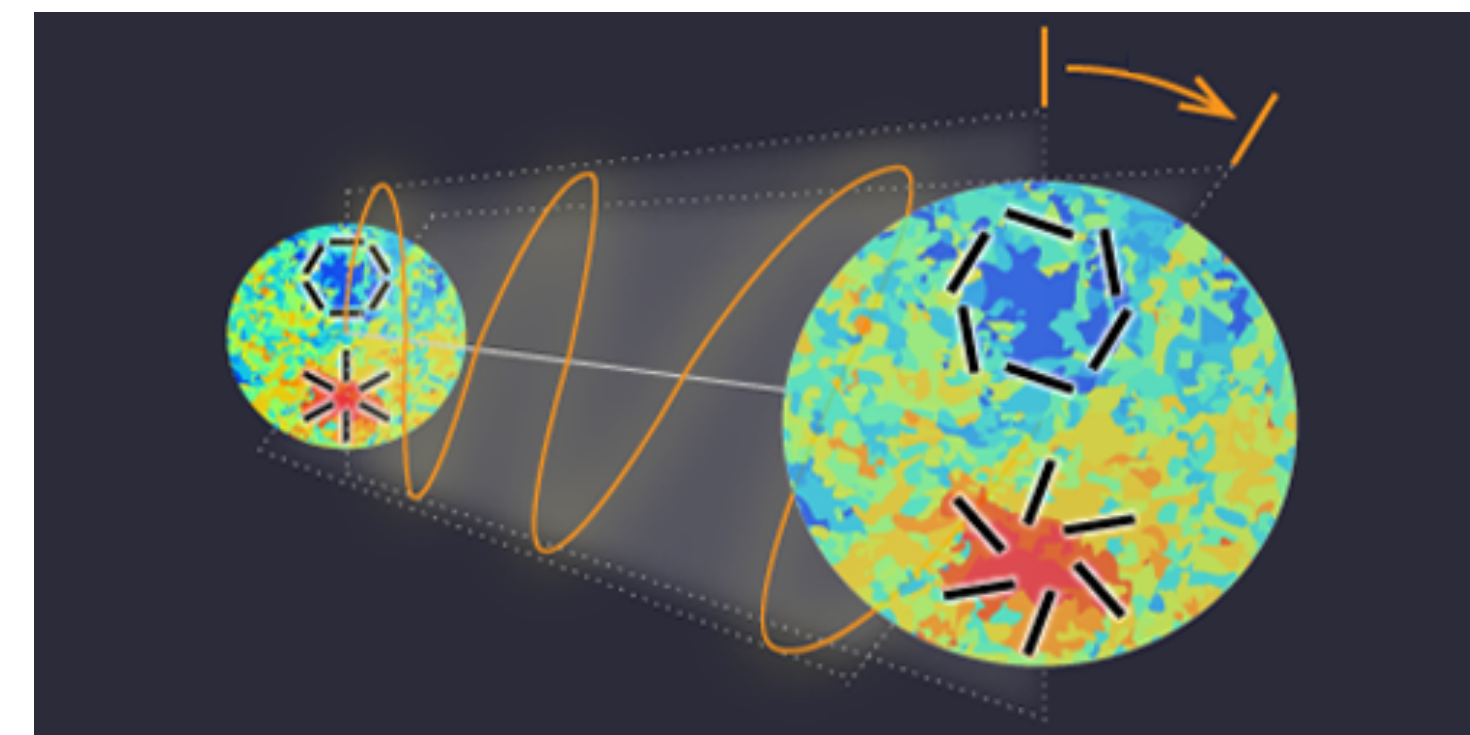


Color: CMB temperature

Texture: direction of polarization

# CMB constraints on the CB

## Isotropic CB



<https://physics.aps.org/articles/v13/s149>

Minami, Komatsu, Phys. Rev. Lett. **125**, 221301 (2020)

P. Diego-Palazuelos et al, Phys. Rev. Lett. **128**, 091302 (2022)

$$\beta = \frac{1}{4\pi} \int d\Omega \Phi(\Omega) = 0.36 \pm 0.11 \text{ deg}$$

from Planck 18 pol. data

based on a new method that uses both the CMB and Galactic foreground to distinguish between CB ( $\beta$ ) and detector orientation miscalibration ( $\alpha$ ).

Minami et al, PTEP 2019 083E02 , Minami PTEP 2020 063E01, Minami and Komatsu PTEP 2020 103E02

cf. The reported isotropic CB in the past:

$$\alpha + \beta = \begin{cases} -0.36 \pm 1.24 \text{ deg} & \text{WMAP} \\ 0.31 \pm 0.05 \text{ deg} & \text{Planck} \\ -0.61 \pm 0.22 \text{ deg} & \text{POLARBEAR} \\ 0.63 \pm 0.04 \text{ deg} & \text{SPTpol} \\ 0.12 \pm 0.06 \text{ deg} & \text{ACT} \\ 0.09 \pm 0.09 \text{ deg} & \text{ACT} \end{cases}$$

$$\sigma_{\text{syst}}(\alpha) = \begin{cases} 1.5 \text{ deg} & \text{WMAP} \\ 0.28 \text{ deg} & \text{Planck} \end{cases}$$

# CMB constraints on the CB

## Anisotropic CB

$$\sqrt{\frac{L(L+1)C_L}{2\pi}} \left( \underset{\uparrow}{=} \frac{g_{\phi\gamma\gamma}}{2} \frac{H_{\text{inf}}}{2\pi} \right) < 0.12 \text{ deg} \quad (95\% \text{ CL})$$

Massless axion

BICEP/Keck Collaboration, arXiv:2210.08038

for a scale-invariant CB; e.g. a massless axion with fluctuations

$$\delta\phi = \frac{H_{\text{inf}}}{2\pi}$$

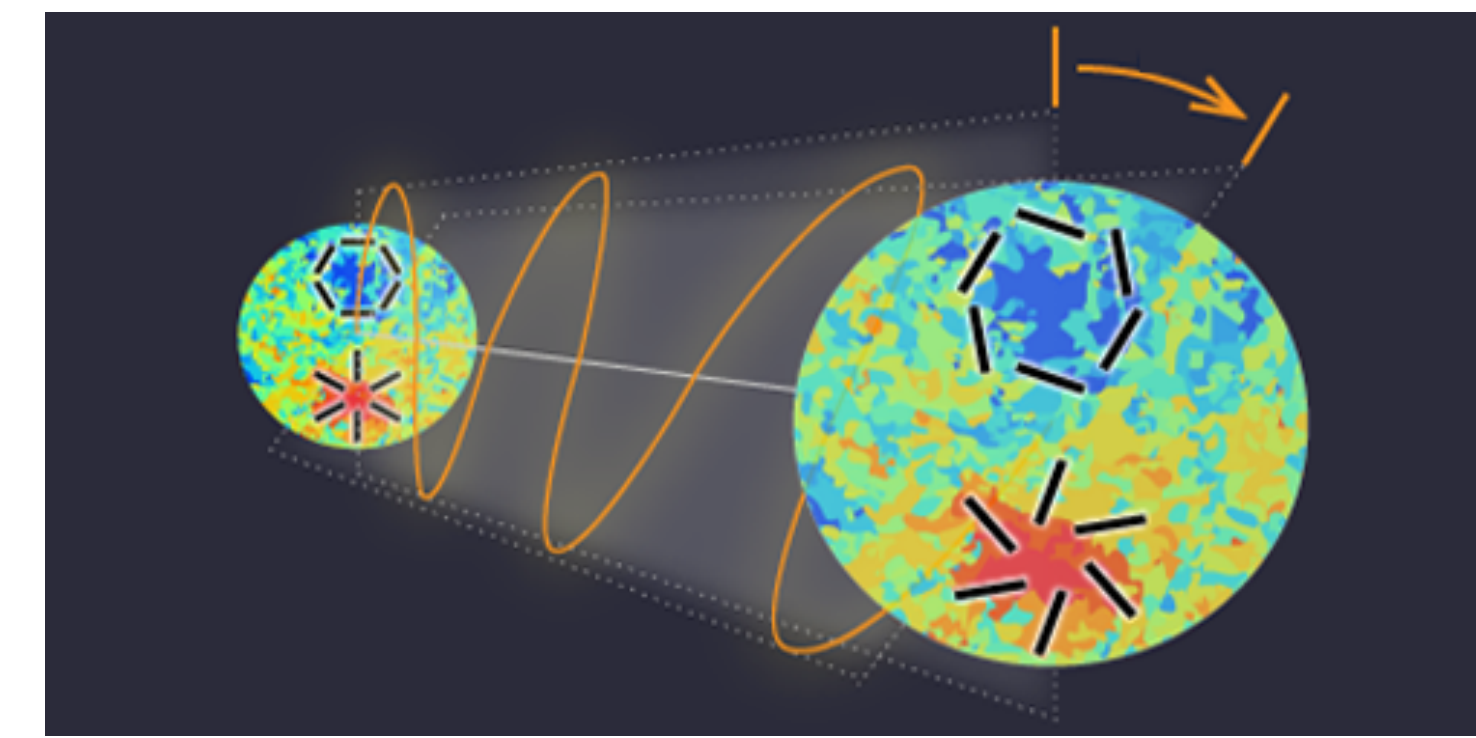
generated during inflation.

$$\left( \text{Recall } \Phi = \frac{g_{\phi\gamma\gamma} \Delta\phi}{2} \right)$$

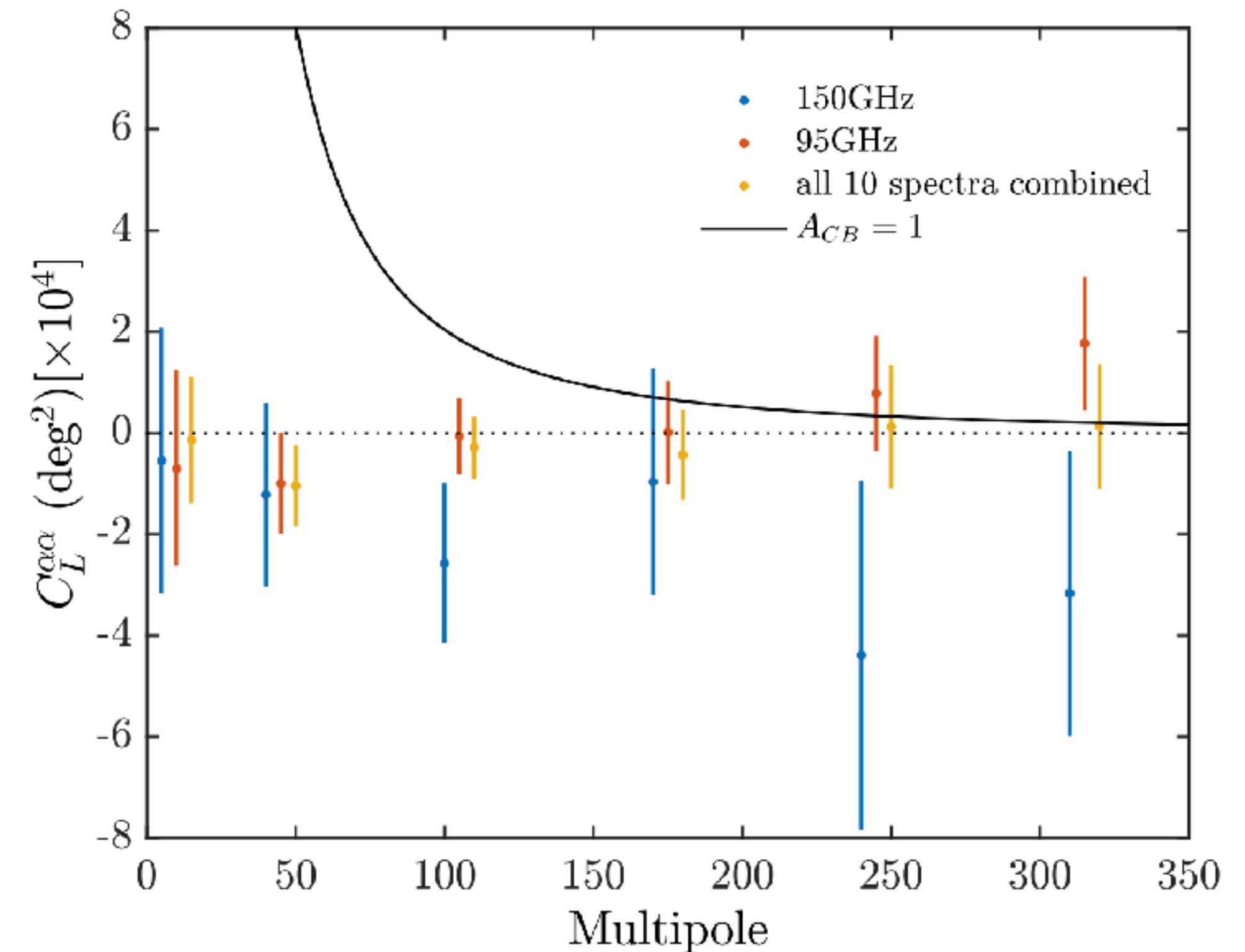
N.B. The limit mainly comes from low multipole  $L < 100$ .

The sensitivity will be improved by a factor of ~30 in the future CMB-S4

Pogosian et al, PRD 100, 023507 (2019)



<https://physics.aps.org/articles/v13/s149>



BICEP/Keck Collaboration, arXiv:2210.08038

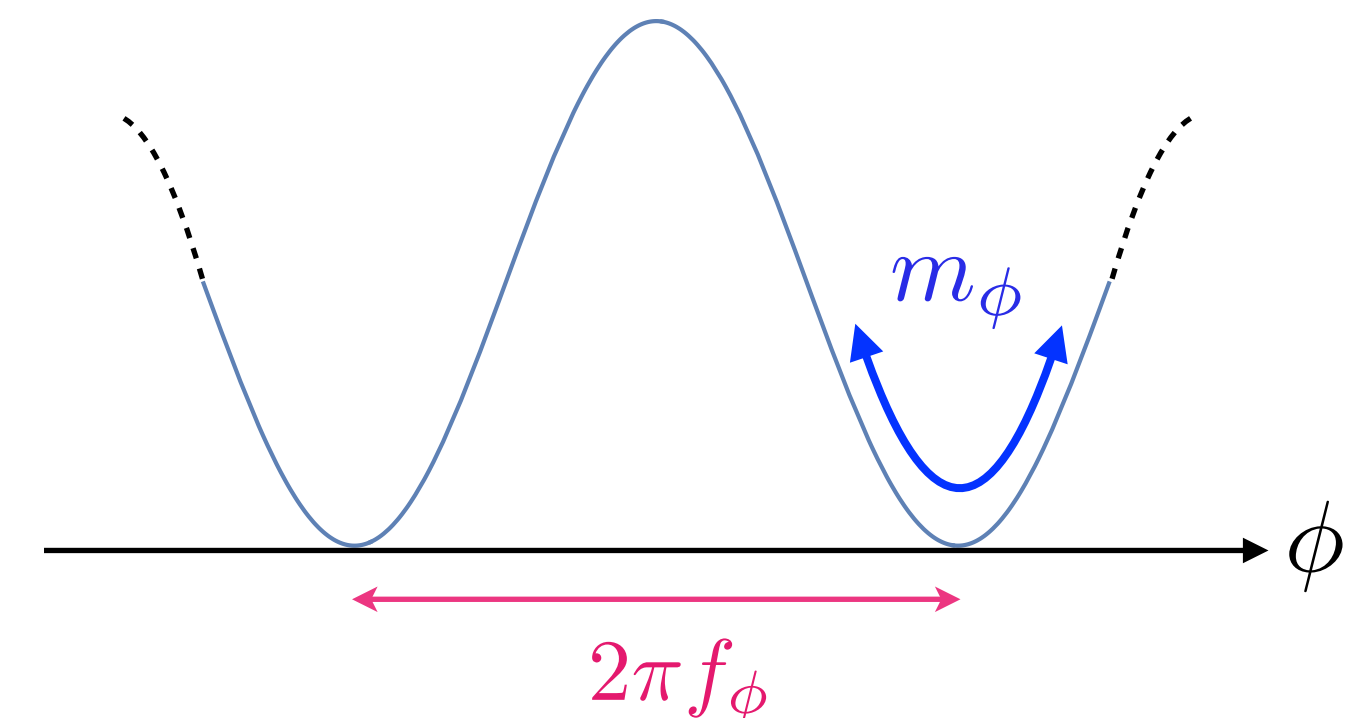
# Implications for ALP

$$\mathcal{L}_{\phi\gamma} = -c_\gamma \frac{\alpha}{4\pi} \frac{\phi}{f_\phi} F_{\mu\nu} \tilde{F}^{\mu\nu} = -\frac{1}{4} g_{\phi\gamma\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu},$$

- The hint of the isotropic CB:  $\beta = \frac{1}{4\pi} \int d\Omega \Phi(\Omega) = 0.35 \pm 0.14$  deg
- The ALP prediction:  $\Phi(\Omega) \simeq 0.42 c_\gamma \left( \frac{\phi_{\text{today}} - \phi_{\text{LSS}}(\Omega)}{2\pi f_\phi} \right)$  deg

➔ The ALP must have moved by  $\Delta\phi = \mathcal{O}(\pi f_\phi)$  for  $c_\gamma = \mathcal{O}(1)$  after recombination

The interpretation in terms of a homogeneous ALP was studied in e.g. [Fujita et al 2011.11894](#)





# ALP domain walls without strings

Let us consider the axion potential

$$V(\phi) = m_\phi^2 f_\phi^2 \left( 1 + \cos \frac{\phi}{f_\phi} \right)$$

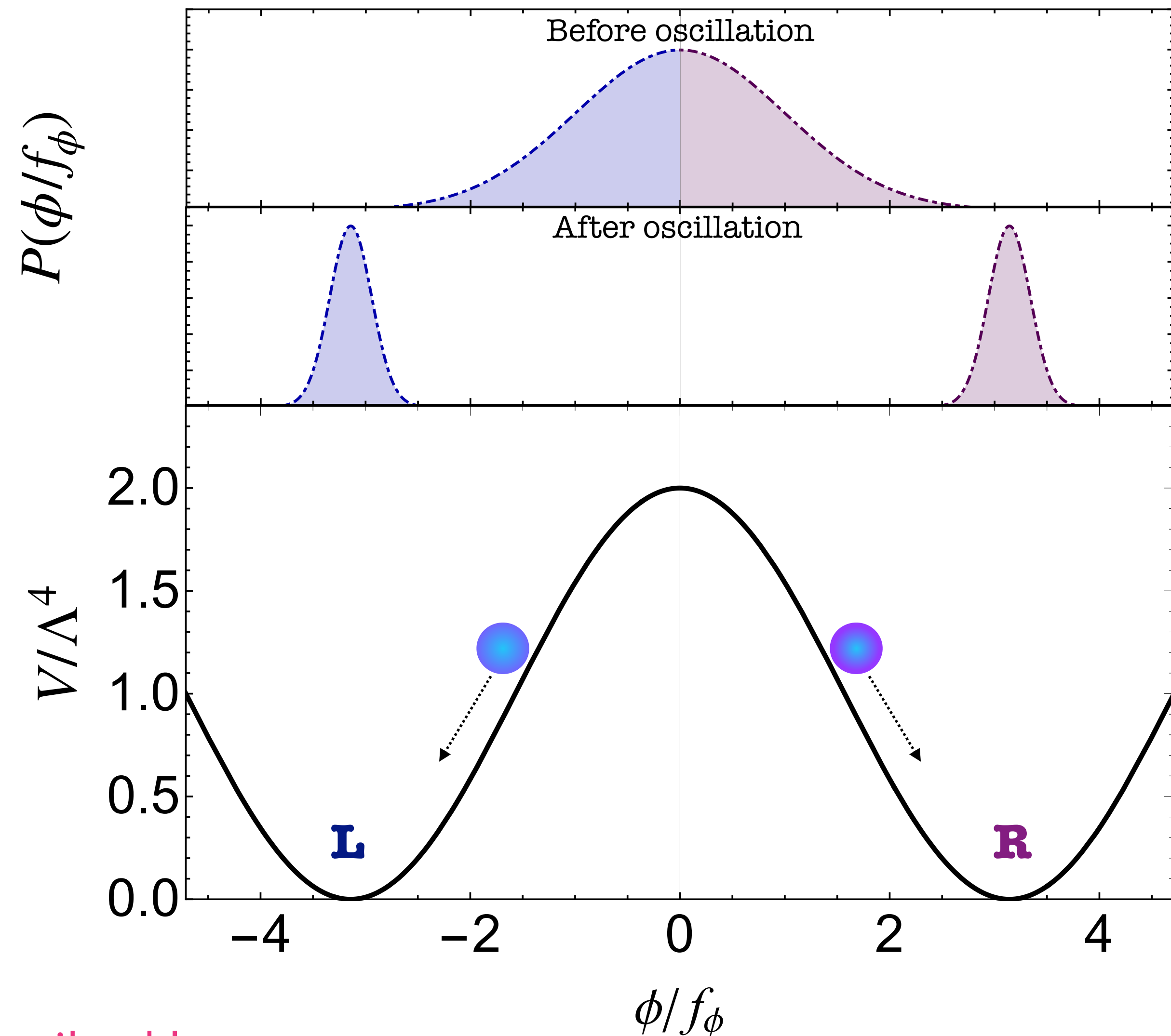
and focus on the adjacent minima,

$$\phi_L = -\pi f_\phi \text{ and } \phi_R = +\pi f_\phi.$$

If both vacua are populated in the early Universe, infinite domain wall (w/o strings) will appear when

$$H \sim m_\phi \gtrsim H_{\text{LSS}}.$$

Specific scenarios to obtain  $\delta\theta = O(1)$  will be described later.



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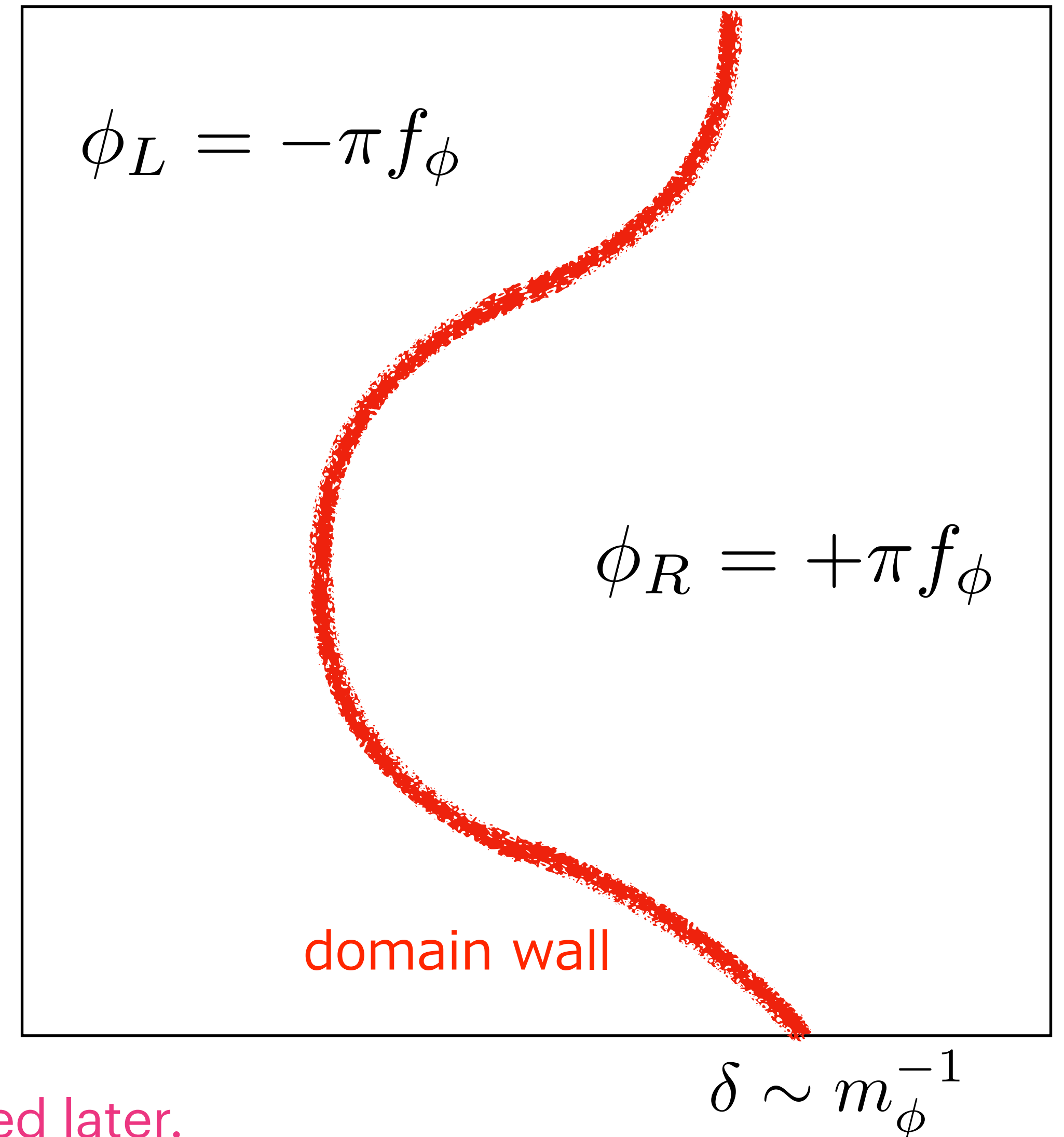
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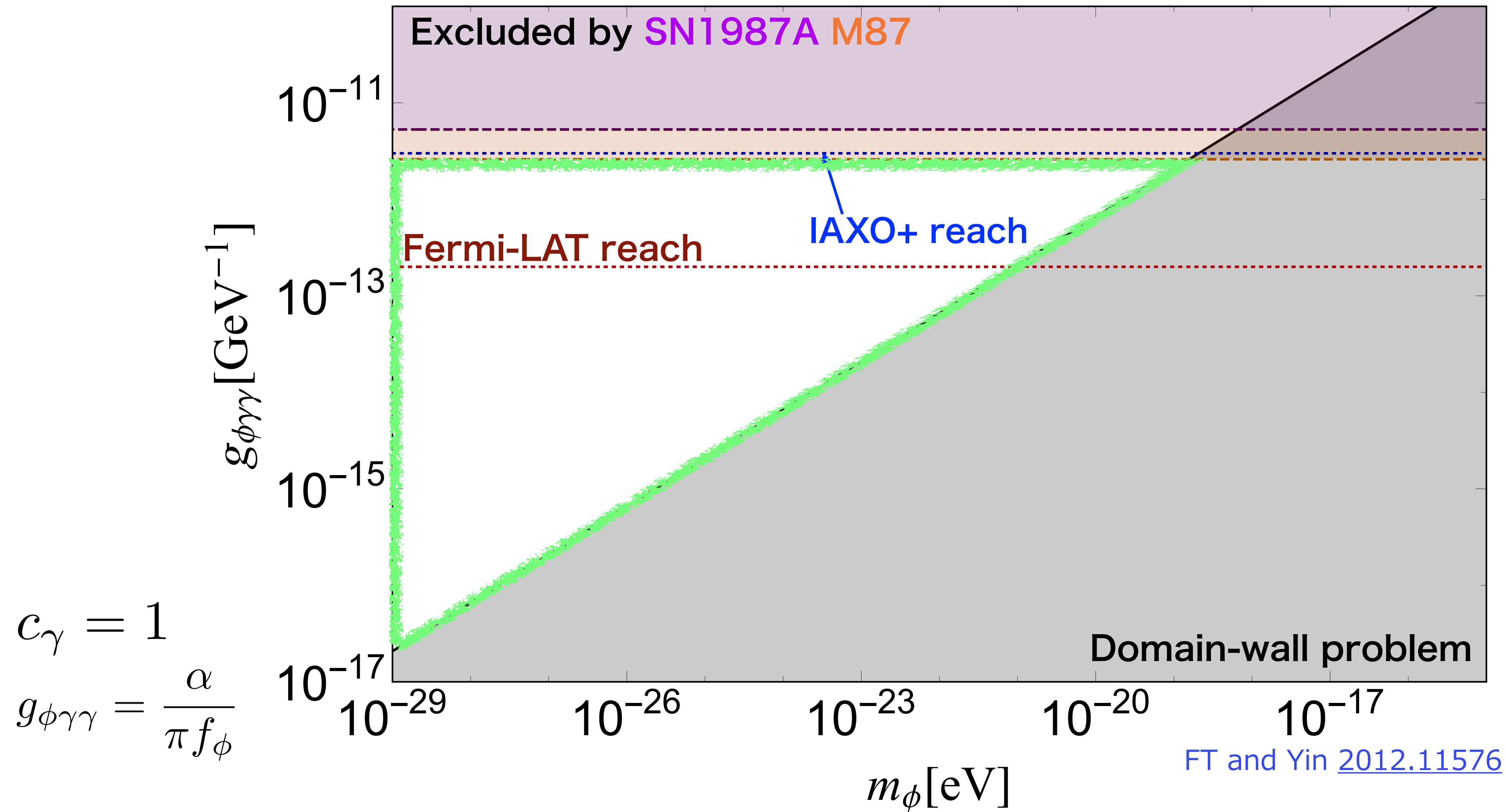
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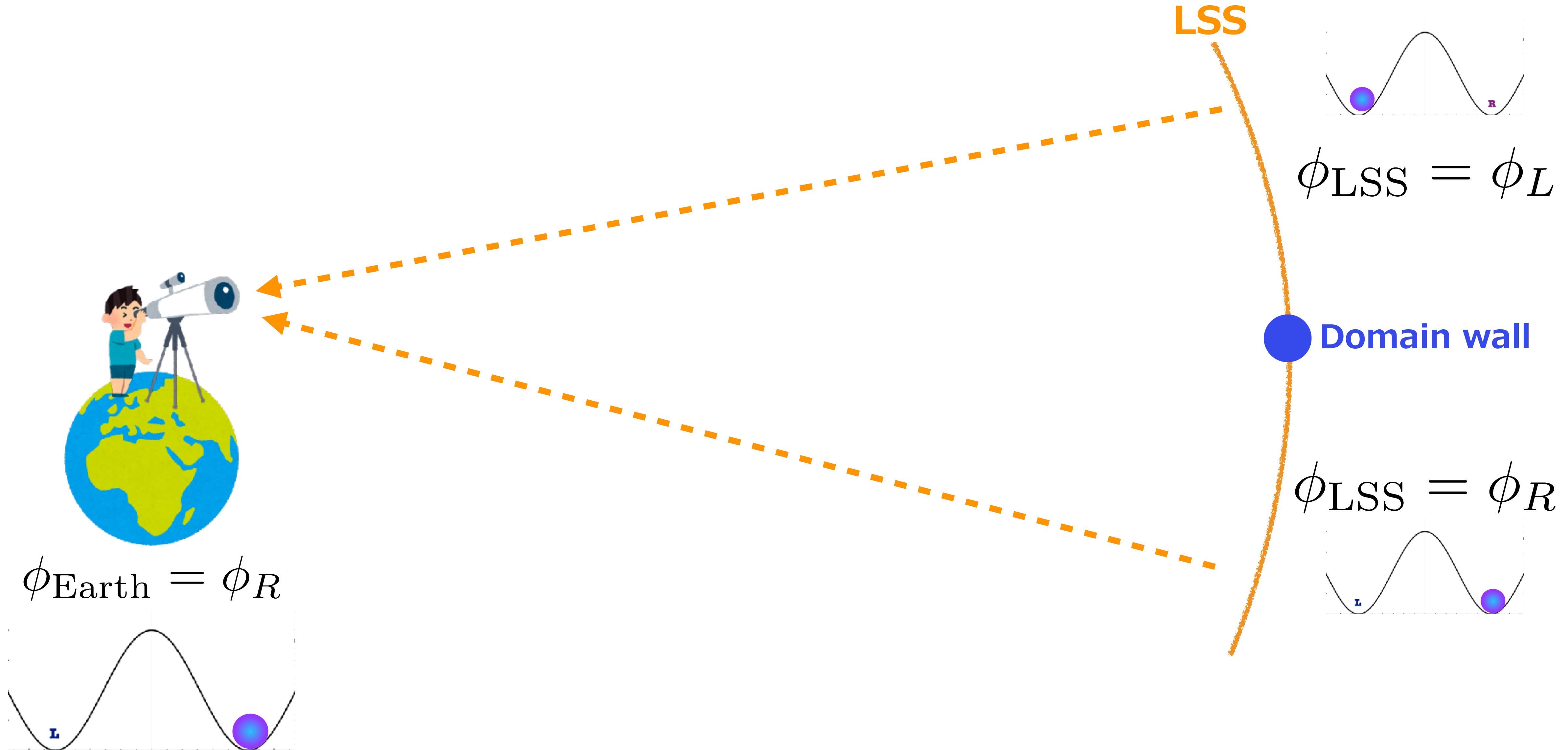


# Various bounds and future sensitivity reach

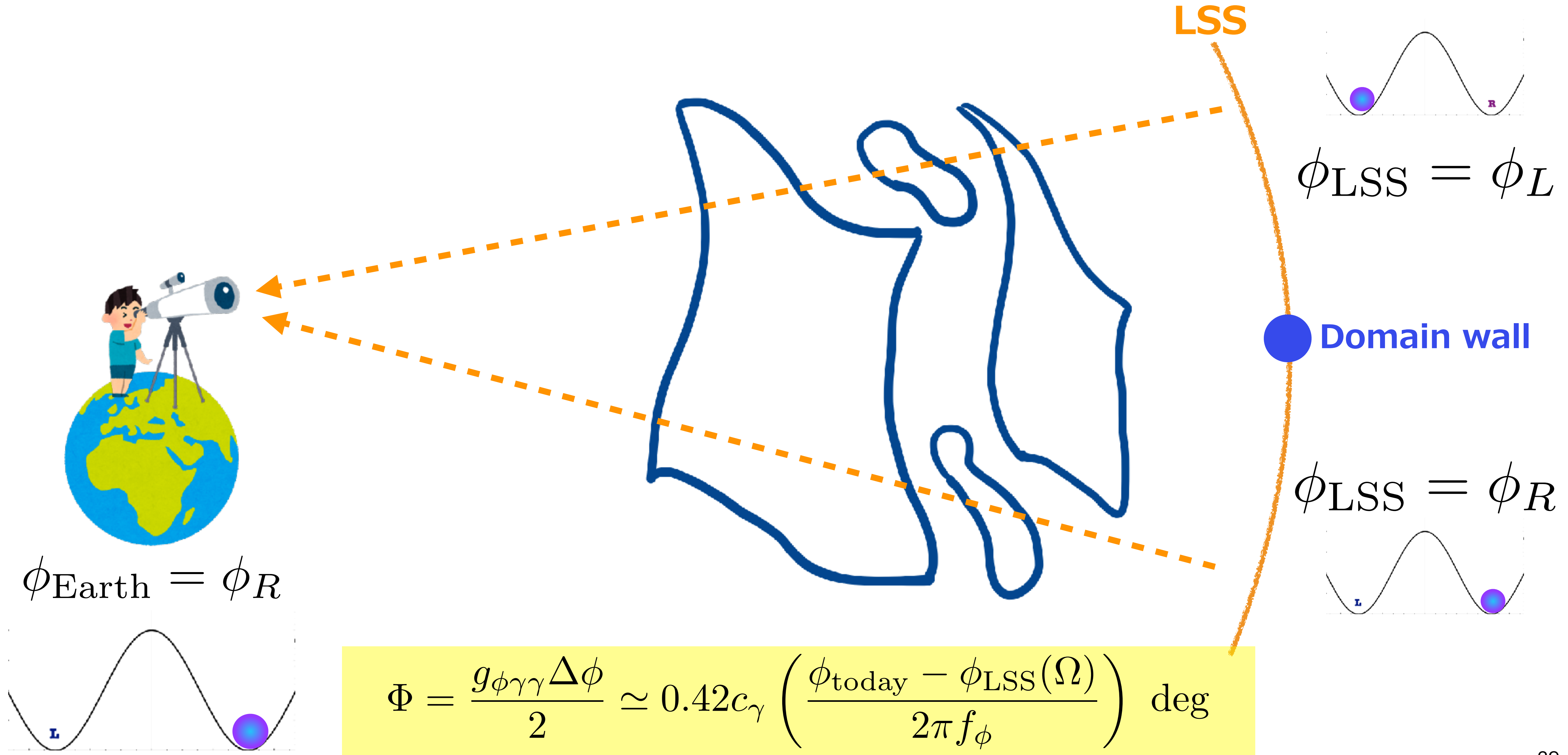


Note that the mass is heavier than  $\sim 10^{-29} \text{eV} \simeq H_{\text{LSS}}$  27

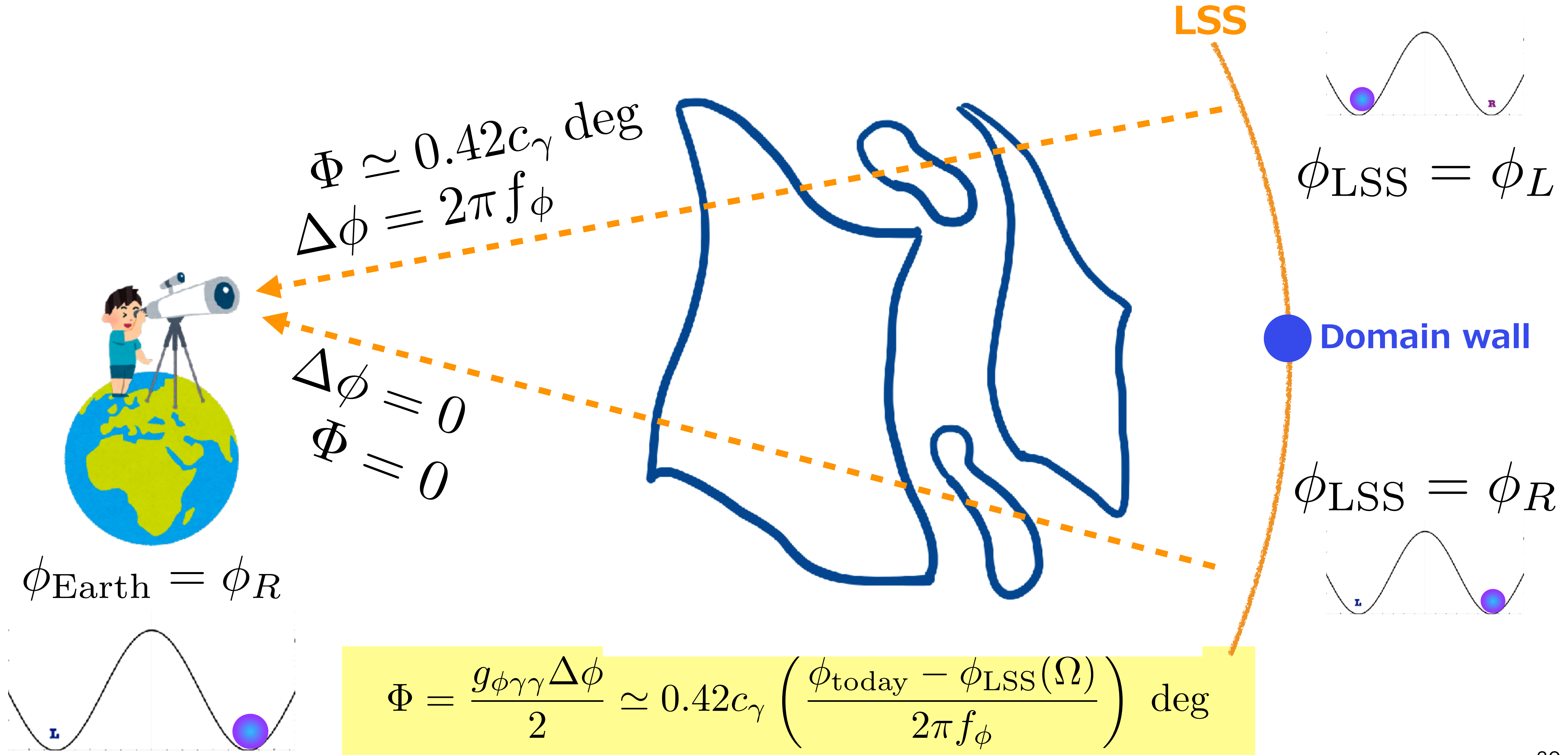
# KiloByte CB from ALP domain walls



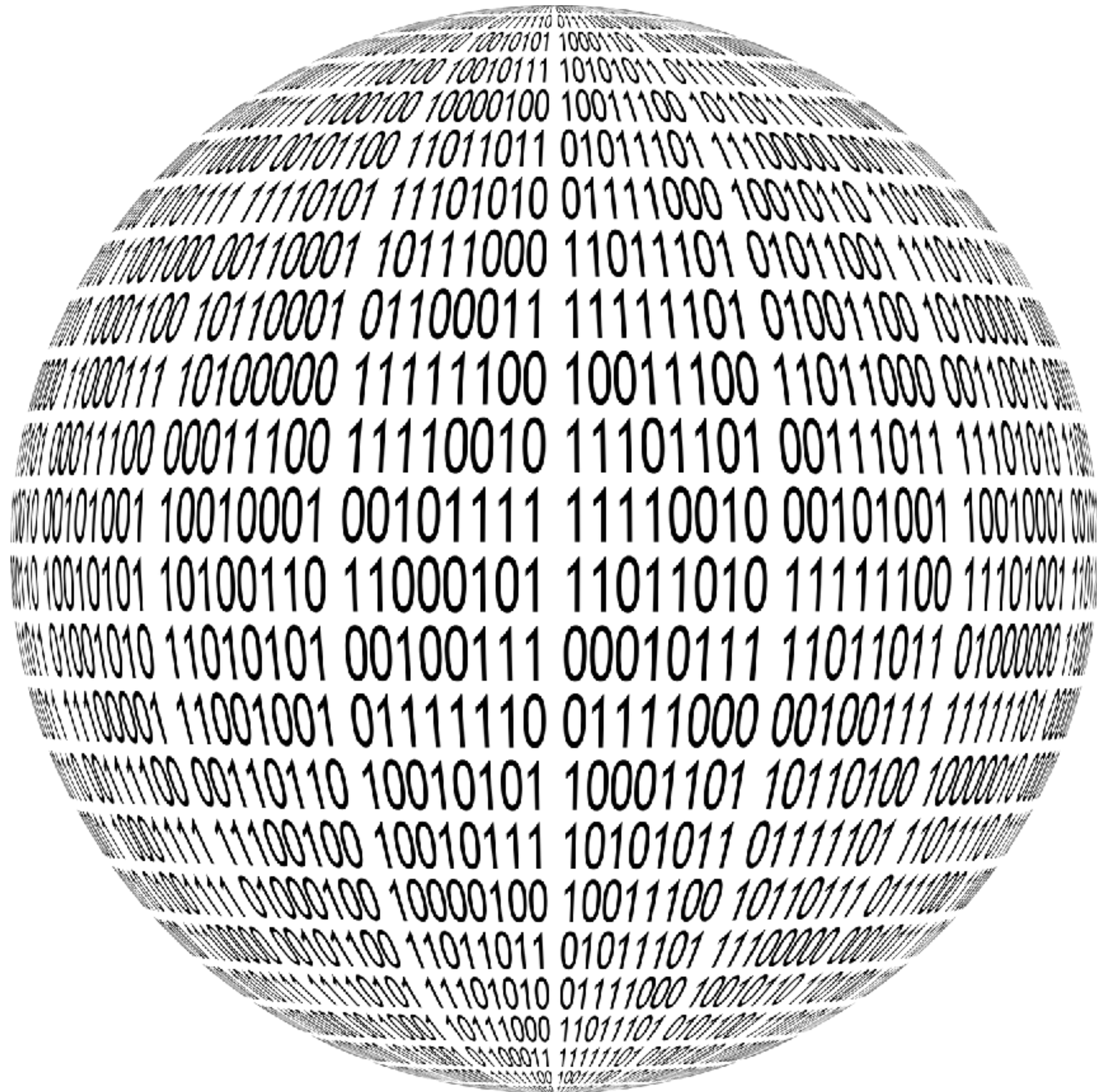
# KiloByte CB from ALP domain walls



# KiloByte CB from ALP domain walls



There will be  $O(10^{3-4})$  domains on the LSS, and the CMB polarization from each domain is either not rotated at all or rotated by a fixed angle,  $\Phi \simeq 0.42c_\gamma$  deg .



$$= 2^N, \quad N = O(10^{3-4})$$

**“KiloByte Cosmic Birefringence”  
(KBCB)**

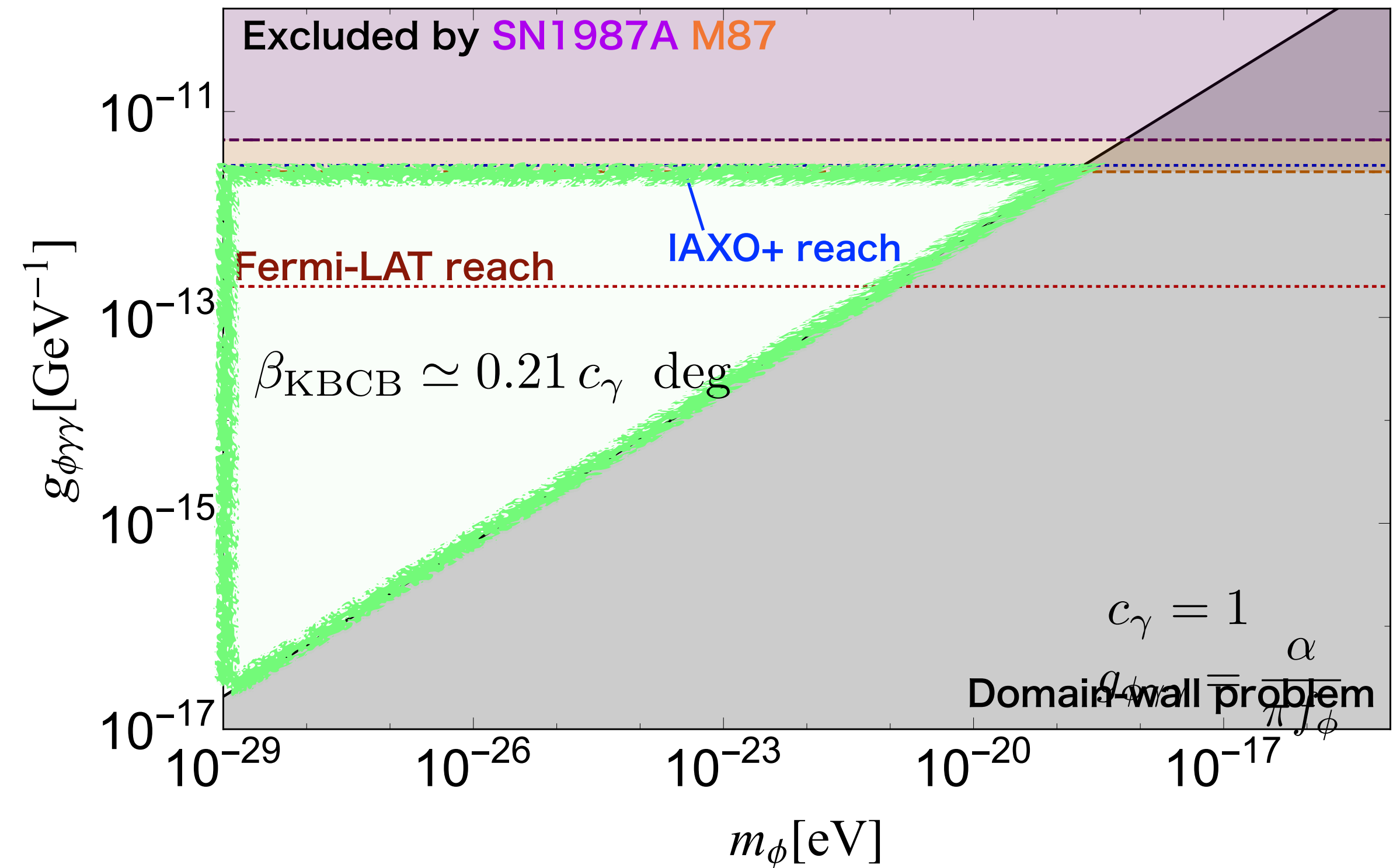
# Predictions of KBCB: isotropic CB

$$\beta = \frac{1}{4\pi} \int d\Omega \Phi(\Omega) = \frac{1}{2} c_\gamma \alpha \simeq 0.21 c_\gamma \text{ deg.}$$

independent of  $m_\phi$  and  $f_\phi$ .

$\beta_{\text{obs}} = 0.36 \pm 0.11 \text{ deg.}$  can be naturally explained for  $c_\gamma = O(1)$ .

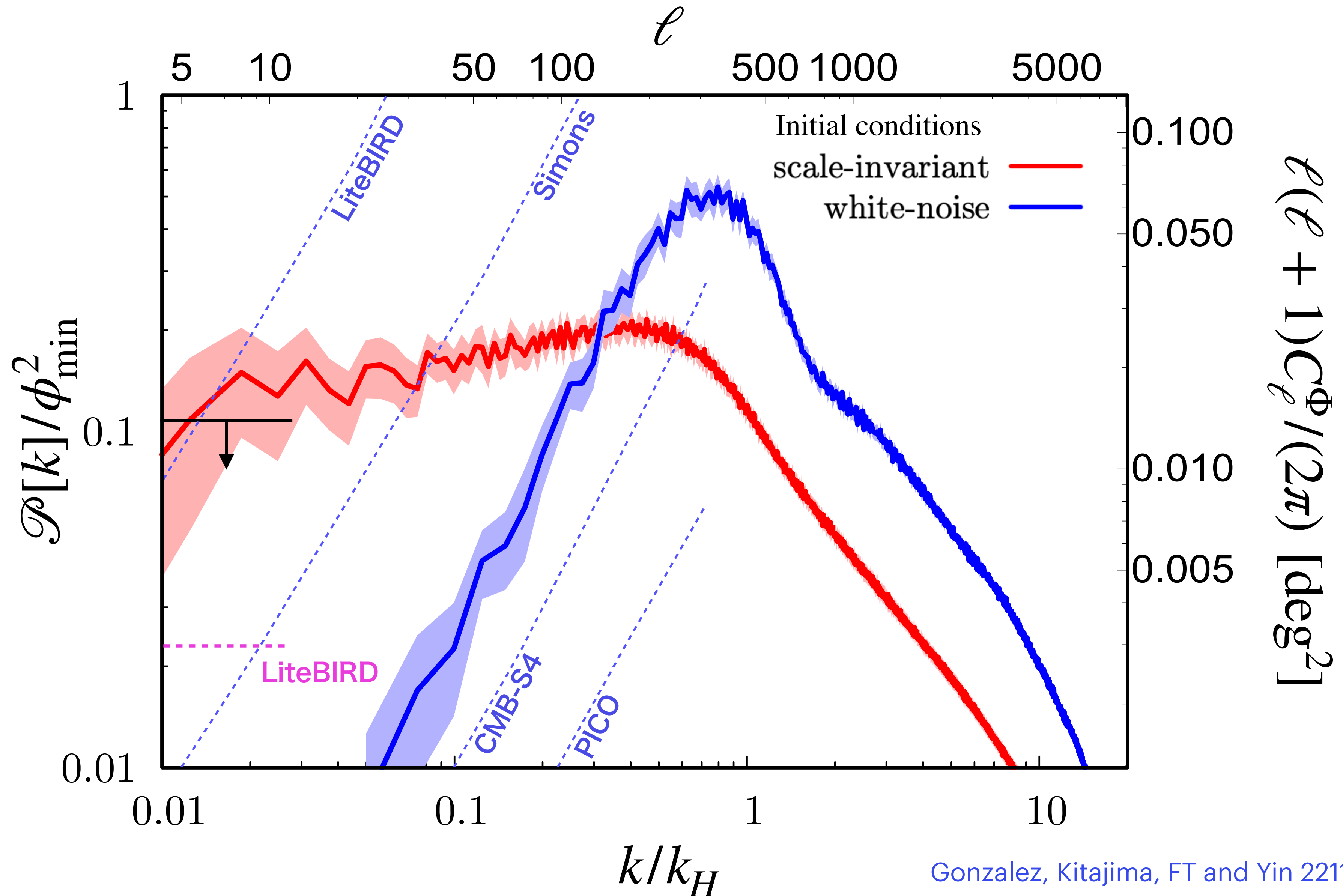
N.B. This naturally explains the closeness of  $\beta$  and  $\alpha \simeq 1/137 \text{ [rad]} \simeq 0.42 \text{ [deg]}$ .



The predicted isotropic CB is the same over the viable parameter space (green triangle).



# Predictions of KBCB: anisotropic CB



# 4. Summary

- DWs with initial inflationary fluctuations are much more stable than previously thought, due to the correlation on super-horizon scales. So, such DWs could play an important role in cosmology.
- The axion DW with inflationary fluctuations not only explains the isotropic CB, but also predicts anisotropic CB that is nearly scale-invariant at large-scales.

