Pancake Physics

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I will talk about stories of pancake solitons in two different systems:

1. QCD
2. QED + monopole

both in 4d.

In QCD, I will argue that the consistent effective theory of η' requires to include **pancakes** in the theory. In addition we need **vector mesons as gauge fields**.

In QED, I will argue that the **pancakes** can describe the final state of the scattering process of an electron and a monopole. This picture solves the problem of the missing final state problem in multi-flavor massless QED.

Let's start with QCD

$$S_{\mathrm{QCD}} = \int d^4x \left(\frac{1}{4g^2} F^2 + \frac{i\theta}{32\pi^2} F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

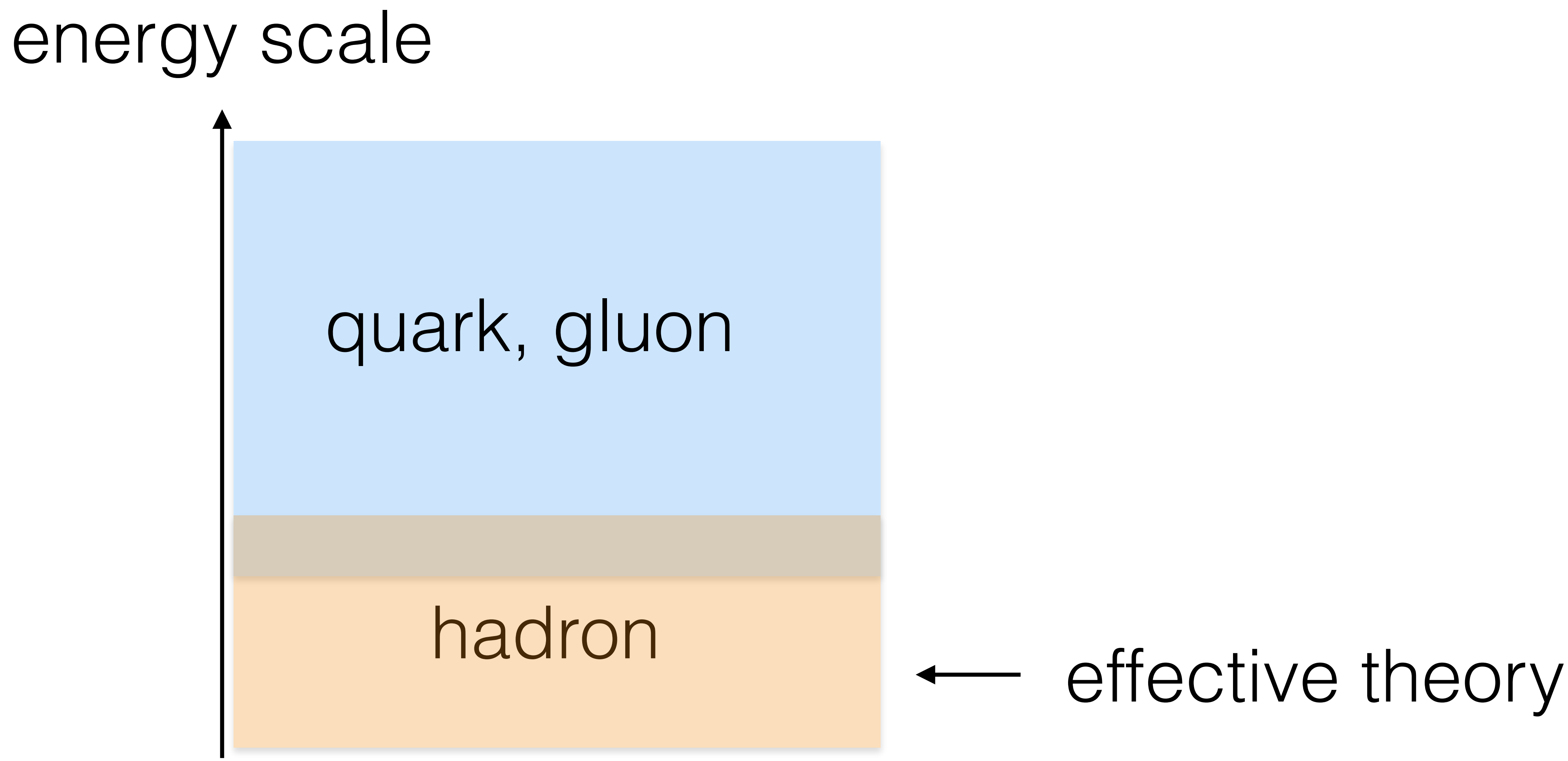
$$Z_{\mathrm{QCD}}(\theta) = \int [dA][d\psi][d\bar{\psi}] e^{-S_{\mathrm{QCD}}}$$

defined as a theory of quarks and gluons.

But,

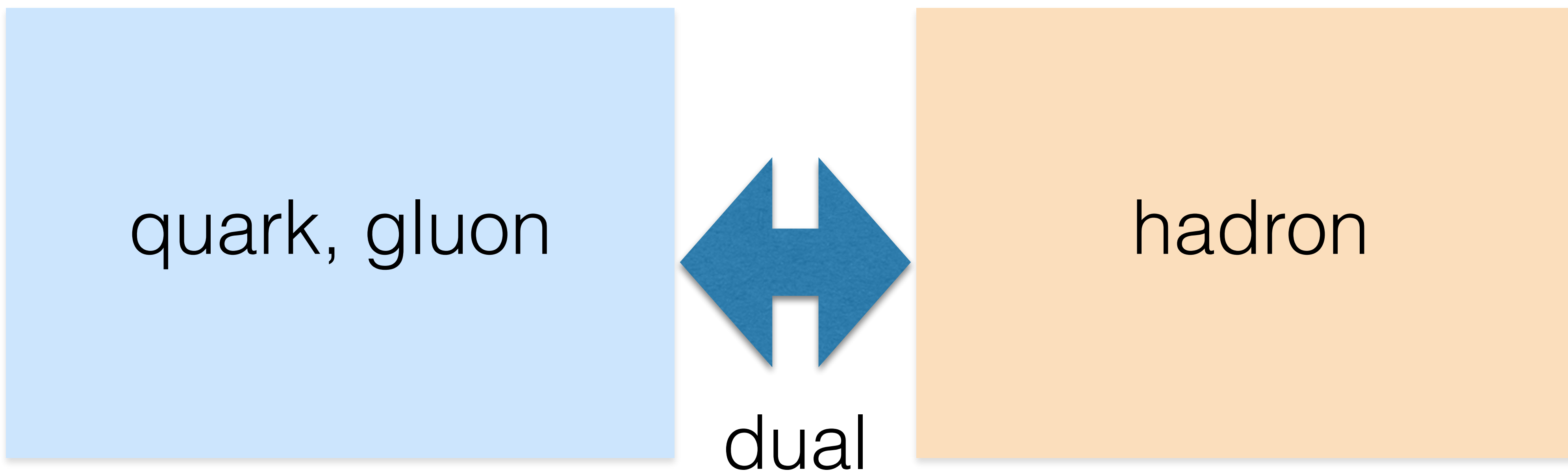
what we see are hadrons.

Usual view



But of course, we have only one physics! Which means there can be definition of the same theory directly by hadrons.

Possible view



gauge theory of gluons

gauge theory of mesons

[Sakurai ’60][Bando, Kugo, Uehara, Yamawaki, Yanagida ’85]

Indeed, AdS/QCD is such an example.

[Sakai, Sugimoto ’04][Erlich, Katz, Son, Stephanov ’05][Da Rold, Pomarol ’05]

There is also an interesting indication from 3d duality.

[Kan, RK, Yankielowicz, Yokokura ’19]

As well as from Seiberg duality. [Komargodski ’10] [RK ’11]

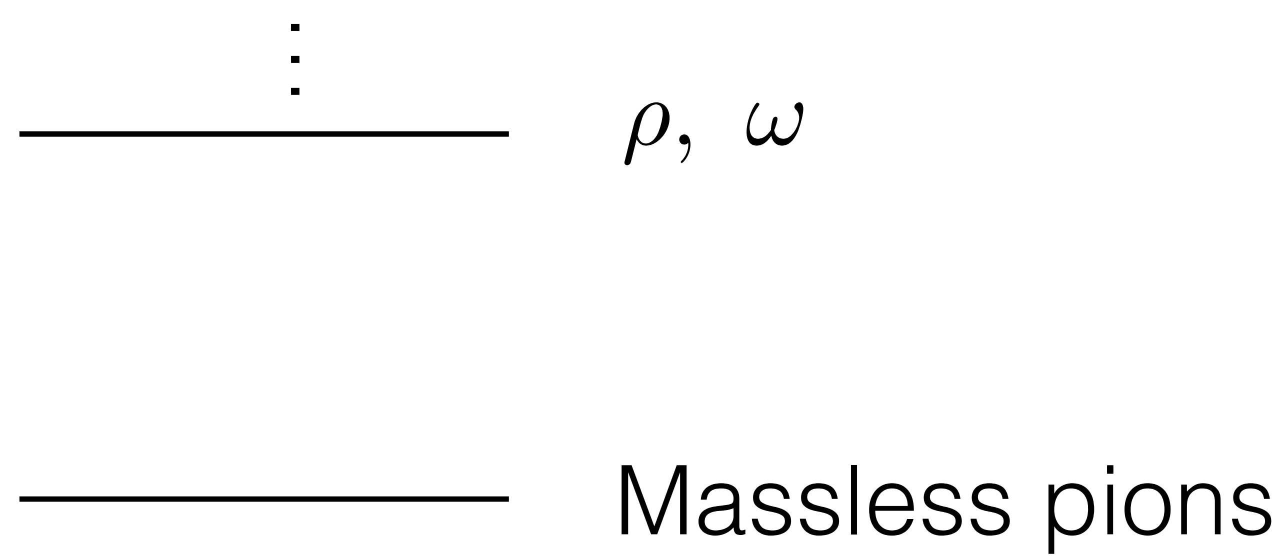
Possible picture of

[Kan, RK, Yankielowicz, Yokokura '19]

S^1 compactified QCD

with winding θ term.

Large radius (4d)



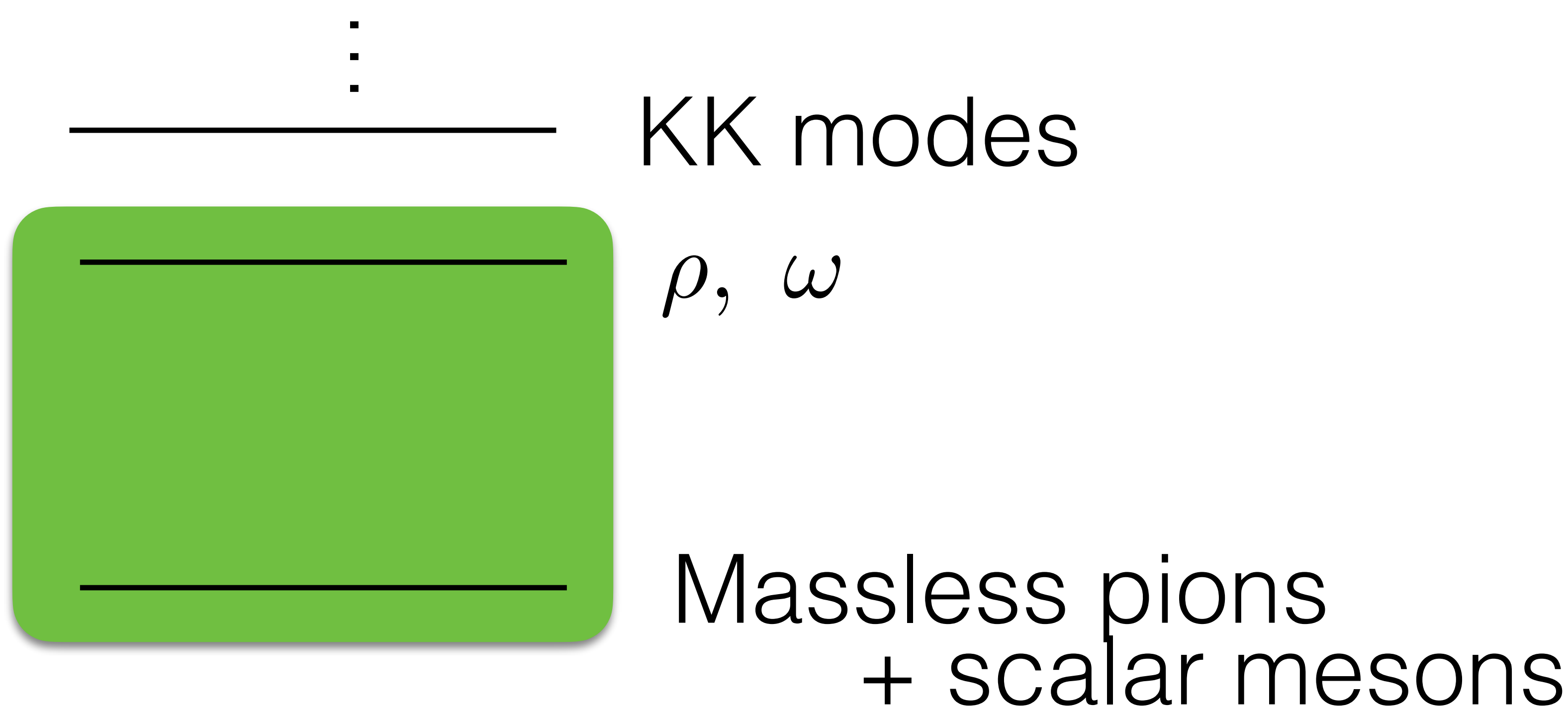
Possible picture of

[Kan, RK, Yankielowicz, Yokokura '19]

S^1 compactified QCD

with winding θ term.

Critical radius



Form a $U(N_f)_{-N}$ gauge theory
+ $2N_f$ scalars (Higgs phase)

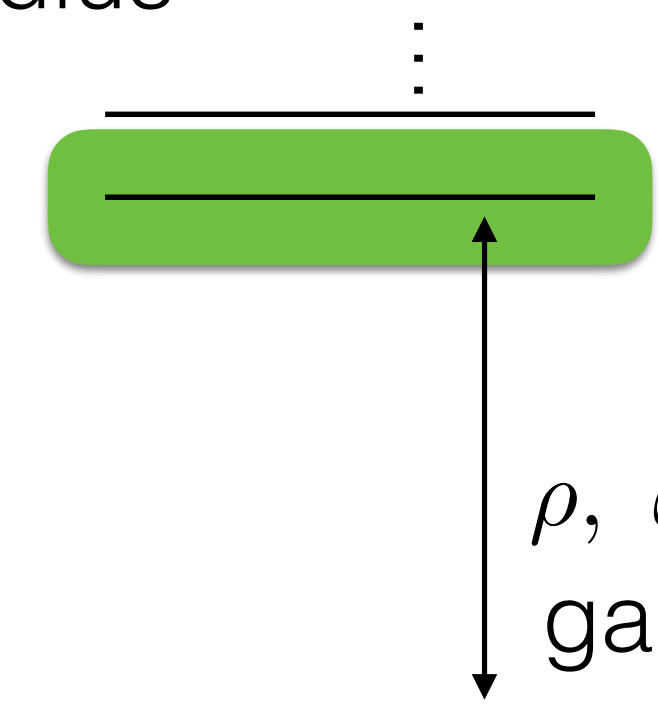
Possible picture of

[Kan, RK, Yankielowicz, Yokokura '19]

S^1 compactified QCD

with winding θ term.

Small radius



KK modes

chiral symmetry restored
(topological phase)

$\rho, \ \omega$

gapped by the CS term

$U(N_f)_{-N}$ theory $\longleftrightarrow$ $SU(N)_{N_f}$

Gluons are replaced by the $\rho, \ \omega$ mesons!?

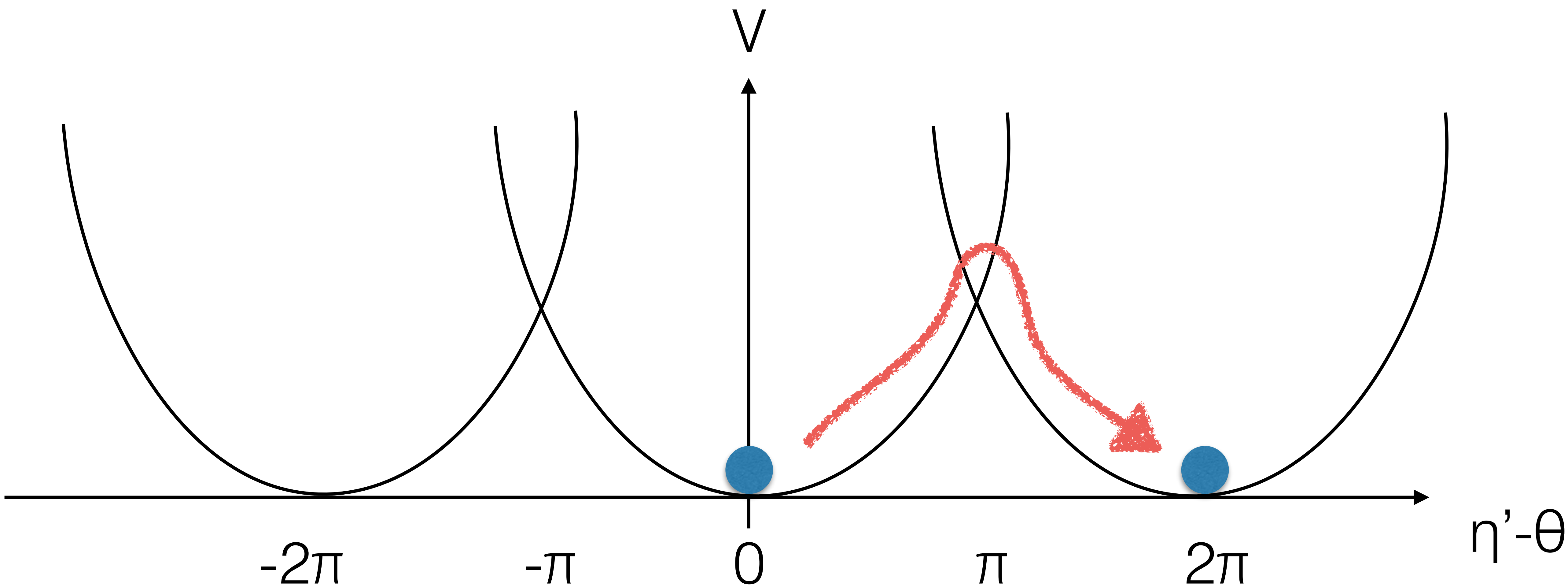
OK, the picture is just a picture.

But, we can actually **“derive”** the hadron gauge theory by using the symmetry of QCD.

The key is an object “pancake” that can bridge QCD and the hadron gauge theory.

η' effective theory

$$\mathcal{L}^{\rm eff}_{\eta'} = \frac{N_f f_\pi^2}{8} d\eta' \star d\eta' + \frac{f_\pi^2}{8N_f} m_{\eta'}^2 \min_{n\in\mathbb{Z}} (N_f \eta' + \theta - 2\pi n)^2,$$



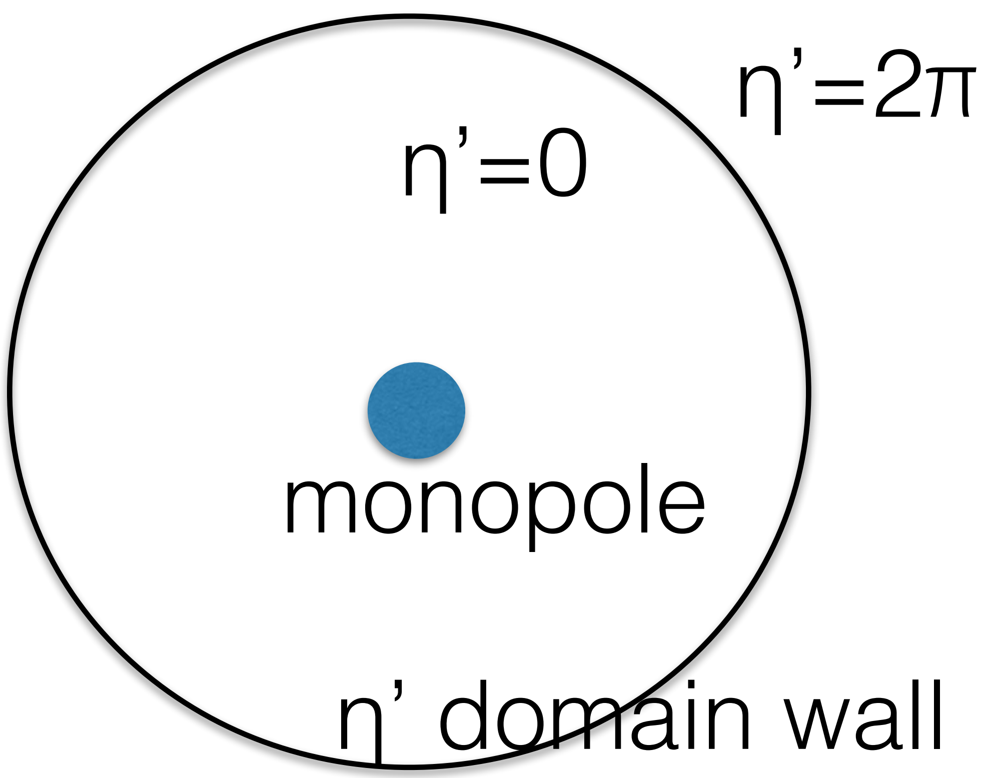
one can find a domain wall configuration.

Impossibility of η' effective theory

WZW term: $\frac{i}{8\pi^2}\frac{N_f}{N_c}\eta' dA_B dA_B$

weakly gauged $U(1)_B$

the monopole background of $U(1)_B$



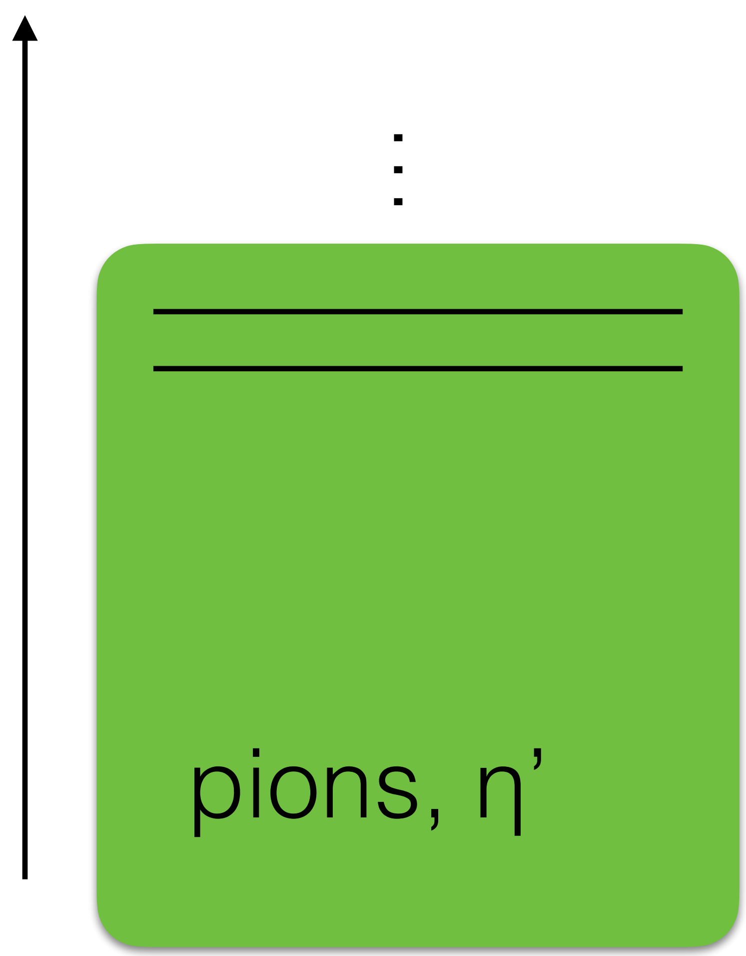
this object carries baryon number N_f/N_c by the Witten effect

not allowed by the Dirac quantization

Therefore, this theory by itself is inconsistent!

At large Nc, η’ is light. There should be some effective theory of pions and η’.

We argue that the vector mesons and baryons … should also be included in the effective theory for consistency.



consistent effective theory requires to include massive modes as well as extended objects.

anomaly matching

$$\mathcal{L}^{\text{eff}}_{\eta'} = \frac{N_f f_\pi^2}{8} d\eta' \star d\eta' + \frac{f_\pi^2}{8N_f} m_{\eta'}^2 \min_{n\in\mathbb{Z}} (N_f \eta' + \theta - 2\pi n)^2,$$

try to couple it to background gauge fields associated with the global symmetry:

$$G_{\text{sub}} := [SU(N_f)_V \times U(1)_V]/[\mathbb{Z}_{N_c} \times (\mathbb{Z}_{N_f})_V],$$

naive guess:

$$\mathcal{L}^{\text{eff}}_{\text{topo}} = i(N_f \eta' + \theta) \frac{1}{N_f} (N_c q_f + N_c N_f q_V).$$

instanton densities

naive guess:

$$\mathcal{L}^{\rm eff}_{\rm topo} = i(N_f\eta' + \theta)\frac{1}{N_f}(N_c q_f + N_c N_f q_V).$$

instanton densities

However, 2π shift of θ would give a phase to the partition function which is different from that in QCD in a background with fractional instanton number allowed by $Z_{N_c} \times Z_{N_f}$.

(Naively, the fermion contribution can be explained but not the gluon part.)

we need something more than η'

we need an object which plays the role of the gluon in the hadron theory!

Consistent effective theory

$$\frac{f_\pi^2}{2} m_{\eta'}^2 (N_f \eta' + \theta - 2\pi n)^2.$$

branches

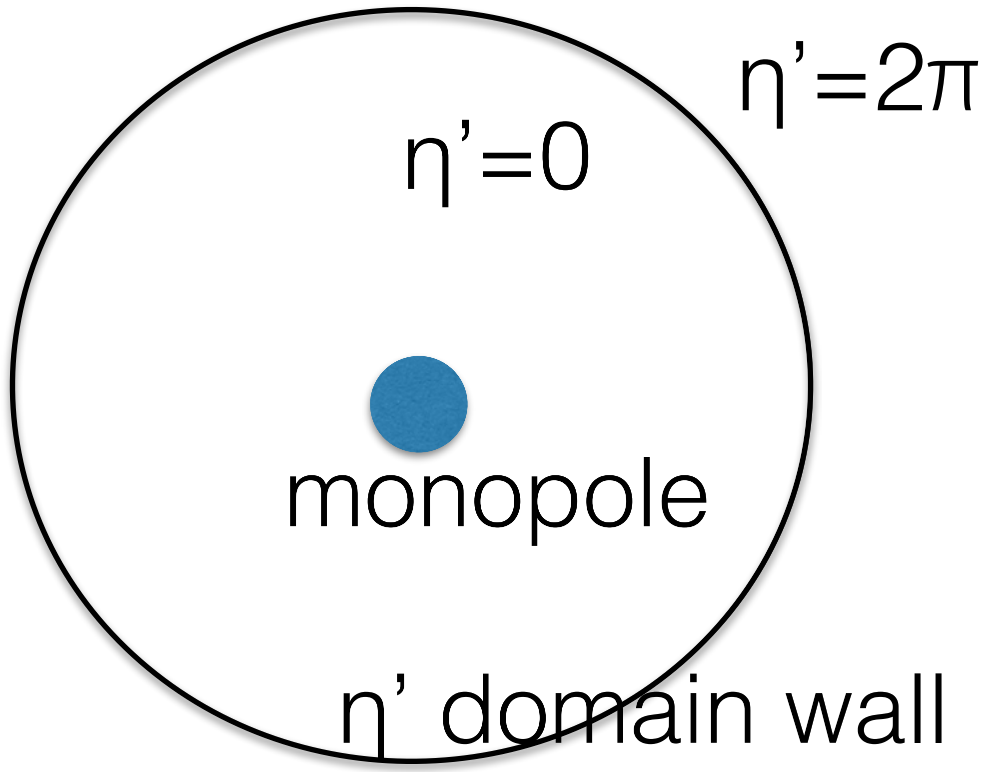
$$-\frac{iN_c}{4\pi}\int_{\mathrm{DW}} \mathrm{tr}\left(cdc - i\frac{2}{3}c^3\right)$$

$U(k)_{-N_c}$ CS theory when we across k branches.

dual to $SU(N_c)_k$ (gluon)

Domain wall attaches to the location where we across branches. Now the background gauge field can couple to DW to match the anomaly.

See also [Dierigl, Pritzel '14]
[Gaiotto, Kapustin, Komargodski, Seiberg '17]
[Gaiotto, Komargodski, Seiberg '17]

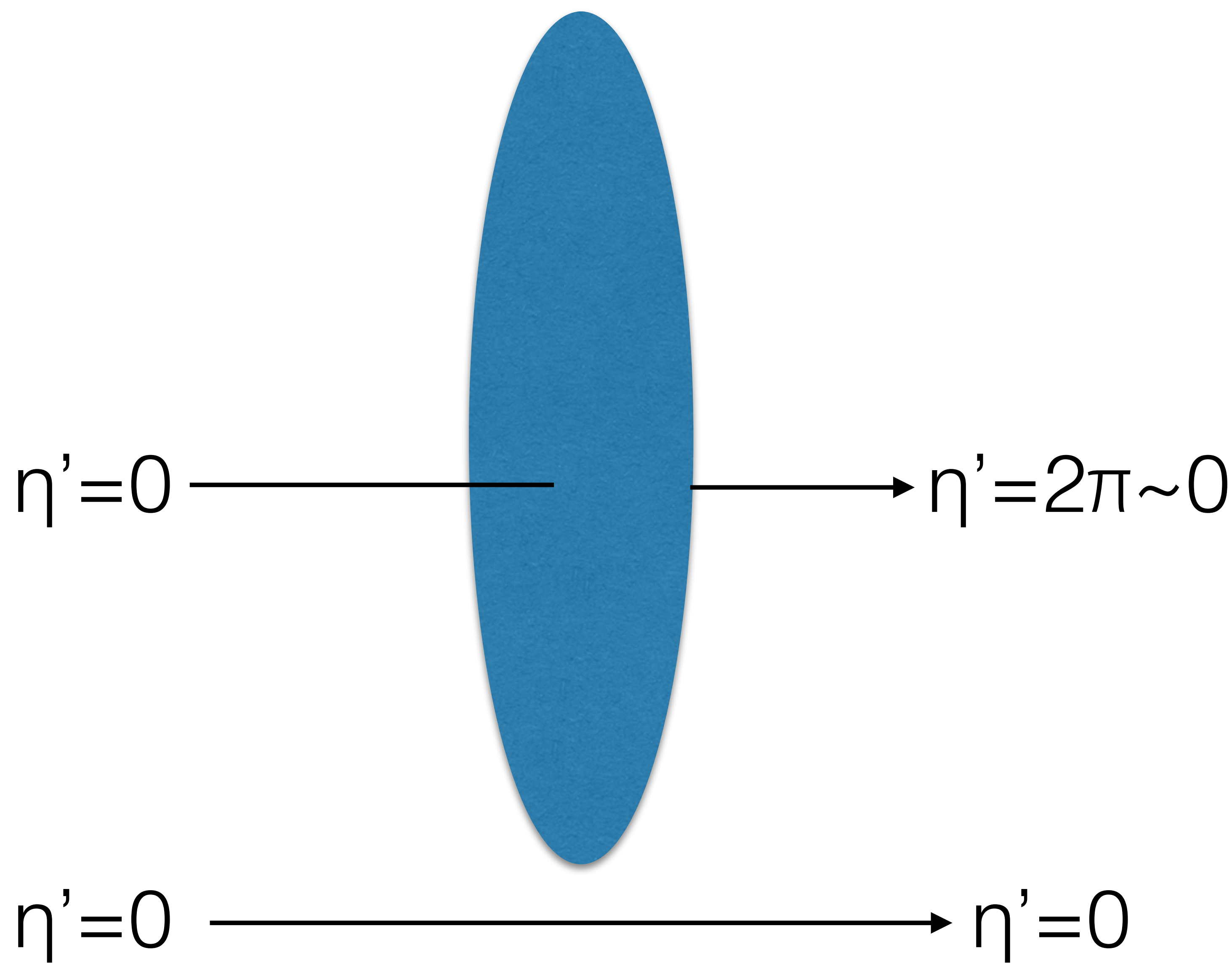


This configuration is now forbidden by the Gauss law constraint on the DW.

⇒ consistent.

[Komargodski ’18]

Pancake?



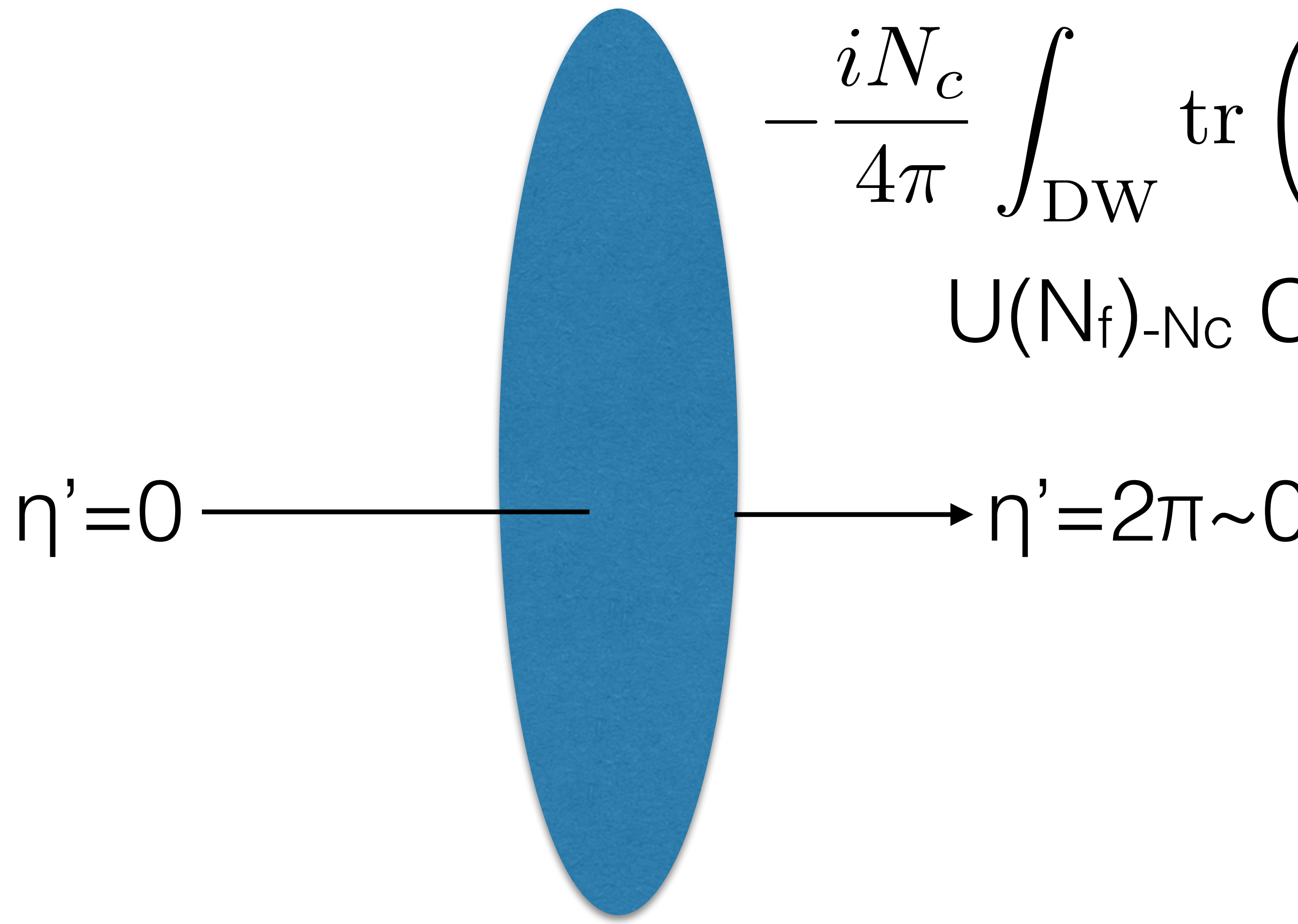
one can consider this object. A DW bounded by a string.

On the string, η’ is singular. That means the chiral symmetry is recovered there.

Pancake?

$$-\frac{iN_c}{4\pi}\int_{\mathrm{DW}} \mathrm{tr}\left(c dc - i\frac{2}{3}c^3\right)$$

$U(N_f)_{-N_c}$ CS theory



We should put a consistent boundary condition for c.

on the boundary c transforms as gauge field of $U(N_f)$ flavor group.

(color-flavor locking)

Pancake?

on the boundary c transforms as gauge field of $U(N_f)$ flavor group.

(color-flavor locking)

Edge mode appears.

quantization of the edge mode gives the baryon number to this object!

pancake = spin $N_c/2$ baryon

Pancake?

Story doesn't end here.

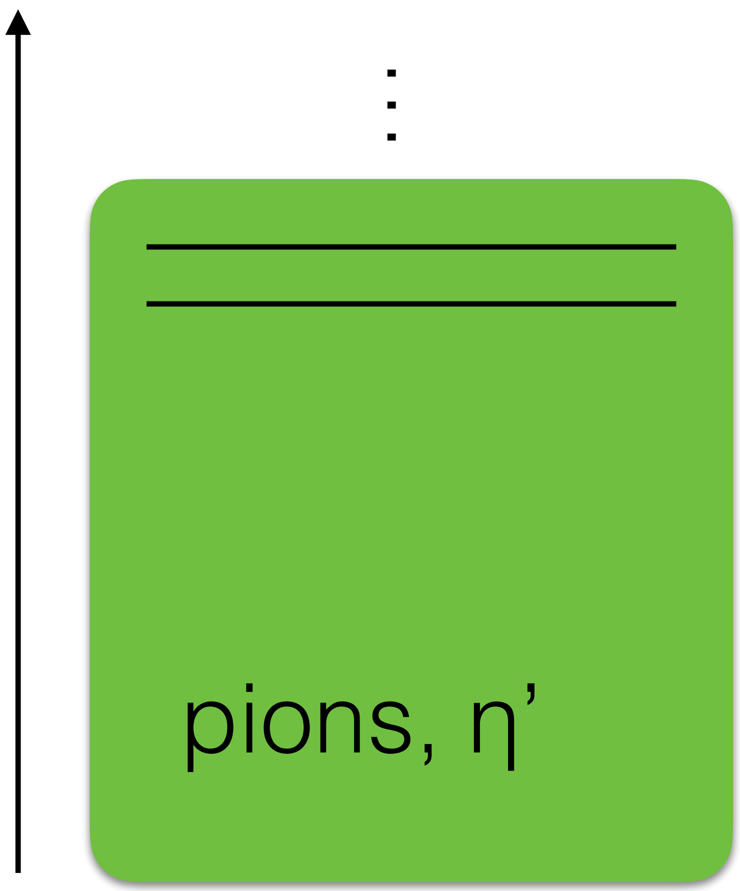
c must be fixed as the background gauge field of vectorial flavor symmetry.

But, the chiral symmetry is recovered on the boundary.

there is no concept of vectorial flavor symmetry there.

→ in order to impose a consistent boundary condition, we need to introduce a field that transform vectorial flavor group, that are the **vector mesons**!

In the end,



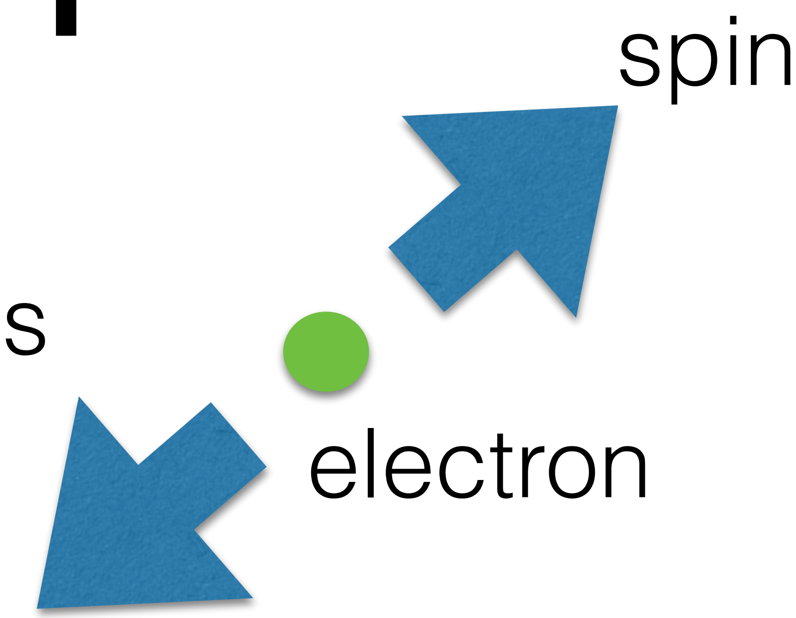
vector mesons,
pancake baryons

We need those for consistent theory.

The relation between the gluon and the vector mesons are now clear.

gluon $SU(N_c)$ ⟷ 3d dual ⟷ c on the wall $U(N_f)_{-N_c}$ ⟷ CFL ⟷ vector meson

QED+monopole



angular momentum by EM fields

$$\vec{J}_{\rm EM} = \frac{1}{4\pi}\int d^3x\, \vec{r} \times (\vec{E} \times \vec{B}) = -\frac{1}{2}\hat{r}_0,$$

magnetic monopole

Fermions can have spherically symmetric wave function.
(s-wave)

2d theory

spherically symmetric part can be reduced to 2d theory.

EM fields are not dynamical in 2d and can be integrated out.

theory of free fermion —> Bosonization

—> Theory of free boson with appropriate boundary conditions

$$S_{\varphi}^{\text{free}} = \int \mathrm{d}r\, \mathrm{d}t\left[\tfrac{1}{2}(\partial_t\varphi)^2 - \tfrac{1}{2}(\partial_r\varphi)^2\right].$$

$\partial_r\varphi = 0 \qquad \text{at } r = 0.$ [Callan ’82][Rubakov ’82] taken from [Rubakov '88]

scattering process

scalar soliton wave

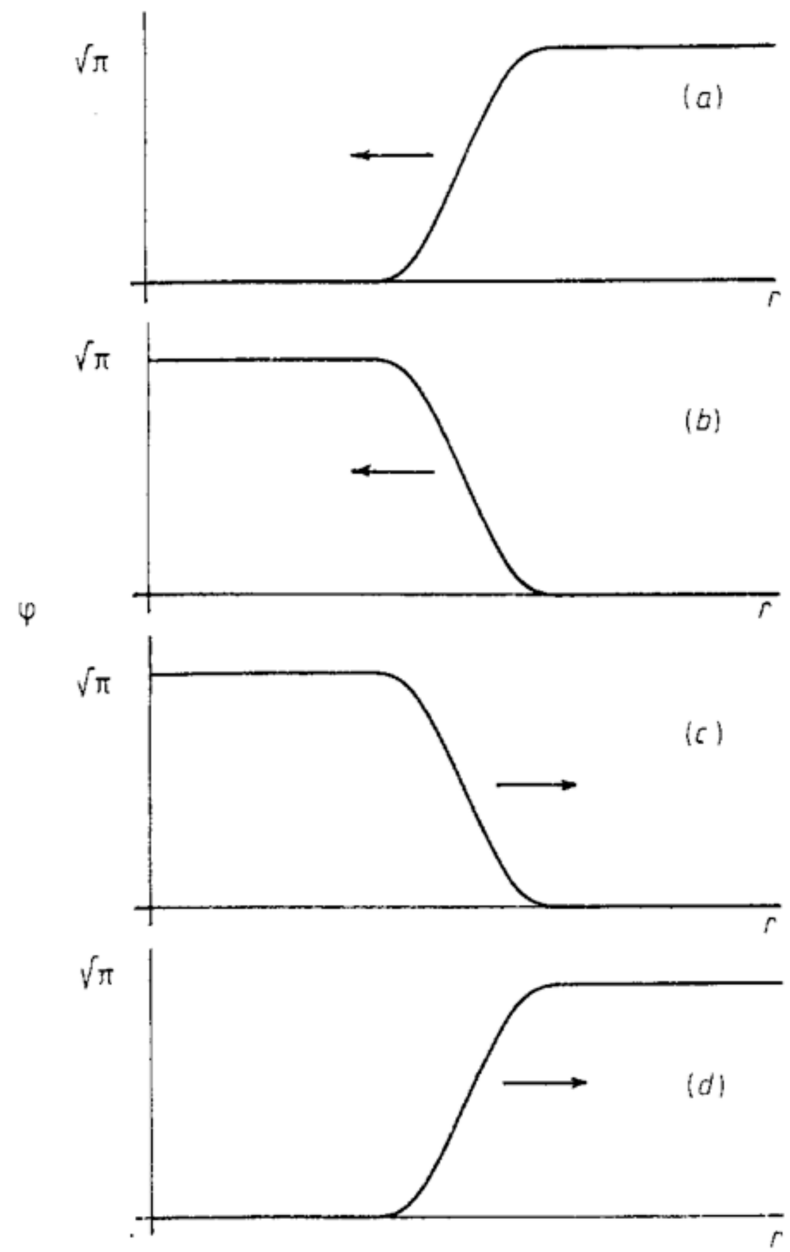


Figure 6. Fermionic excitations in the free bosonised theory ((*a*) *a*; (*b*) $\bar{a}$; (*c*) *b*; (*d*) $\bar{b}$). As before, *a* and *b* are positive and negative charge left-handed fermions, $\bar{a}$ and $\bar{b}$ are their antiparticles. The arrows indicate the direction of motion.

fermion

$$\psi_L = \begin{pmatrix} a_+ \\ b_- \end{pmatrix}_L .$$

incoming a

incoming $\bar{a}$

outgoing b

outgoing $\bar{b}$

a+M —> $\bar{b}$+M

no simple scattering. always a—>b

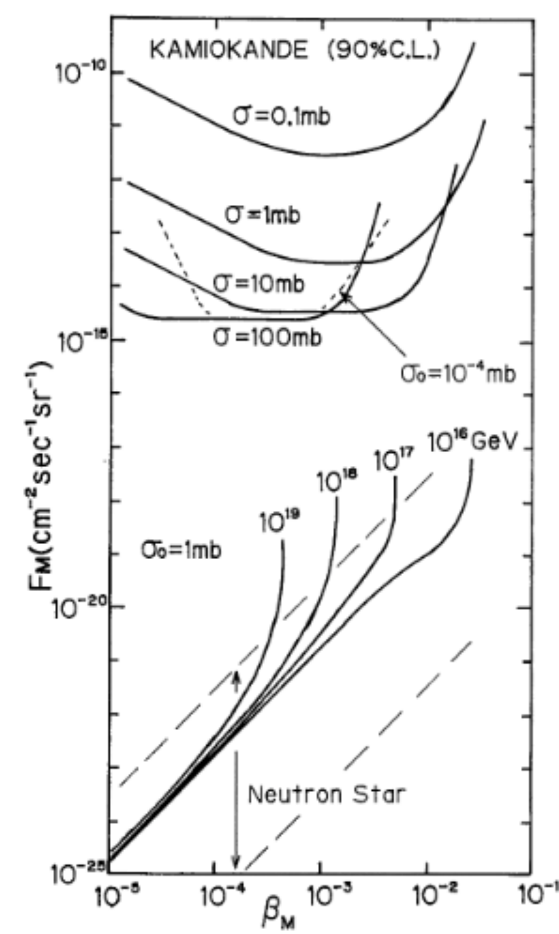
no simple scattering. always a—>b

with GUT monopole [Callan-Rubakov effect]

$$\bar{5} = (\bar{d}_1, \bar{d}_2, \boxed{\bar{d}_3, e^-,} \nu_e)_L$$

baryon number violating process at O(1) rate!

Indeed, this process is giving the severest constraints on the abundance of the monopole in the Universe.



[Kamiokande '85]

Puzzle?

Once we extend the discussion to more flavors (4 doublets):

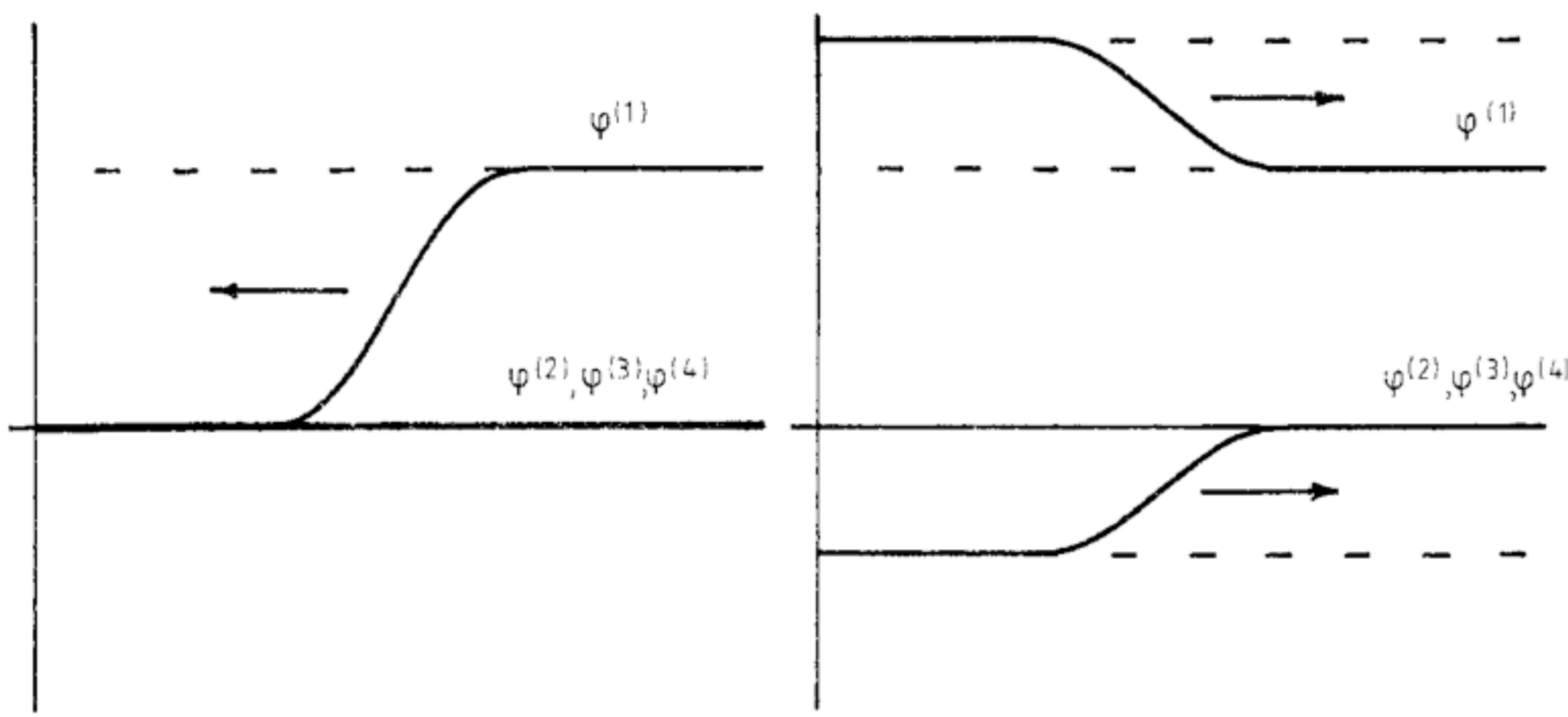


Figure 9. The fate of $a^{(1)}$ in the model with four massless left-handed doublets.

We can find a final state in the bosonic picture.

Puzzle?

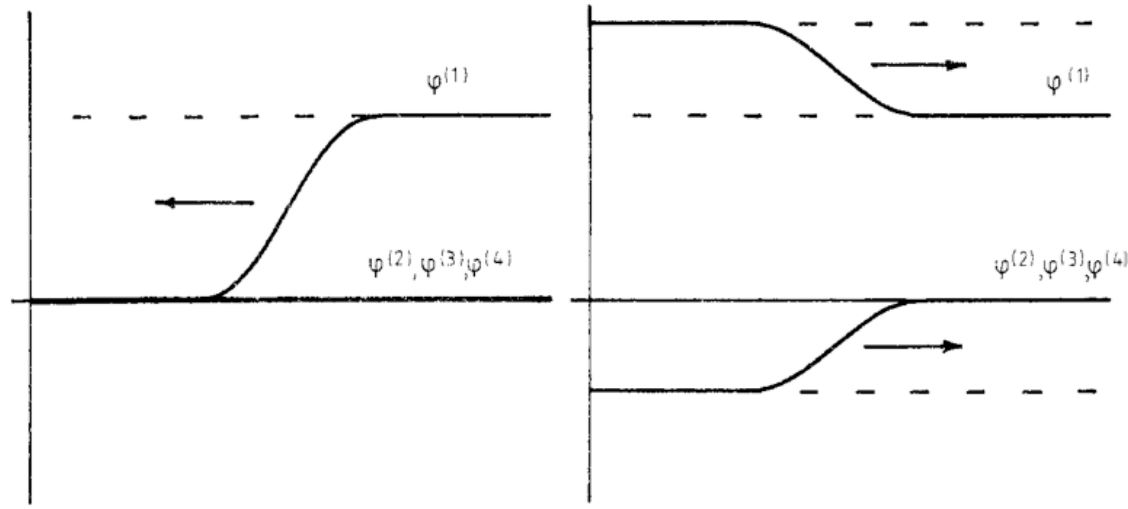


Figure 9. The fate of $a^{(1)}$ in the model with four massless left-handed doublets.

This final state, if translated into fermions, it is

$$a^{(1)} + \text{mon} \to \tfrac{1}{2} b^{(1)} + \tfrac{1}{2}(\bar{b}^{(2)} + \bar{b}^{(3)} + \bar{b}^{(4)}) + \text{mon}.$$

We find fractional fermions as the final state.

What is this?

Indeed, one cannot find a final state consistent with symmetry in the massless limit of fermions.

Puzzle?

Indeed, if a is the initial state, one cannot find a candidate of final state particle in the spectrum, consistent with the symmetry.

(helicity, charge, SU(2N_f))

$$\boldsymbol{a}: \ (L, +1, \Box), \quad \boldsymbol{b}: \ (L, -1, \Box), \quad \overline{\boldsymbol{a}}: \ (R, -1, \overline{\Box}), \quad \overline{\boldsymbol{b}}: \ (R, +1, \overline{\Box}).$$

final state should be (R, +1, □).

Puzzle?

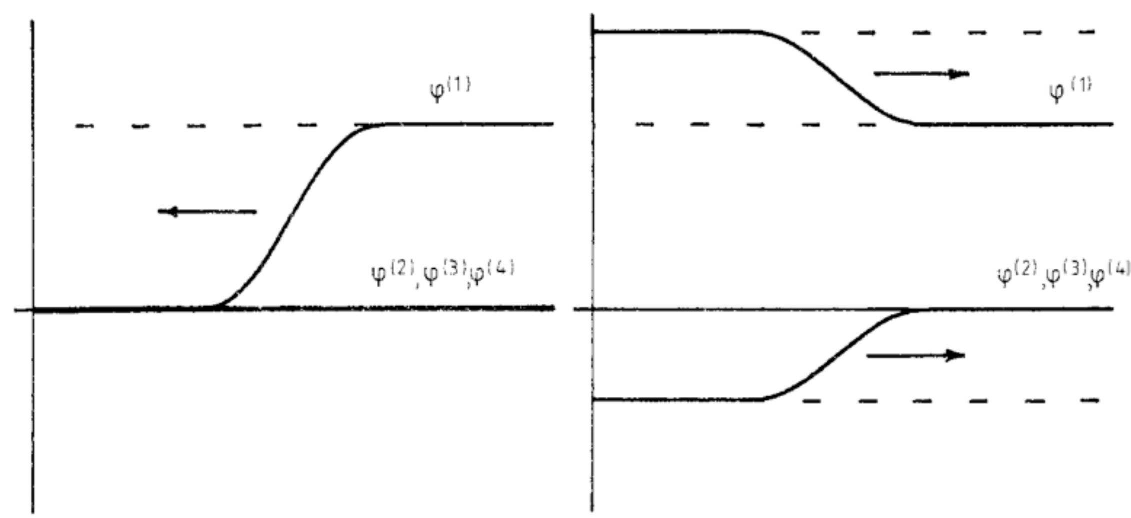


Figure 9. The fate of $a^{(1)}$ in the model with four massless left-handed doublets.

In any case, the final state should be this.

$$a^{(1)} + \text{mon} \to \tfrac{1}{2}b^{(1)} + \tfrac{1}{2}(\bar{b}^{(2)} + \bar{b}^{(3)} + \bar{b}^{(4)}) + \text{mon}.$$

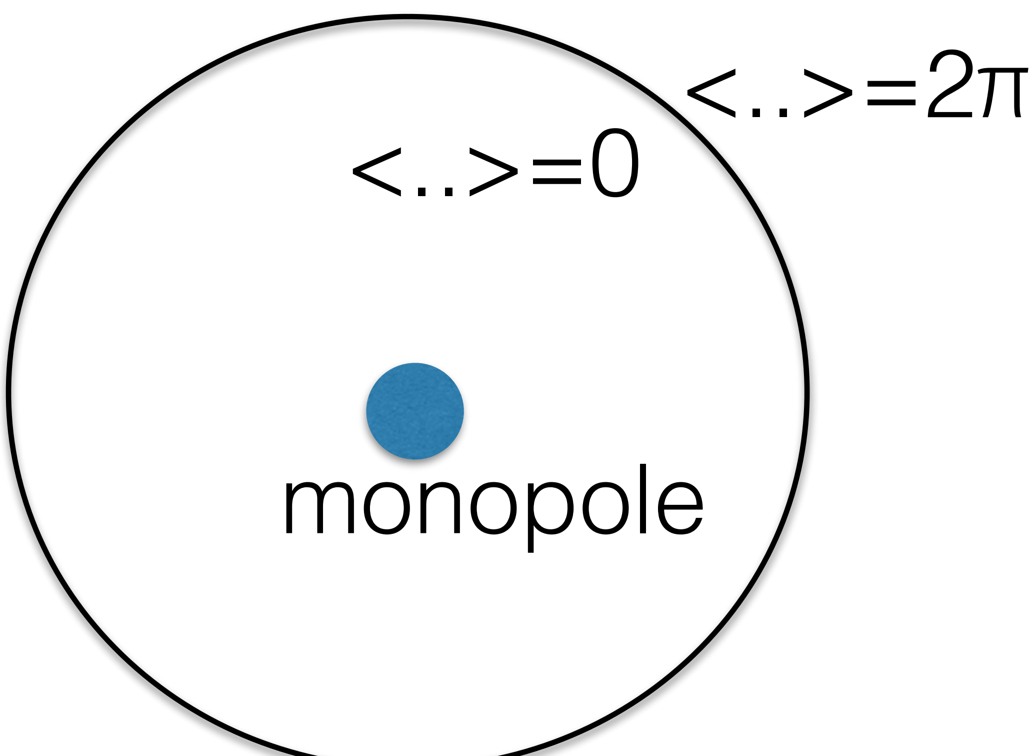
What we need is an interpretation.

Fermion as a soliton

In the monopole background, there is a condensation of the fermion product operators:

$$\left\langle (a_{i_1} b_{i_2}) \cdots (a_{i_{2N_f-1}} b_{i_{2N_f}}) \right\rangle = \frac{1}{r^{3N_f}} c \varepsilon_{i_1 \ldots i_{2N_f}}$$

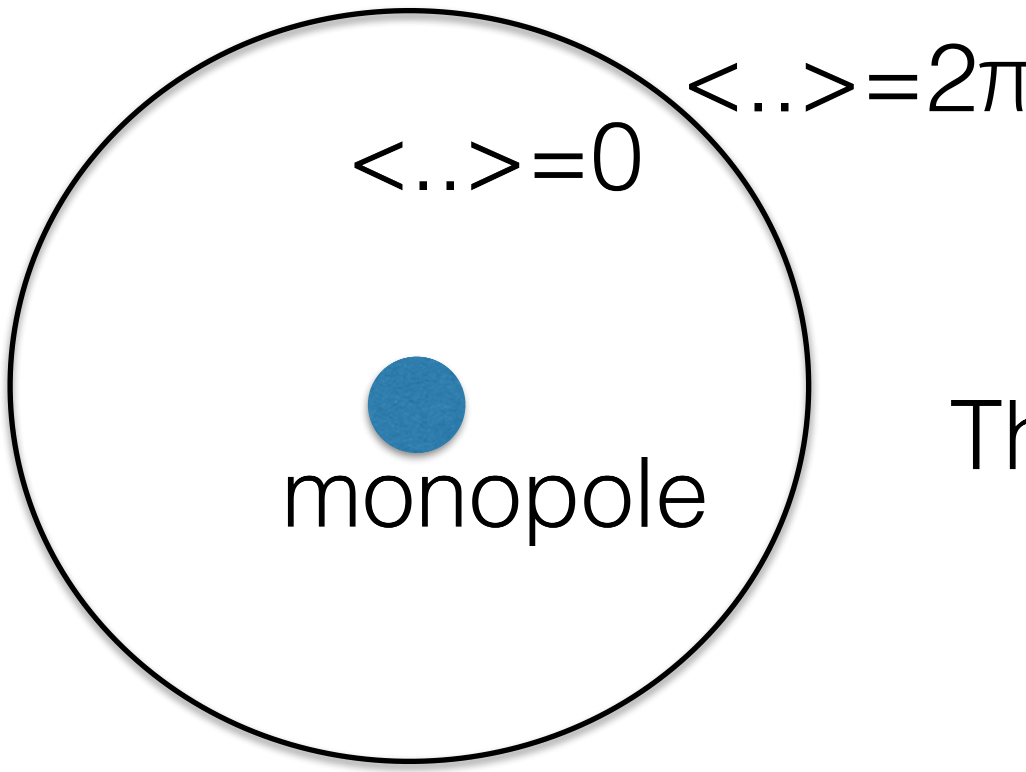
We can think of this as the origin of the a->b scatterings.



This is similar to QCD η' story.

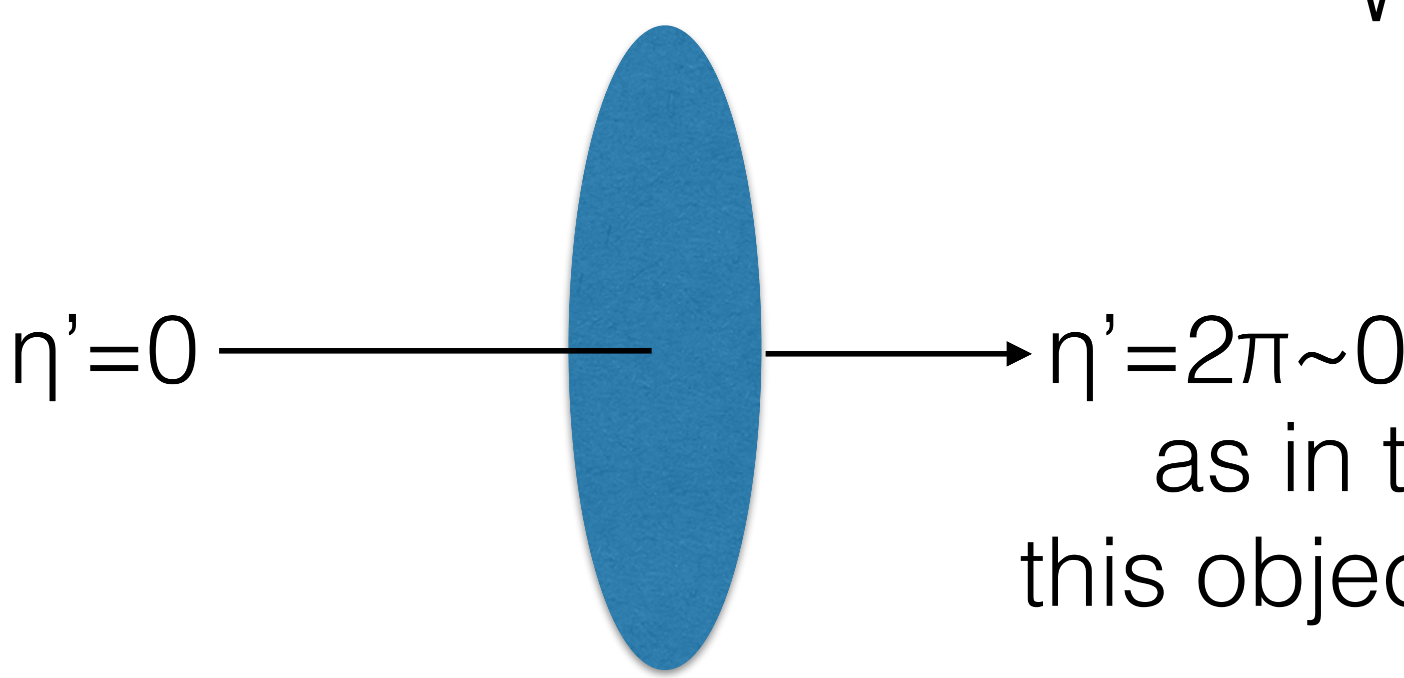
One can consider a domain wall configuration where the phase of the condensate changes from 0 to 2π.

Fermion as a soliton



This object has the unit electric charge by the Witten effect.

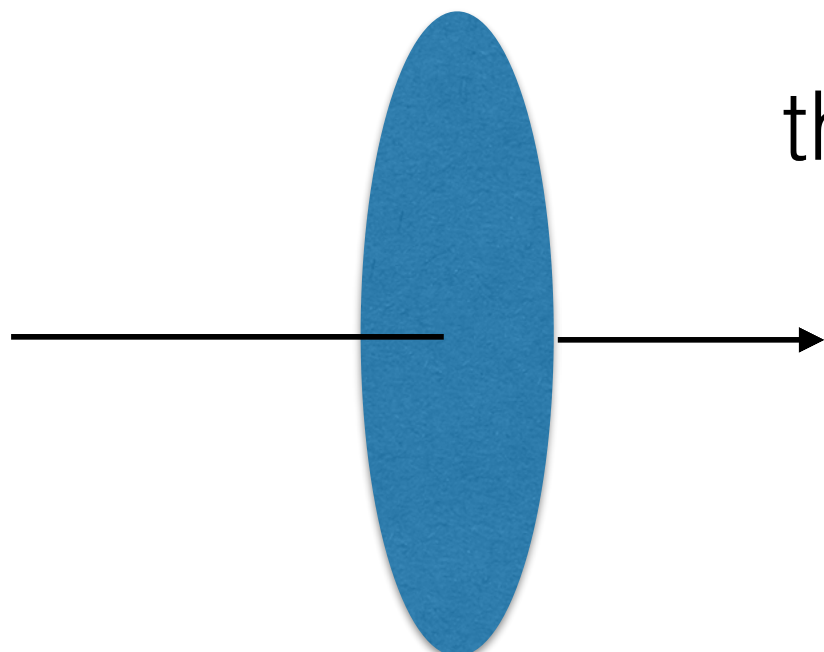
We can also consider a pancake soliton.



as in the same as QCD case, this object carries spin, charge and flavors.

Fermion as a soliton

as in the same as QCD case,
this object carries spin, charge and flavors.

$\eta' = 0$  $\eta' = 2\pi \sim 0$

It turns out, original fermions **a** and **b** can be described by the **solitons**!

This picture gives us the identification of the 2d bosonized theory to the theory of pancakes.

By the way, this object makes sense near the monopole where there is a condensation. However, in the massless limit of the fermions, there is no scale. There is no near or far. Everywhere is near the monopole.

Fermion as a soliton

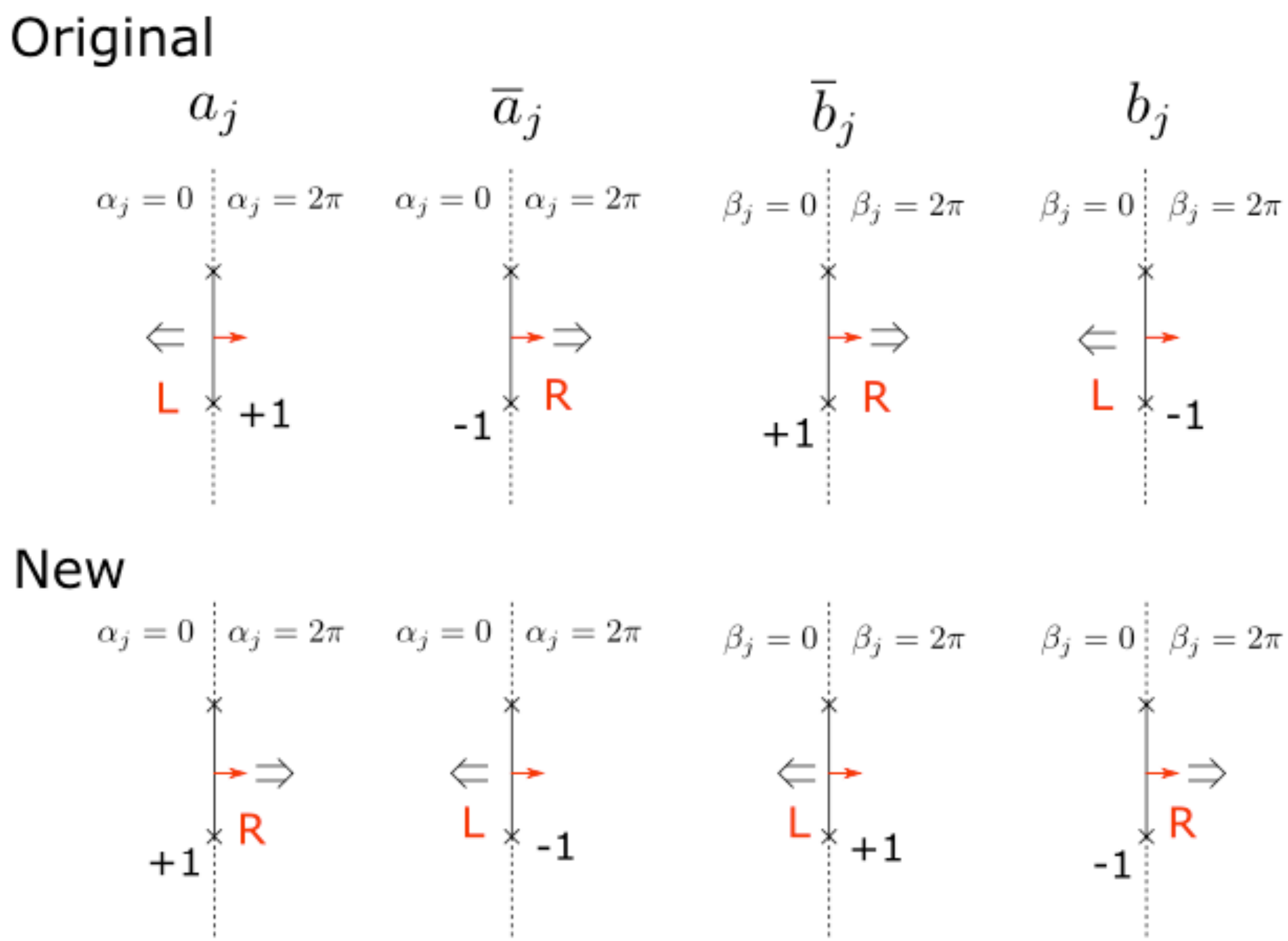


Figure 3: Pancakes corresponding to the original fermions (upper panel) and the new fermions (bottom panel). The double arrows denote the direction of the motion. The red arrows denote the direction of the spin.

We can find not only original fermions in the theory but new fermions with opposite helicity!

fermion scattering off a monopole

fermion incoming

monopole bag

collide with appropriate boundary condition

new fermion outgoing

monopole bag with a hole

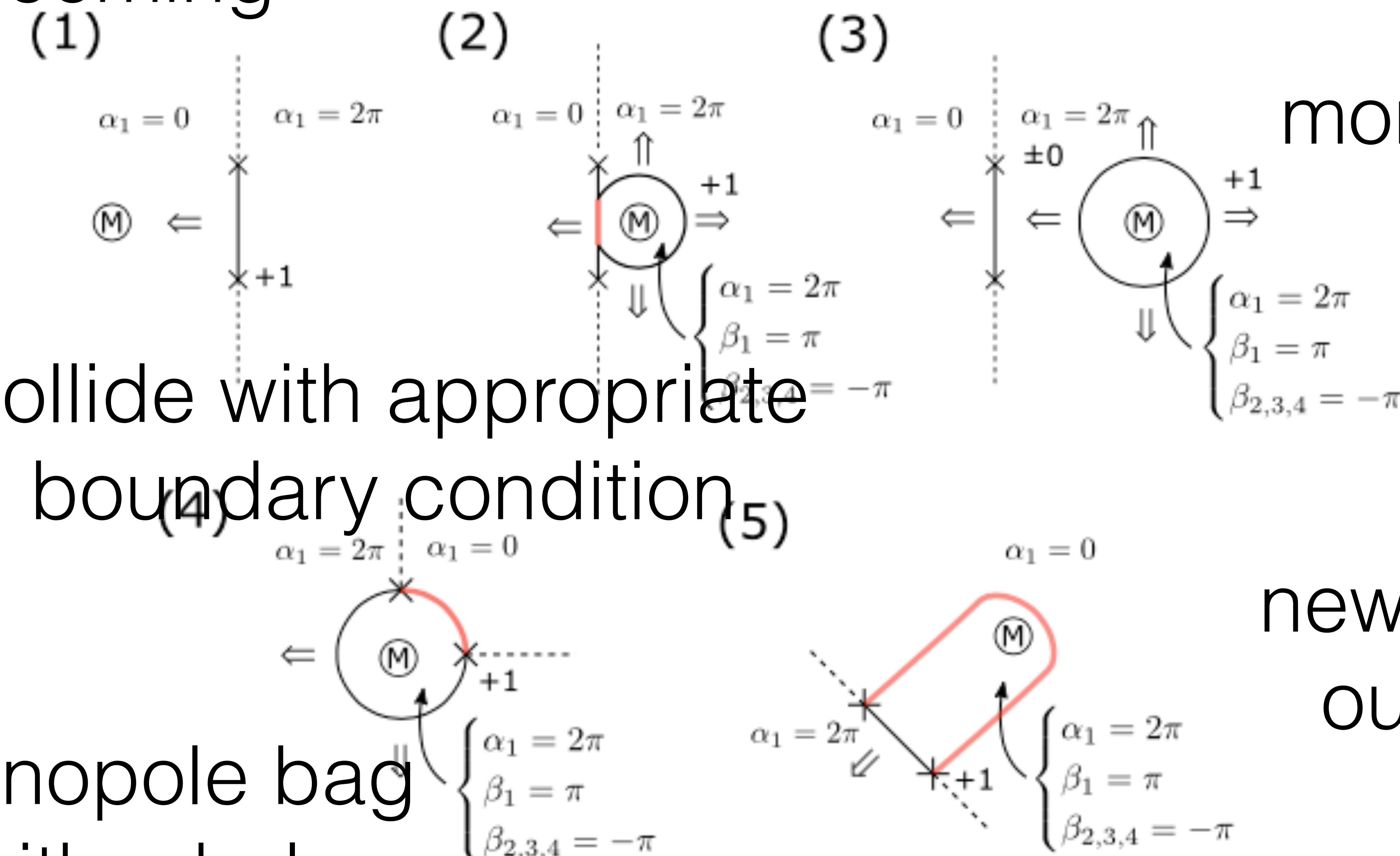


Figure 4: The soliton picture of the monopole scattering. The black solid line shows the wall where α_1 changes.

We argue that the final state of the scattering is a particle but with exotic quantum numbers. Such states are in the spectrum as solitons even though there is no such field in the original theory.

Summary

Pancake is fun.

The pancake object give a connection between the defining theory such as QCD/QED and an effective degrees of freedom.

We find vector mesons are actually identified as dual gluons on the boundary of the pancake, and also the electron can be described as a pancake around the monopole.