Gauge Kinetic Mixing and Dark Topological Defects

Masahiro Ibe (ICRR) @ YU Workshop 2022/11/27

[*JHEP* 12 (2021) 122 : Takashi Hiramatsu, MI, Motoo Suzuki, Soma Yamaguchi]

Dark Photon (1983 Holdom ~)

- A simple extension of the SM with a massive “dark photon (γ')” that mixes with the QED (or $U(1)_Y$ gauge boson).

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{\epsilon}{2}F_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m^2_{\gamma'}A'^{\mu}A'_{\mu} + eA_{\mu}J^{\mu}_{\rm QED} + gA'_{\mu}J^{\mu}_{\rm DP}$$

Photon   Dark Photon   Kinetic Mixing   Gauge couplings

In the canonical base, i.e, $(A_\mu, A'_\mu) \to (A_\mu + \epsilon A'_\mu, A'_\mu)$

only the “dark photon - QED current coupling” appears!

- Recently, dark photon has attracted attention as it plays various roles in light dark matter models.

Typically : $m_{\gamma'} \ll \mathcal{O}(100)\mathrm{GeV}$ $\epsilon \ll 1$

Origin of Dark Photon Mass ?

- A massive "dark photon (γ')" as a Stuckelberg vector boson?

$$\mathcal{L} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{\gamma'}^2(A'_\mu - \partial_\mu\pi/m_{\gamma'})^2$$

Gauge symmetry : $A'_\mu \to A'_\mu + \partial_\mu\alpha \quad \pi \to \pi + m_{\gamma'}\alpha$

- Coupling to a Higgs boson : $\lambda_0 H^\dagger H X'_\mu X'^\mu \quad (X'^\mu = A'_\mu - \partial_\mu\pi/m_{\gamma'})$

→ perturbative unitarity of $\gamma'\gamma' \to H^\dagger H$ is violated for

$$s^{1/2} \gtrsim \sqrt{\frac{1}{\lambda_0}}m_{\gamma'}$$ [see e.g. 2204.01755 Kribs et. al.]

(Longitudinal mode : $\varepsilon_L^\mu \propto \dfrac{\sqrt{s}}{m_{\gamma'}}$)

- It seems more likely that the massive dark photon arises from spontaneous breaking of *U(1)* gauge symmetry.

What if *U(1)* gauge symmetry is embedded in *SU(2)* ?

✔ Two-Step Spontaneous Symmetry Breaking (*SU(2)* breaking → *U(1)* breaking)

SU(2) gauge symmetry breaking

$$\mathcal{L}_{\rm mix} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ -\frac{1}{4}F'^{a}_{\mu\nu}F'^{a\mu\nu} + \frac{\phi_1^a}{2\Lambda}F'^{a}_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi_1^a D^\mu\phi_1^a - \frac{\lambda_1}{4}(\phi_1\cdot\phi_1 - v_1^2)$$

Photon Dark SU(2) Kinetic Mixing SU(2) adjoint scalar ϕ_1^a $(a = 1{,}2{,}3)$

SU(2) → *U(1)* by the VEV of the adjoint scalar

$$\langle\phi_1^a\rangle = v_1\delta^{a3}$$

→ effective kinetic mixing is induced : $\epsilon = \dfrac{v_1}{\Lambda}$

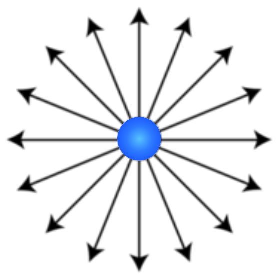
Two-step SSB model is advantageous to explain tiny kinetic mixing.

✔ The possibility that the origin of dark photons is due to two-step SSB may be an interesting story.
(*SU(2)* breaking scale $\gg$ *U(1)* breaking scale)

Topological Defects in two-step symmetry breaking ?

- At $SU(2) \rightarrow U(1)$ Breaking

  We expect "dark" Monopole as a topological defect.

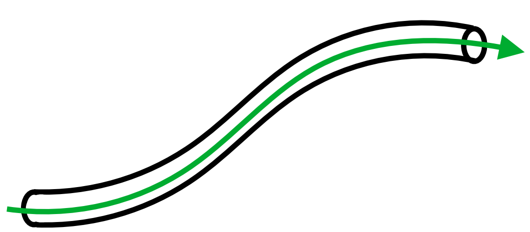


The dark monopole sources dark magnetic field

[1974 t'Hooft, Polyakov]

- At $U(1)$ Breaking

  We expect "dark" Cosmic Strings as a topological defect.

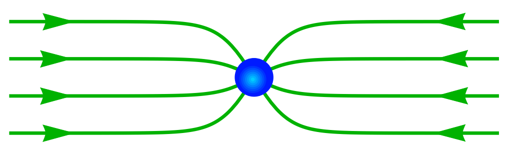


The dark magnetic field is confined along with the dark cosmic string.

[1973 Nielsen Olsen]

- At $SU(2) \rightarrow U(1) \rightarrow Z_2$ breaking (depending on how $U(1)$ is broken)

  We expect "dark" bead solution as a topological defect.



The dark magnetic field from the magnetic monopole flows into the attached cosmic strings.

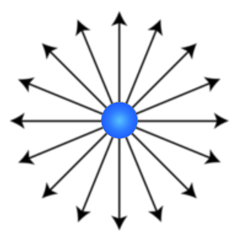
[1985 Hindmarsh & Kibble]

How do the defects in the dark sector look from the QED ?

Topological defects in the dark sector

How do they look in QED ?

SU(2) → U(1)
Dark Magnetic Monopole

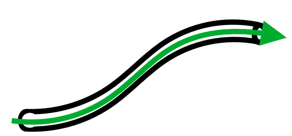


Through Kinetic Mixing

εFF'

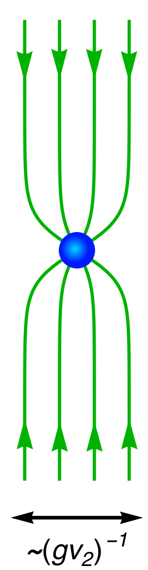
?

U(1) breaking
Dark Cosmic String



?

SU(2) → U(1) → Z_2
Dark Bead Solution



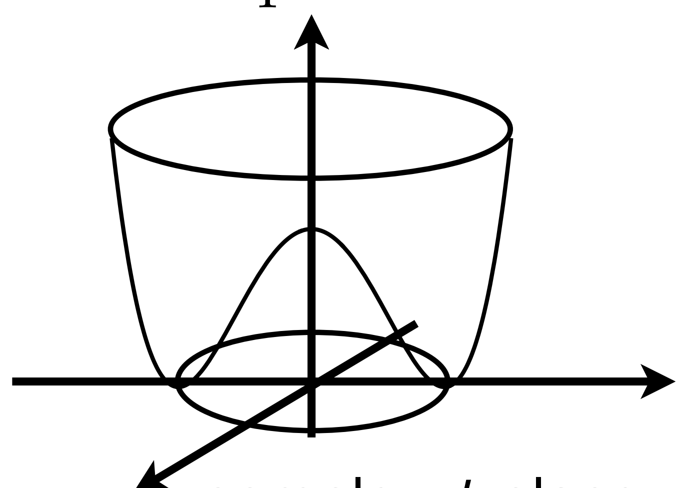
Bead solution

?

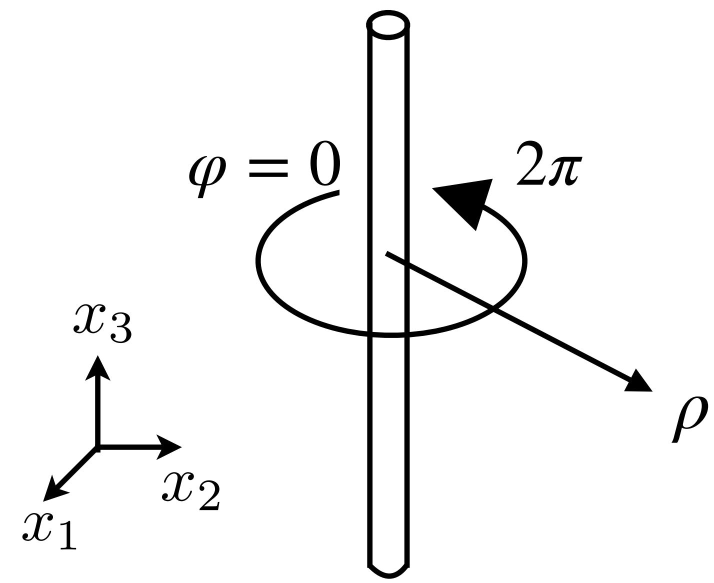
Cosmic string for $\epsilon = 0$

✓ At *U(1)* breaking, cosmic strings can be formed [1973 Nielsen Olsen]

$$V = \frac{\lambda}{4}(|\phi^2| - v^2)^2$$



complex ϕ plane



Width ~ $(gv)^{-1}$ ($g^2 \sim \lambda$)

$$\begin{cases} \phi = v h(\rho) e^{in\varphi} \ , \\ A_i' = -\dfrac{n}{g}\dfrac{\epsilon_{ij}x^j}{\rho^2} f(\rho) \ , \qquad (i,j = 1,2) \ , \end{cases} \qquad \begin{cases} h(\rho) \to 0 \ , \ (\rho \to 0) \ , \qquad h(\rho) \to 1 \ , \ (\rho \to \infty) \\ f(\rho) \to 0 \ , \ (\rho \to 0) \ , \qquad f(\rho) \to 1 \ , \ (\rho \to \infty) \end{cases}$$

U(1) symmetry is broken at $\rho \to \infty$

✓ An isolated cosmic string is stable due to the topological charge :

Π₁(U(1)) = Z

Cosmic string for $\epsilon = 0$

For $\rho \to \infty$ $\quad \partial_\varphi \phi \to in \times v e^{in\varphi}$

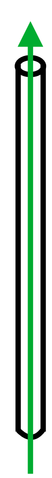
$$D_\varphi \phi = (\partial_\varphi - igA_\varphi)\phi \to 0$$ (exponentially dumped)

✔ Local string has a finite tension (= string weight per unit length)

$$\mathcal{E} = \int d^2x \left[\frac{1}{4} F_{ij} F^{ij} + |D_i \phi|^2 + V(\phi) \right] = 2\pi v^2 \times \mathcal{F}(2\lambda/g^2)$$

$$\left(\mathcal{F}(1) = 1 \quad \mathcal{F}'(x) < 0 \right)$$

✔ Dark Magnetic Flux inside the cosmic string



$$\int d^2x B'_z = \oint_{\rho\to\infty} A'_i dx^i = \frac{2\pi n}{g}$$

Cosmic string for $\epsilon \neq 0$?

- Equation of motion :

$$\left.\begin{aligned} &\partial_\mu F^{\mu\nu} - \epsilon \partial_\mu F'^{\mu\nu} = e J^\nu_{\rm QED} \ , \\ &\partial_\mu \tilde{F}^{\mu\nu} = 0 \ , \end{aligned}\right\} \text{QED field strength}$$

$$\left.\begin{aligned} &\partial_\mu F'^{\mu\nu} - \epsilon \partial_\mu F^{\mu\nu} = g J^\nu_{\rm D} \ , \\ &\partial_\mu \tilde{F}'^{\mu\nu} = 0 \ . \end{aligned}\right\} \text{Dark photon field strength}$$

We are interested in a vacuum configuration → $J^\mu_{\rm QED} = 0$

EOM is reduced to $\begin{cases} \partial F^{\mu\nu} = \epsilon \partial F'^{\mu\nu} \\ (1-\epsilon^2)\partial_\mu F'^{\mu\nu} = g J^\nu_{\rm D} \end{cases}$

$$J^i_{\rm D} = i\phi D_i \phi^\dagger - i\phi^\dagger D_i \phi = 2v^2 n \frac{\epsilon_{ij} x_j}{\rho^2} h^2 (f-1)$$

- The cosmic string solution for $\epsilon \neq 0$ is obtained by just rescaling

$$g_s = \frac{g}{\sqrt{1-\epsilon^2}} \qquad g A'_\mu = g_s A'_{s\mu}$$

(EOM of ϕ is not changed)

Cosmic string for $\epsilon \neq 0$?

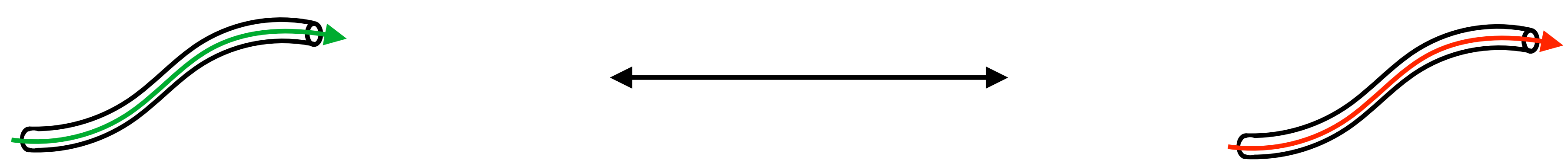
✔ The magnetic flux of $F^{\mu\nu}$ is induced ($F^{\mu\nu} = \epsilon F'^{\mu\nu}$)

$$\oint A'_{s\mu} dx^\mu = \frac{2\pi n}{g_s} \quad \rightarrow \quad W_{\rm QED} = \oint e A_\mu dx^\mu = \frac{g_s \epsilon e}{g} \oint A'_{s\mu} dx^\mu = \frac{2\pi n \epsilon e}{g}$$

✔ For $\epsilon \neq 0$, dark string is associated with non-vanishing QED magnetic flux !

U(1) breaking
Dark Cosmic String

QED magnetic flux is
Induced



$$F^{\mu\nu} = \epsilon F'^{\mu\nu}$$

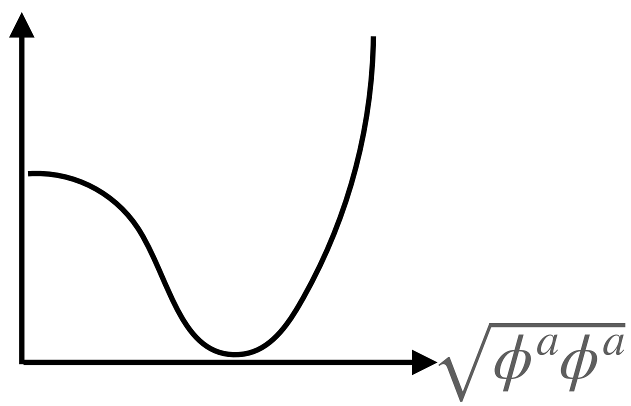
✔ Around the dark cosmic string, a QED charged particle feels through the Aharonov-Bohm phase !

$$e^{iqW_{\rm QED}} \qquad\qquad qW_{\rm QED} = \frac{2\pi n q \epsilon e}{g}$$

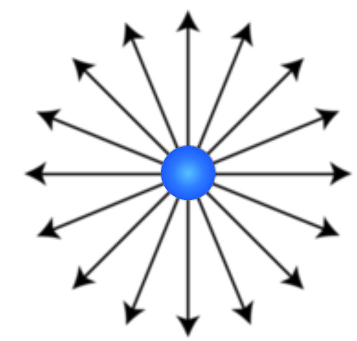
Magnetic Monopole for $\epsilon = 0$

✔ At *SU(2)* → *U(1)* breaking, magnetic monopole can be formed [1974 t'Hooft, Polyakov]

$$V = \frac{\lambda}{4}(\phi^a\phi^a - v^2)^2$$



(dark) magnetic monopole



Size ~ (gv)$^{-1}$ (g^2 ~ λ)

$$\begin{cases} \phi^a = vH(r)\dfrac{x^a}{r} \ , \\ A_i'^a = \dfrac{1}{g}\dfrac{\epsilon^{aij}x^j}{r^2}F(r) \ , \quad (i,j = 1,2,3) \end{cases}$$

$$\begin{cases} H(r) \to 0 \ , \ (r \to 0) \ , \quad H(r) \to 1 \ , \ (r \to \infty) \ , \\ F(r) \to 0 \ , \ (r \to 0) \ , \quad F(r) \to 1 \ , \ (r \to \infty) \ . \end{cases}$$

SU(2) symmetry is broken down to U(1) at $r \to \infty$

✔ An isolated magnetic monopole is stable due to the topological charge :

Π₂ (SU(2)/U(1)) = Z

Magnetic Monopole for $\epsilon = 0$

- Dark magnetic field around the dark monopole :

  Effective *U(1)$_D$* field strength : $\mathcal{F}'_{\mu\nu} \equiv \frac{1}{v}\phi^a F'^a_{\mu\nu}$,

$$\mathcal{F}'^{ij} = -\frac{1}{g}\frac{\epsilon^{ijk}x^k}{r^3}(2F - F^2)H \ , \quad (i,j = 1,2,3)$$

- Dark magnetic charge :

$$Q'_M = \frac{1}{2}\int_{r\to\infty} dS_{ij}\mathcal{F}'^{ij} = -\frac{4\pi}{g}$$

- The Bianchi Identity of the effective *U(1)$_D$* field strength

$\partial_\mu \tilde{\mathcal{F}}'^{\mu\nu} = 0$ Is satisfied only at $r \gg (gv)^{-1}$

- Dark magnetic monopole mass :

$$M_M = \frac{4\pi v}{g^2}\mathcal{F}_M(\lambda/g^2) \quad (\mathcal{F}_M(0) = 1 \quad \mathcal{F}'_M(x) > 0 \ )$$

Magnetic Monopole for $\epsilon \neq 0$?

- Equation of motion around the monopole :

satisfied in the entire space

$$\partial_\mu F^{\mu\nu} - \epsilon \partial_\mu \mathcal{F}'^{\mu\nu} = 0$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \ .$$

} QED field strength

$$\partial_\mu \mathcal{F}'^{\mu\nu} = 0$$

$$\partial_\mu \tilde{\mathcal{F}}'^{\mu\nu} \neq 0$$

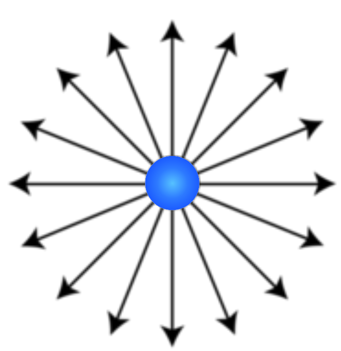
} Effective dark photon field strength

($\partial_\mu \tilde{\mathcal{F}}'^{\mu\nu} = 0$ only for $r \gg (gv)^{-1}$)

- Nothing is induced to the QED sector…

dark magnetic monopole

QED sector



Nothing…

QED satisfies the usual Bianchi identity → cannot have monopoles

[see also arXiv:0902.3615 Brummer and Jaeckel]

Bead solution for $\epsilon = 0$

U(1) breaking in a model with two *SU(2)* adjoint scalars

$$V = \frac{\lambda_1}{4}(\phi_1 \cdot \phi_1 - v_1^2) + \frac{\lambda_2}{4}(\phi_2 \cdot \phi_2 - v_2^2) + \frac{\kappa}{2}(\phi_1 \cdot \phi_2)^2$$

$$\kappa > 0$$

Hierarchical Breaking : $v_1 \gg v_2$

✔ Trivial vacuum configuration :

Step 1 : *SU(2)* → *U(1)* by $\langle \phi_1^a \rangle = v_1 \delta^{a3}$

$\tilde{\phi} = \frac{1}{\sqrt{2}}\left(\phi_2^1 - i\phi_2^2\right)$ has the *U(1)* charge 1

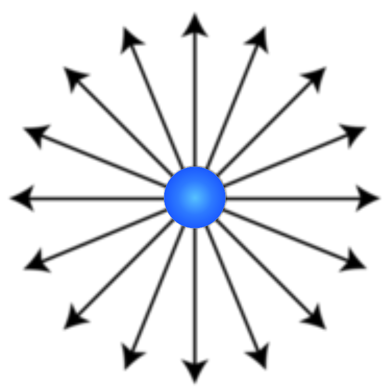
For $\kappa > 0$, $\phi_1 \cdot \phi_2 = 0$ direction is preferred :

Step 2 : *U(1)* → Z_2 (center of *SU(2)*) by $\langle \phi_2^a \rangle = v_2 \delta^{a1}$

Bead solution for $\epsilon = 0$

✔ What happens to the monopole solution ?

Step 1 : Monopole solution at the energy scale v_1



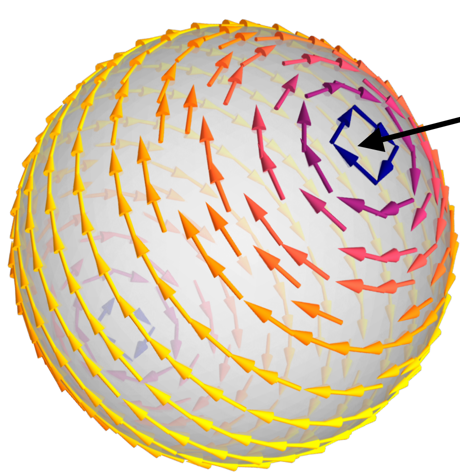
$$\phi_1^a = v_1 H(r) \frac{x^a}{r}$$

$$|\phi_1(r \to \infty)| \to v_1$$

Step 2 : *U(1) breaking at the energy scale* v_2

$\phi_1 \cdot \phi_2 = 0$ && $|\phi_2| \to v_2$ at $r \to \infty$?

→ Such a configuration conflicts with the Hairy-ball theorem
(on S^2 , there is no tangent vector field with a constant magnitude)



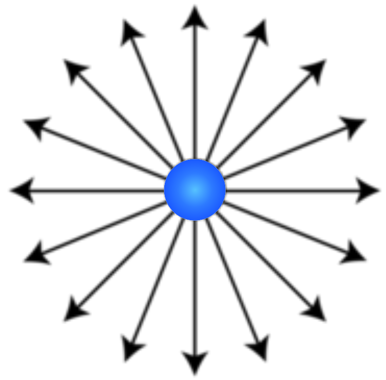
$\phi_2^a = 0$

U(1) symmetry restoration at some points on S^2 !

Bead solution for $\epsilon = 0$

- ✓ Consider "Combed" gauge

Hedgehog gauge



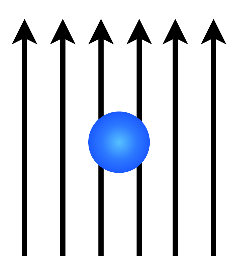
$$A'^a_r \to 0 \ ,$$

$$A'^a_\theta \to \frac{1}{g}(s_\varphi, -c_\varphi, 0) \ ,$$

$$A'^a_\varphi \to \frac{1}{g}(s_\theta c_\theta c_\varphi, s_\theta c_\theta s_\varphi, -s_\theta^2) \ ,$$

$$\phi_1^a \to v(s_\theta c_\varphi, s_\theta s_\varphi, c_\theta) \ ,$$

Northern hemisphere U_N



$$\phi_1^a \to \phi_N^a = v\delta^{a3} \ ,$$

$$A'^a \to A'^a_N = \frac{1}{g}\delta^{a3}(\cos\theta - 1)d\varphi$$

U(1) is given by $A_N^{3\mu}$

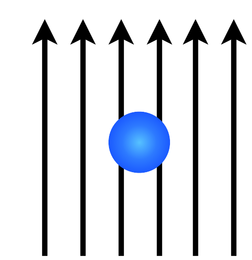
At the equator $\theta \sim \pi/2$

$$A'^3_S = A'^3_N + \frac{2}{g}d\varphi \ .$$

Transition function of U(1) on S² at $\theta \sim \pi/2$

$$t_{NS} = e^{2i\varphi}$$

Southern hemisphere U_S



$$\phi_1^a \to \phi_S^a = v\delta^{a3} \ ,$$

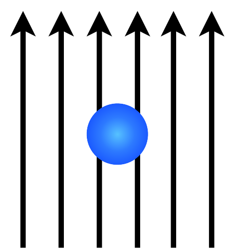
$$A'^a \to A'^a_S = \frac{1}{g}\delta^{a3}(\cos\theta + 1)d\varphi$$

U(1) is given by $A_S^{3\mu}$

Bead solution for $\epsilon = 0$

✔ Trivial ϕ_2^a configuration in the northern hemisphere $\tilde{\phi} = \frac{1}{\sqrt{2}}(\phi_2^1 + i\phi_2^2)$

Northern hemisphere U_N



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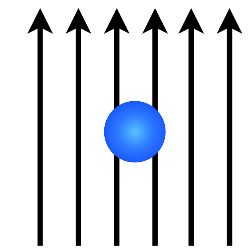


$$\phi_1^a \to \phi_S^a = v\delta^{a3} \ ,$$

$$A'^a \to A'^a_S = \frac{1}{g}\delta^{a3}(\cos\theta + 1)d\varphi$$

U(1) is given by $A_S^{3\mu}$

Trivial configuration in U_N

$$\tilde{\phi}_N = \frac{v_2}{\sqrt{2}}$$

$$A'^3_{Ni} = 0$$

Transited at the equator $\longrightarrow$

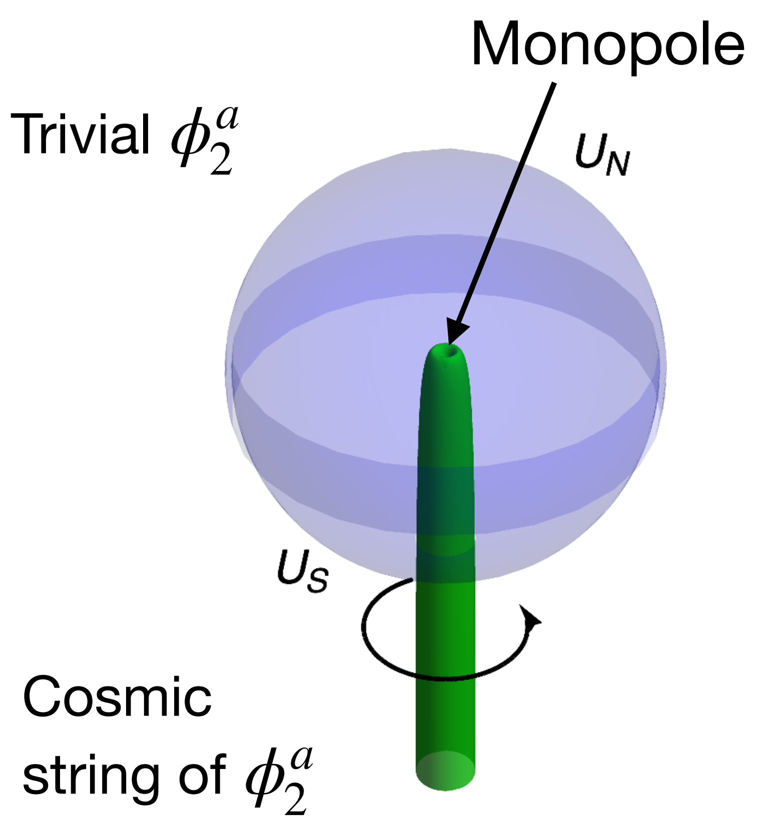
Non trivial winding in U_S

$$\tilde{\phi}_S = e^{2i\varphi}\tilde{\phi}_N = e^{2i\varphi}\frac{v_2}{\sqrt{2}} \ ,$$

$$A'^3_{Si}dx^i = \frac{2}{g}d\varphi \ ,$$

Bead solution for $\epsilon = 0$

✔ Trivial ϕ_2^a configuration in the northern hemisphere $\tilde{\phi} = \frac{1}{\sqrt{2}}(\phi_2^1 + i\phi_2^2)$

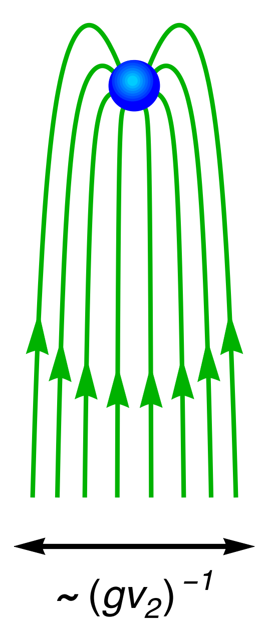


✔ Cosmic string in the southern hemisphere has the winding number 2.

$$\tilde{\phi}_S \to e^{2i\varphi}\frac{v_2}{\sqrt{2}}$$

✔ Dark magnetic flux of the magnetic monopole is confined in the half cosmic string

$$Q'_M = -\oint A^3_{S\varphi} d\varphi = -\frac{4\pi}{g}$$

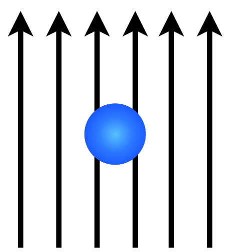


✔ This configuration is not stable.
The monopole is pulled by the string.

Bead solution for $\epsilon = 0$

✔ Cosmic string of ϕ_2^a configuration in the northern hemisphere

Northern hemisphere U_N



$$\phi_1^a \to \phi_N^a = v\delta^{a3} ,$$

$$A'^a \to A'^a_N = \frac{1}{g}\delta^{a3}(\cos\theta - 1)d\varphi$$

U(1) is given by $A_N^{3\mu}$

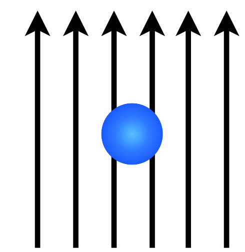
At the equator $\theta \sim \pi/2$

$$A'^3_S = A'^3_N + \frac{2}{g}d\varphi \ .$$

Transition function of U(1) on S² at $\theta \sim \pi/2$

$$t_{NS} = e^{2i\varphi}$$

Southern hemisphere U_S



$$\phi_1^a \to \phi_S^a = v\delta^{a3} ,$$

$$A'^a \to A'^a_S = \frac{1}{g}\delta^{a3}(\cos\theta + 1)d\varphi$$

U(1) is given by $A_S^{3\mu}$

Cosmic string in U_N

$$\tilde{\phi}_N \to e^{-i\varphi}\frac{v_2}{\sqrt{2}}$$

$$A'^3_{Ni}dx^i \to -\frac{1}{g}d\varphi$$

(winding number -1)

Transited at the equator $\longrightarrow$

Non trivial winding in U_S

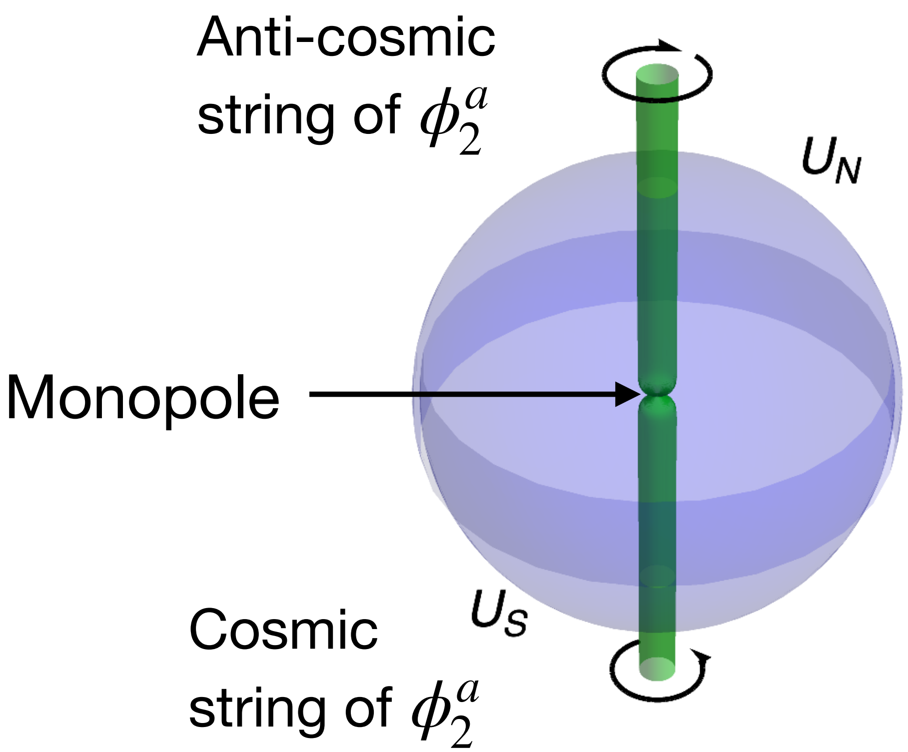
$$\tilde{\phi}_S \to e^{2\varphi}\tilde{\phi}_N = \frac{v_2}{\sqrt{2}}e^{i\varphi} ,$$

$$A'^3_{Si}dx^i \to A'^3_{Ni}dx^i + \frac{2}{g}d\varphi = \frac{1}{g}d\varphi ,$$

(winding number +1)

Bead solution for $\epsilon = 0$

- Cosmic string of ϕ_2^a configuration in the northern hemisphere



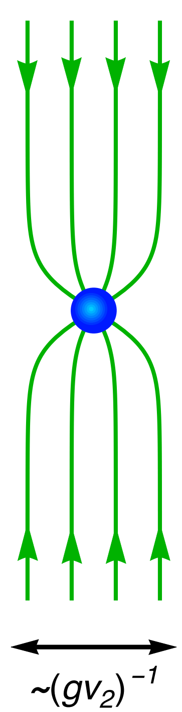
- Monopole is attached by cosmic and anti-string

$$\tilde{\phi}_N \to e^{-i\varphi}\frac{v_2}{\sqrt{2}} \qquad \text{in } U_N$$

$$\tilde{\phi}_S \to e^{i\varphi}\frac{v_2}{\sqrt{2}} \qquad \text{in } U_S$$

- Dark magnetic flux of the magnetic monopole is confined in the two opposite cosmic strings

$$Q'_M = \oint A^3_{N\varphi} d\varphi - \oint A^3_{S\varphi} d\varphi = -\frac{4\pi}{g}$$



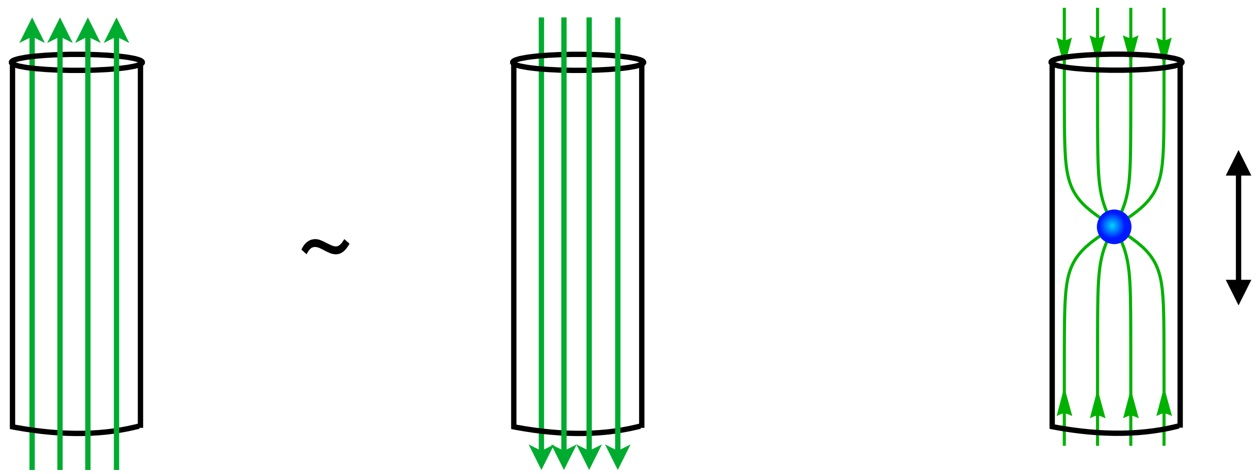
- This configuration is stable !

bead solution ! [1985 Hindmarsh & Kibble]

$\Pi_1(Z_2) = Z_2$

Bead solution for $\epsilon = 0$

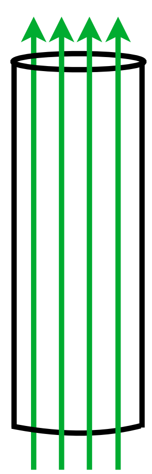
- ***How $\Pi_1(Z_2) = Z_2$ is realized ?***
  - String solution is equivalent with an anti-string solution



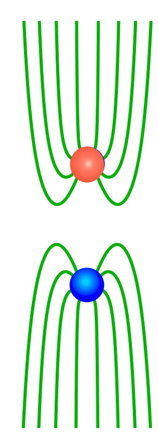
Moving monopole at $x_3 = -\infty$ to $x_3 = \infty$.

Winding # 1 $\sim$ Winding # -1

  - String solution with even winding number is broken by pair creation of the monopole anti-monopole



Winding # 2 $\rightarrow$



Monopole-Anti-Monopole pair creation

Bead solution for $\epsilon = 0$

- What if *U(1)* is broken by a VEV of fundamental representation ?

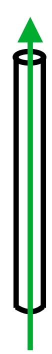
SU(2) → *U(1)* → *Nothing* → No stable soliton is expected

For $\rho \to \infty$

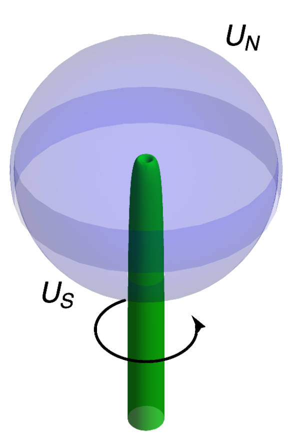
$$\partial_\varphi \phi_F = in \times \frac{1}{\sqrt{2}} v_2 e^{in\varphi}$$

$$D_\varphi \phi_F = \left(\partial_\varphi - \frac{1}{2} g A_\varphi \right) \phi_F \to 0$$

Magnetic flux of is twice larger than the case of the adjoint scalar !



$$\int d^2 x B'_z = \oint_{\rho\to\infty} A'_i dx^i = \frac{4\pi n}{g}$$

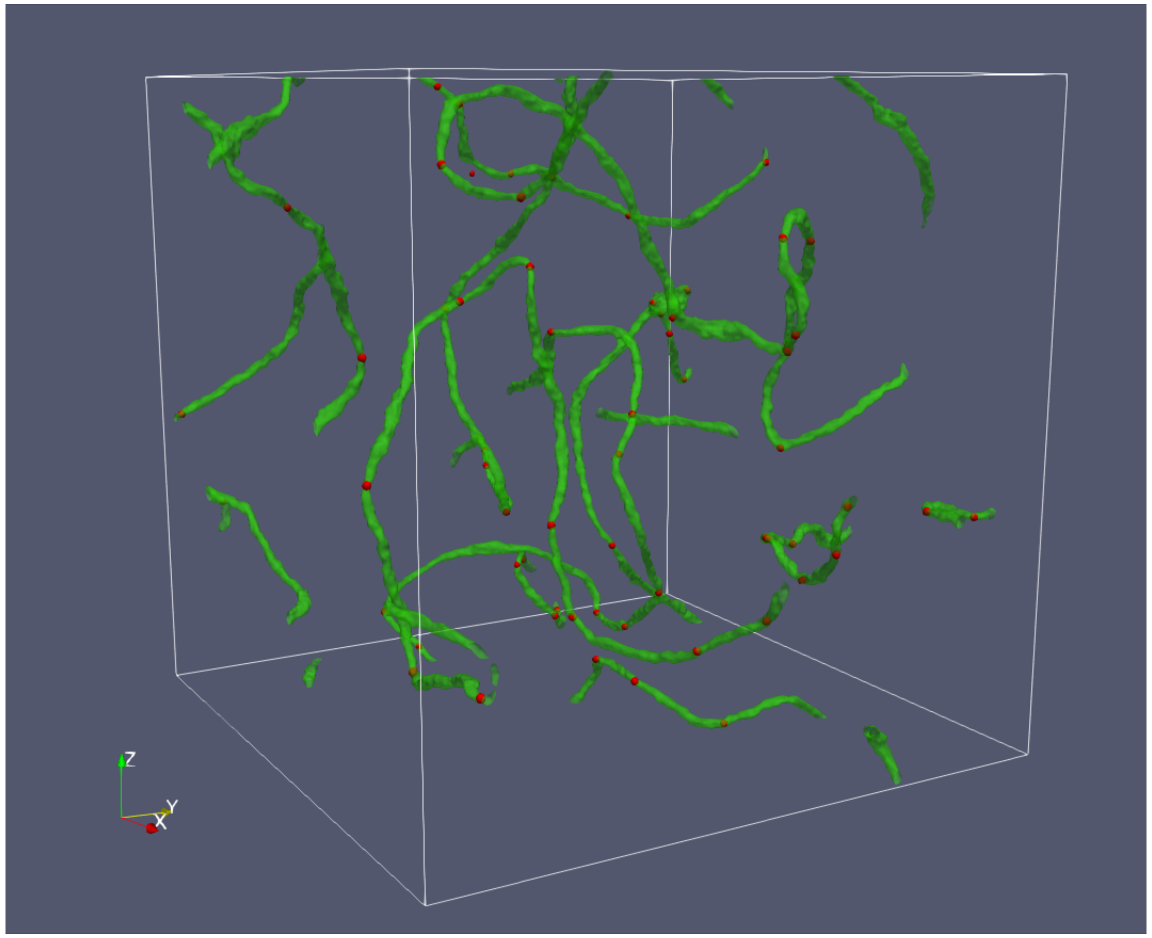


Only unstable composite monopole-cosmic string can be formed !

Bead solution for $\epsilon = 0$

- ✔ Classical Lattice Simulation

v_2/v_1	0.3
λ_{10}	1
λ_{20}	1
κ_0	2
ϵ	0.2
g	$1/\sqrt{2}$



Red points
= Monopole

Green Lines
= Cosmic Strings

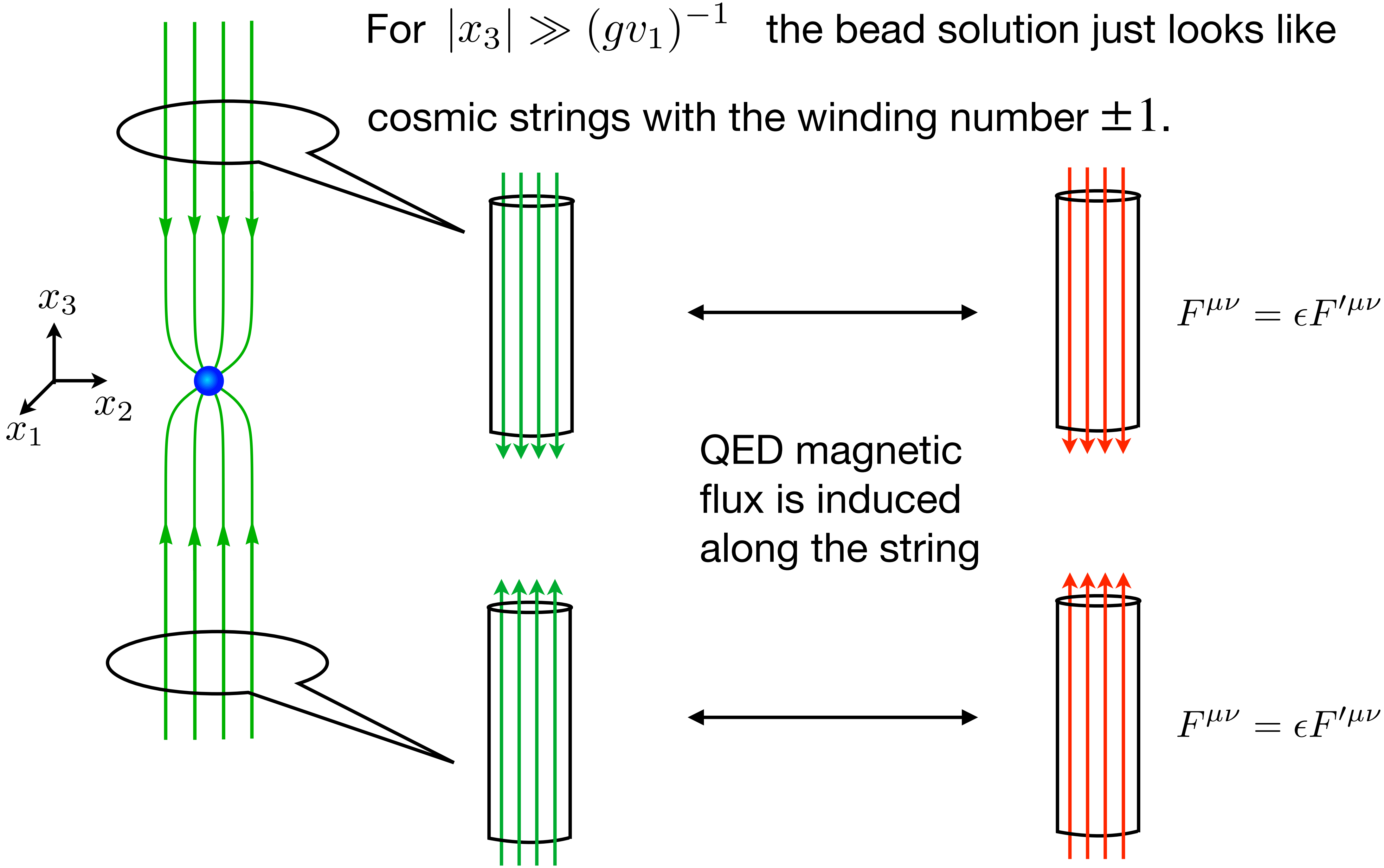
Figure 6. *Cosmic beads network, i.e., the necklace. The red and green surfaces are the isosurface of* $|\phi_1| = 0.5v_1$ *and* $|\phi_2| = 0.06v_1$*, respectively. The figure shows that the magnetic monopoles (or the beads) appearing as red points are connected by the cosmic strings.*

Starting from random configuration (i.e., ~ thermalized configuration),
we confirmed the formation of the beads network = necklace

Bead solution for $\epsilon \neq 0$?

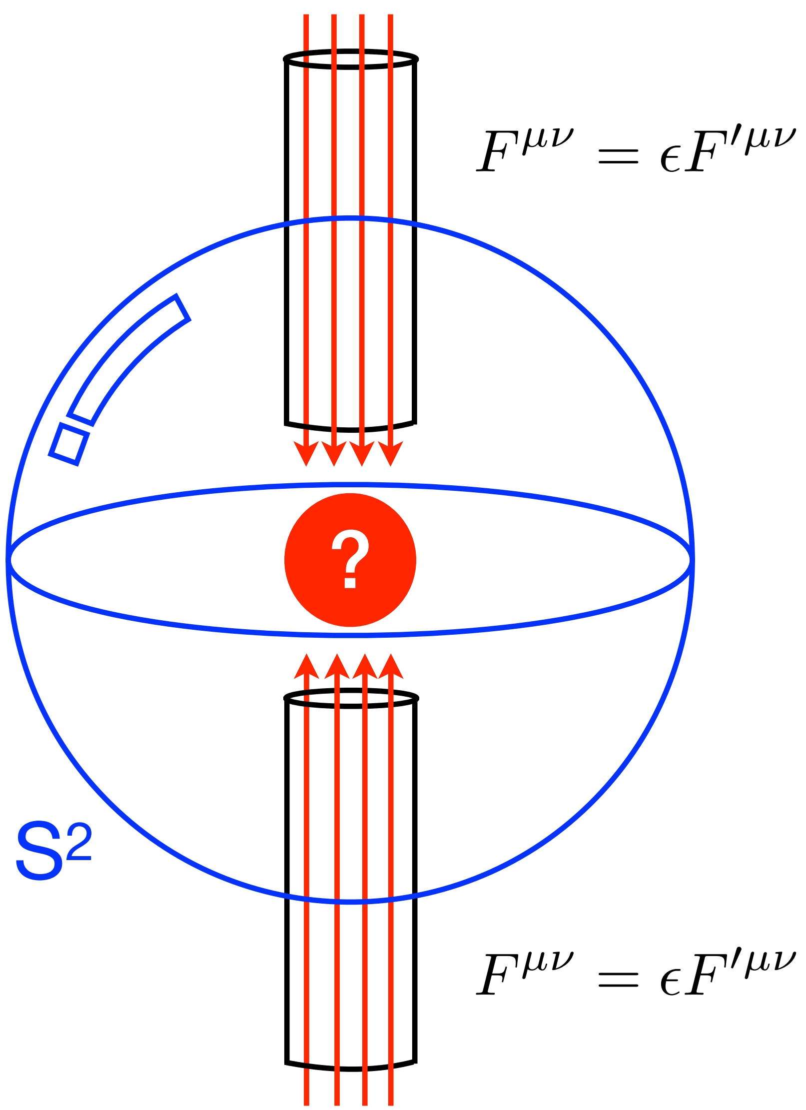
- How does the necklace look like from QED sector ?

For $|x_3| \gg (gv_1)^{-1}$ the bead solution just looks like cosmic strings with the winding number ± 1.



Bead solution for $\epsilon \neq 0$?

- How does the necklace look like from QED sector ?



- Flux in string and anti-string :

$$\int_{S^2} F_{\rm QED} = \epsilon Q'_M = -\epsilon \frac{4\pi}{g} \quad ?$$

- Bianchi identity of QED is satisfied "everywhere" in R^3 :

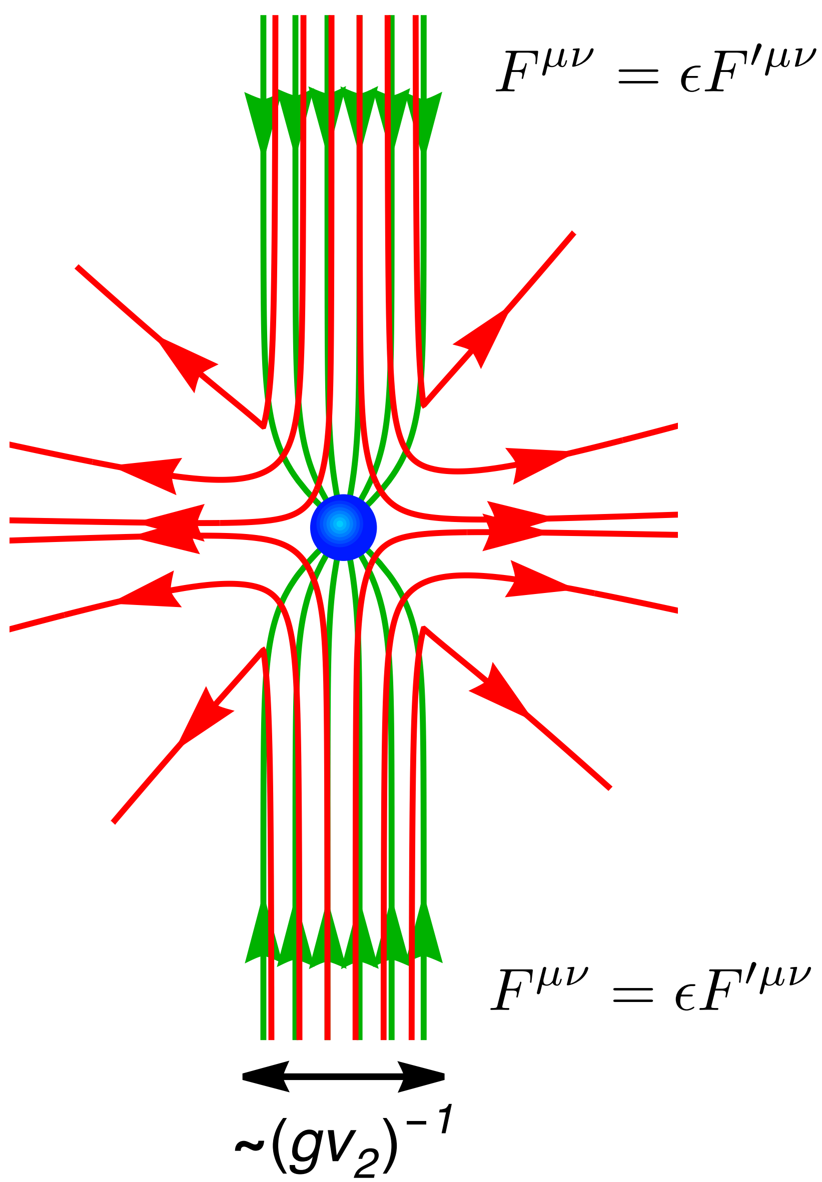
$$\int_{S^2} F_{\rm QED} = 0 \quad !$$

- QED magnetic flux needs to source out from the monopole !

Bead solution for $\epsilon \neq 0$?

- How does the necklace look like from QED sector ?

Rough sketch of the
QED magnetic flux (RED)



- Due to the Bianchi identity of QED magnetic flux lines are not broken

$$\int_{S^2} F_{\rm QED} = 0$$

- QED magnetic flux sources out from the monopole.

- It looks like a QED monopole from a distance !

- It is attached by the visible strings in which QED magnetic fields flow.

→ Pseudo QED monopole

Bead solution for $\epsilon \neq 0$?

- ✔ Magnetic necklace

If the two step symmetry breaking takes place for $v_1 \gg v_2$,
we expect a network of pseudo-magnetic monopole-network

Magnetic necklace

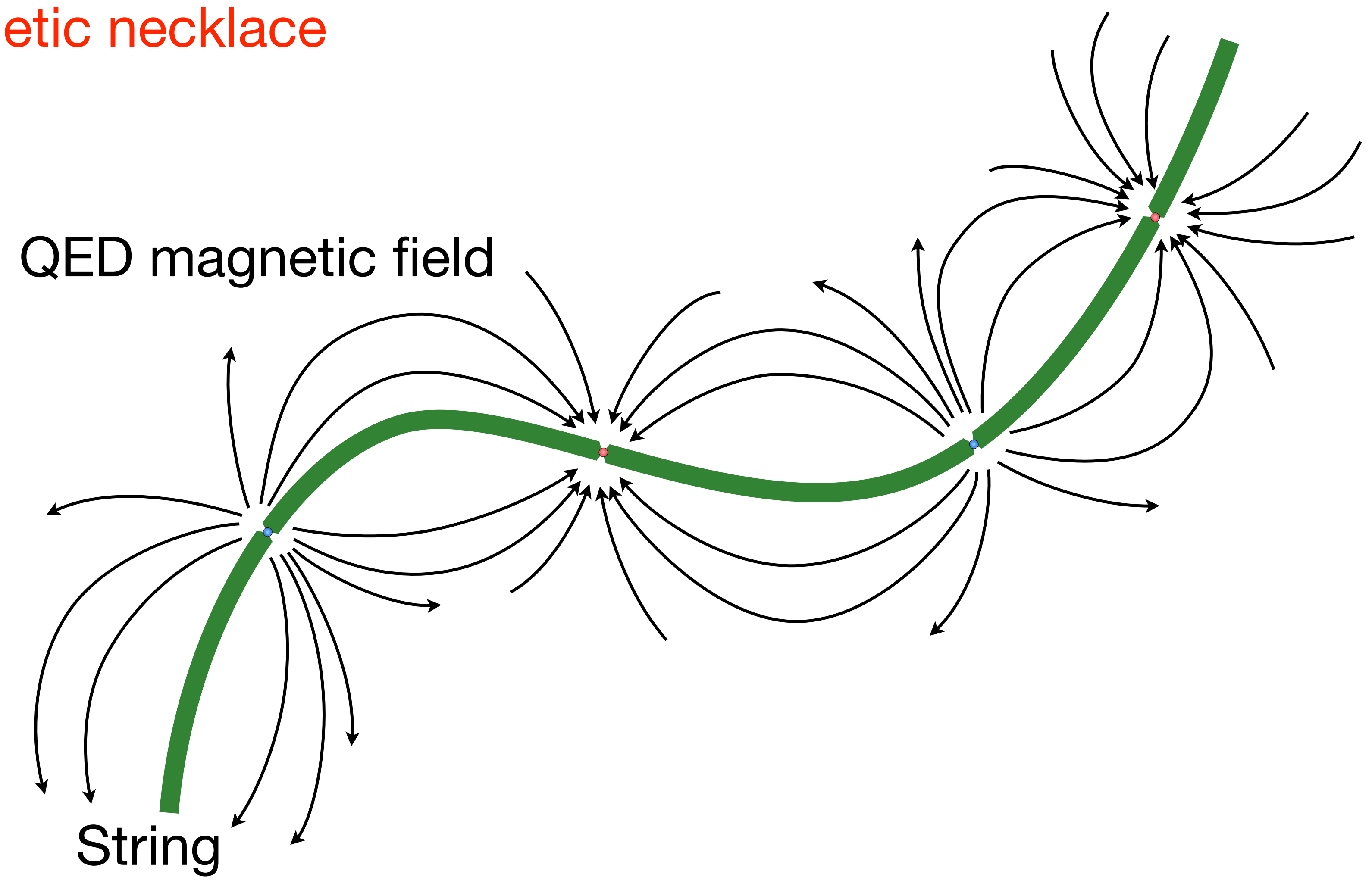
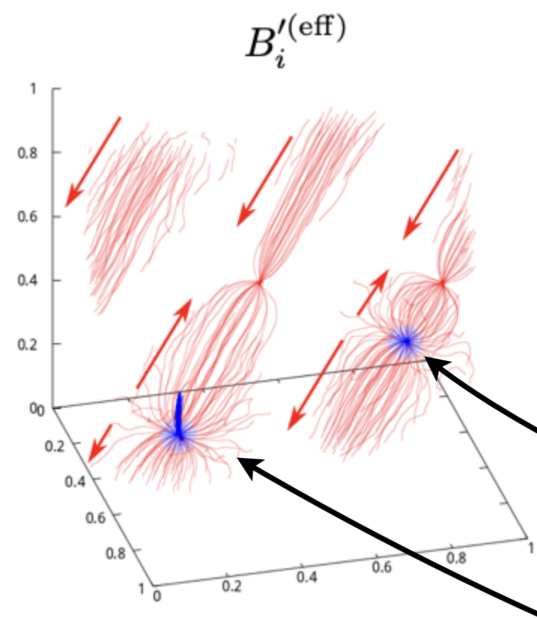


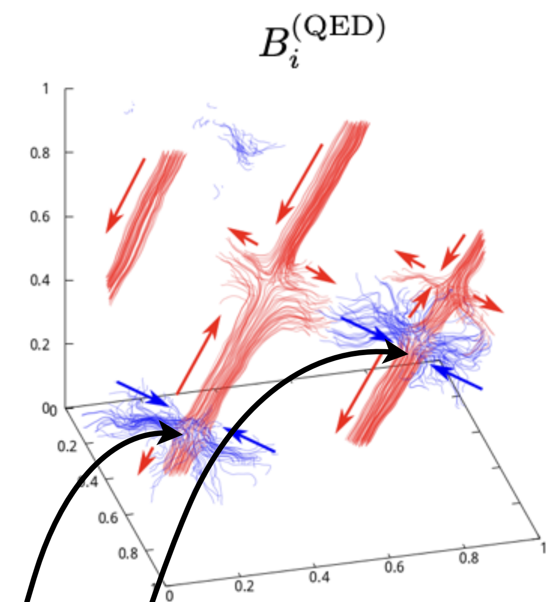
Figure 5. *A schematic picture of the magnetic necklace. The dark magnetic flux is trapped inside the necklace (the green line). In the presence of the kinetic mixing, the QED magnetic flux (the black lines) leaks out from the positions of the (anti-)monopole.*

Classical Lattice Simulation

Stream lines around the monopoles



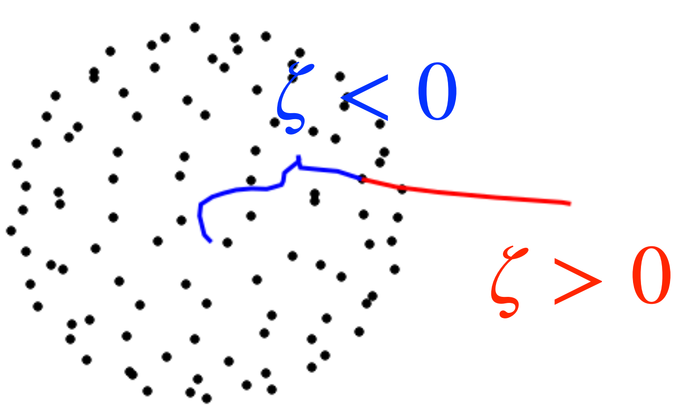
dark magnetic flux



QED magnetic flux

After obtaining the final configuration, we choose random points in a sphere of a radius about 3x of the monopole

(starting from those monopoles)

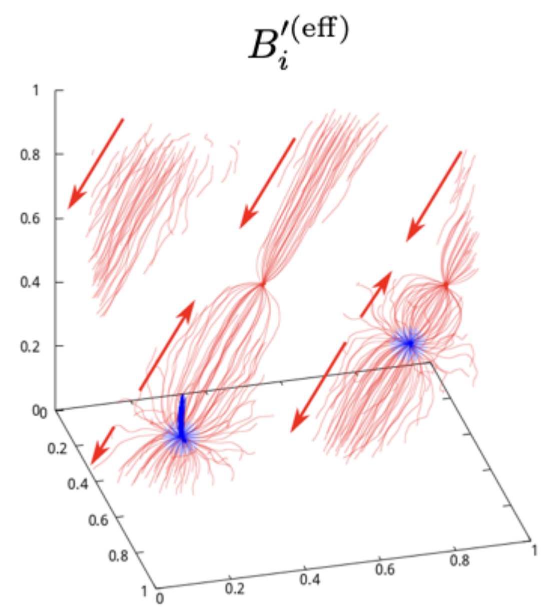


Then, we solve vector flow

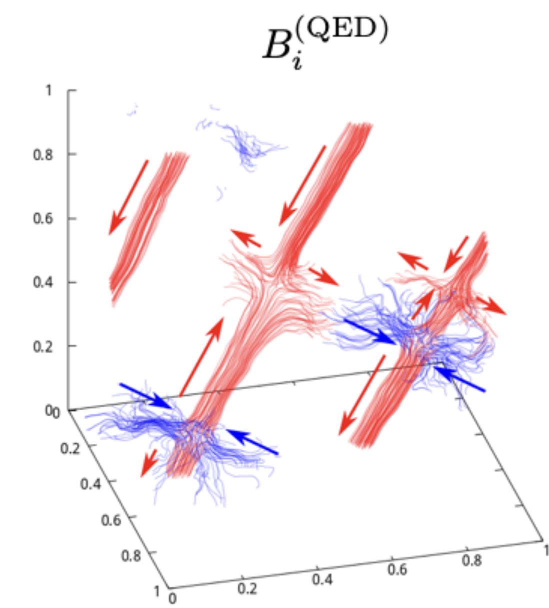
$$\frac{d\boldsymbol{x}_s}{d\zeta} = \boldsymbol{B}_i^{(\mathrm{eff})}(\boldsymbol{x}_s(\zeta))$$

Classical Lattice Simulation

Stream lines around the monopoles



dark magnetic flux



QED magnetic flux

- ✓ dark magnetic flux : converge to the monopole points
- ✓ QED magnetic flux : flowing out or absorbed out

→We confirmed the formation of pseudo monopoles

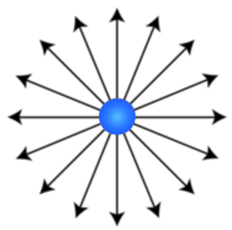
Summary

Topological defects in the dark sector

Through Kinetic Mixing
εFF'

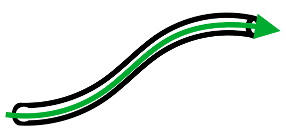
How do they look in QED ?

SU(2) → U(1)
Dark Magnetic Monopole

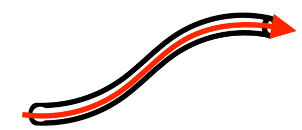


Nothing

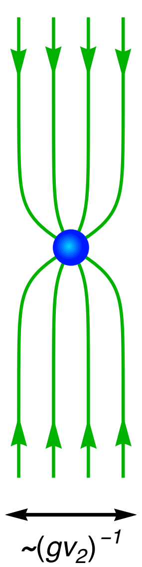
U(1) breaking
Dark Cosmic String



$$F^{\mu\nu} = \epsilon F'^{\mu\nu}$$

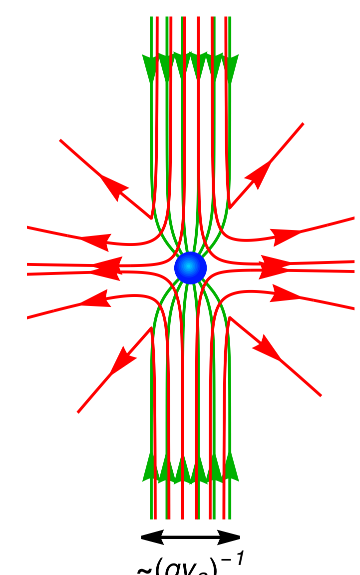


SU(2) → U(1) → Z_2
Dark Bead Solution



Bead solution

Pseudo QED monopole



Back UP

Cosmic string for $\epsilon \neq 0$ in canonical base

We can move to the canonical base (X_μ, X'_μ) by

$$A_\mu = X_\mu + \epsilon A'_\mu \ ,$$
$$A'_\mu = \frac{1}{\sqrt{1-\epsilon^2}} X'_\mu \ .$$

In this basis, the EOM of X_μ and X'_μ decouple :

$$\left.\begin{aligned} \partial_\mu F_X^{\mu\nu} &= e J^\nu_{\mathrm{QED}} \\ \partial_\mu \tilde{F}_X^{\mu\nu} &= 0 \end{aligned}\right\} \text{ QED field strength}$$

$$\left.\begin{aligned} \partial_\mu F_X'^{\mu\nu} &= g_s J^\nu_D + \epsilon e J^\nu_{\mathrm{QED}} \\ \partial_\mu \tilde{F}_X'^{\mu\nu} &= 0 \end{aligned}\right\} \text{ Dark photon field strength}$$

Cosmic string for $\epsilon \neq 0$ in canonical base

The magnetic flux of the dark string solution

$$\oint X'_{s\mu} dx^\mu = \frac{2\pi n}{g_s}$$

The AB phase the test QED charged particle (charge = q) feels

$$qW_{\mathrm{QED}} = \frac{q\epsilon e}{\sqrt{1-\epsilon^2}} \oint X'_{\mu} dx^\mu = \frac{2\pi n q \epsilon e}{g}$$

which is the same with the non-canonical basis analysis.

Combing gauge transformation

To comb the hedgehog, we use the gauge transformation in U_N and U_S

$$g_N = \begin{pmatrix} c_{\theta/2} & e^{-i\varphi} s_{\theta/2} \\ -e^{i\varphi} s_{\theta/2} & c_{\theta/2} \end{pmatrix} , \qquad g_S = \begin{pmatrix} e^{i\varphi} c_{\theta/2} & s_{\theta/2} \\ -s_{\theta/2} & e^{-i\varphi} c_{\theta/2} \end{pmatrix} .$$

$$\phi^a \tau^a \to \phi^a_{N,S} \tau^a = g_{N,S} \phi^a \tau^a g^\dagger_{N,S} \ ,$$
$$A'^a_i \tau^a \to A'^a_{N,S\,i} \tau^a = g_{N,S} A'^a_i \tau^a g^\dagger_{N,S} - \frac{i}{g} (\partial_i g_{N,S}) g^\dagger_{N,S}$$

Bead solution in hedgehog gauge

The hedgehog chart is globally defined as an SU(2) theory

How does the bead solution look like in this gauge ?

Combed gauge in northern hemisphere

$$\tilde{\phi}_N \to e^{-i\varphi}\frac{v_2}{\sqrt{2}}$$

$$A'^3_{Ni}dx^i \to -\frac{1}{g}d\varphi$$

Combed gauge in southern hemisphere

$$\tilde{\phi}_S \to e^{i\varphi}\frac{v_2}{\sqrt{2}}$$

$$A'^3_{Si}dx^i \to \frac{1}{g}d\varphi$$

$$\phi^a_1 \to v_1\big(s_\theta c_\varphi, s_\theta s_\varphi, c_\theta\big)\ ,$$

$$\phi^a_2 \to v_2\big(c_\theta c_\varphi, c_\theta s_\varphi, -s_\theta\big)\ ,$$

$$A'^a_r \to 0\ ,$$

$$A'^a_\theta \to \frac{1}{g}(s_\varphi, -c_\varphi, 0)\ ,$$

$$A'^a_\varphi \to \frac{1}{g}(0, 0, -1)\ .$$