Link
~~Knot~~ soliton in models with B-L and Peccei-Quinn symmetries

Yu Hamada (KEK)

arXiv: 2301.XXXXX (work in progress)

w/ M. Eto (Yamagata U.) and M. Nitta (Keio U.)

26th Nov. 2022, YU Workshop 2022 @ Yamagata U

Introduction

Topological Soliton

- **Non-perturbative object in field theories**
  - monopole, vortex string, skyrmion, instanton, etc..

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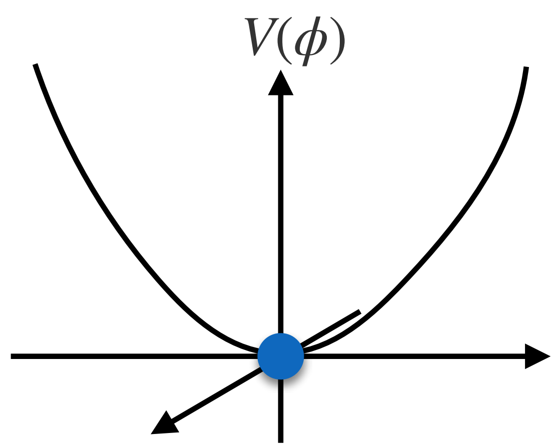
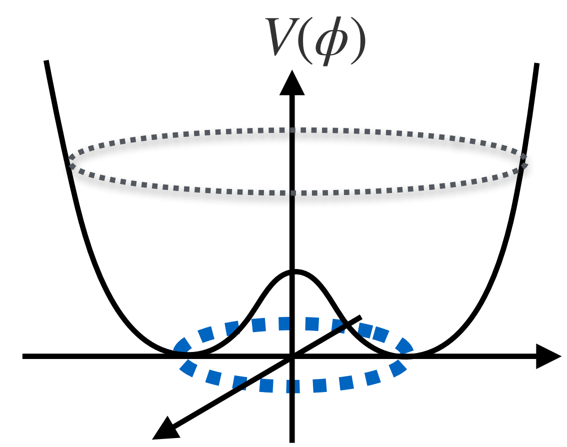
- It appears if **vacuum has non-trivial topology.**



trivial

(point)



non-trivial

(circle S^1)

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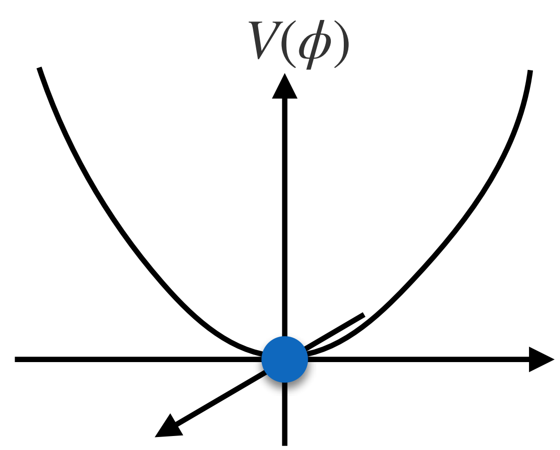

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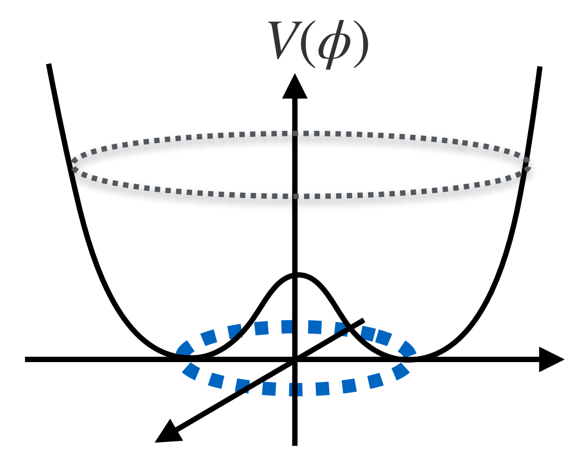

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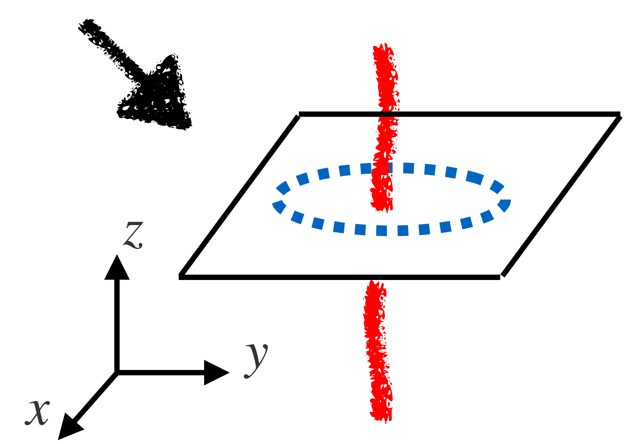

vortex string

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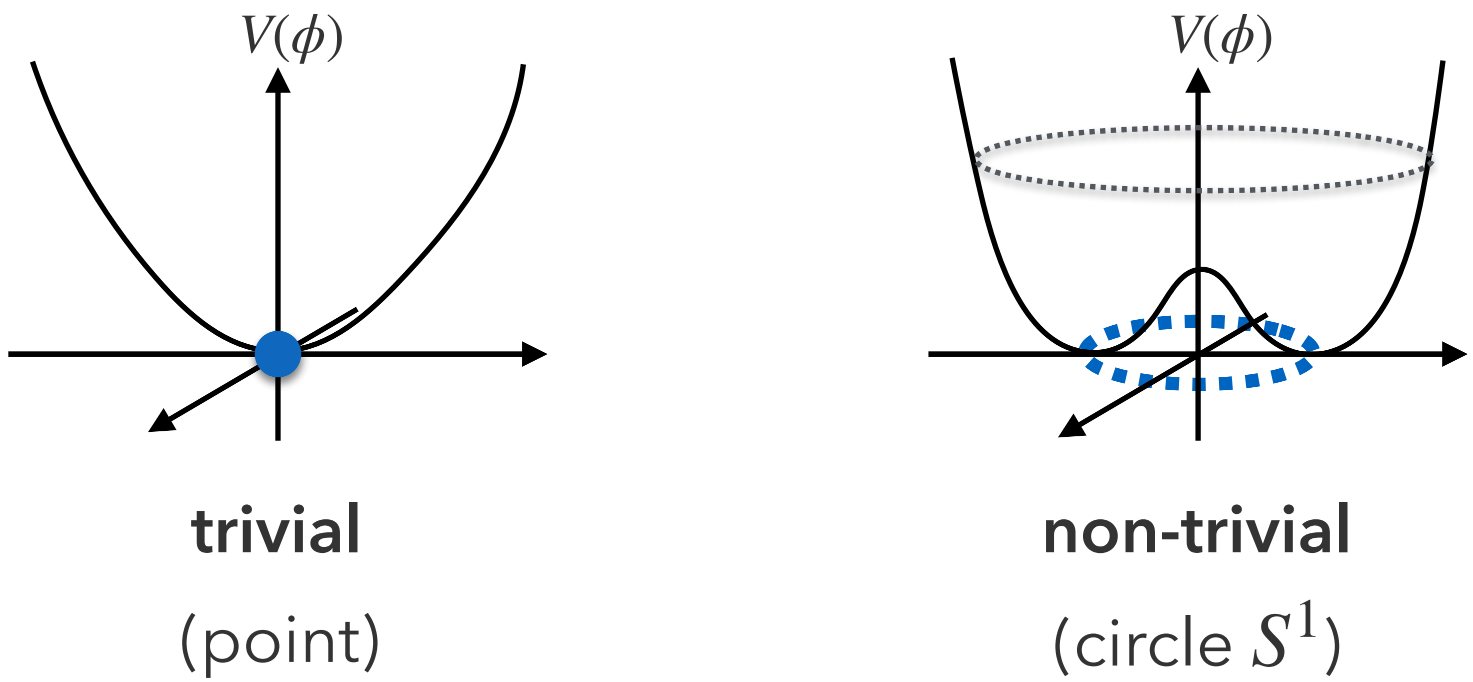
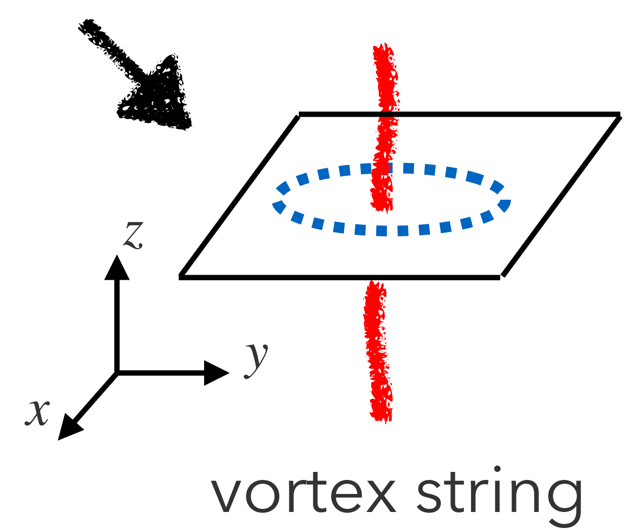




- **Vortex string appears in many systems:**
  - cosmic string, superconductor, neutron star, etc.

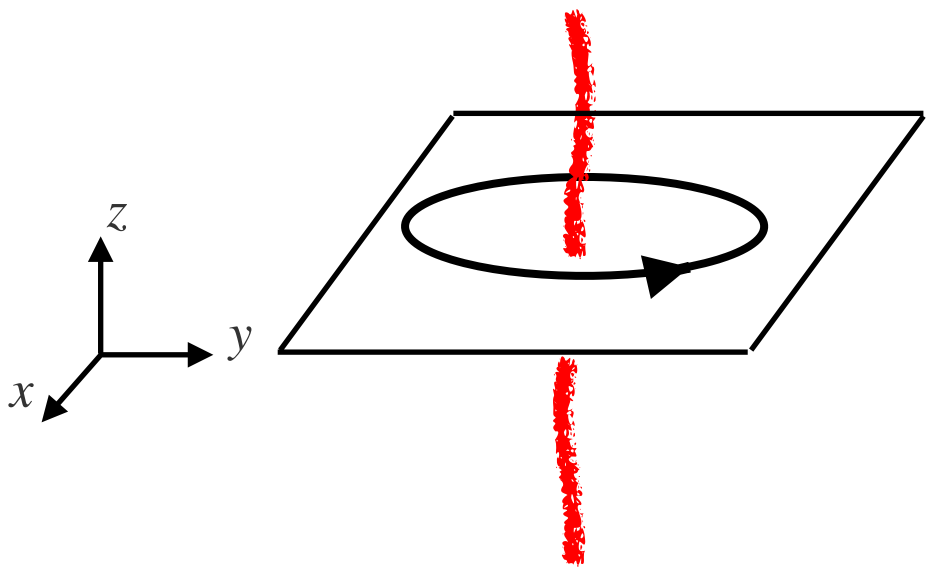
Global vs Local strings

- SSB of **gauged** $U(1)$ sym → **local** vortex string

  → **magnetic flux is squeezed w/ finite width**

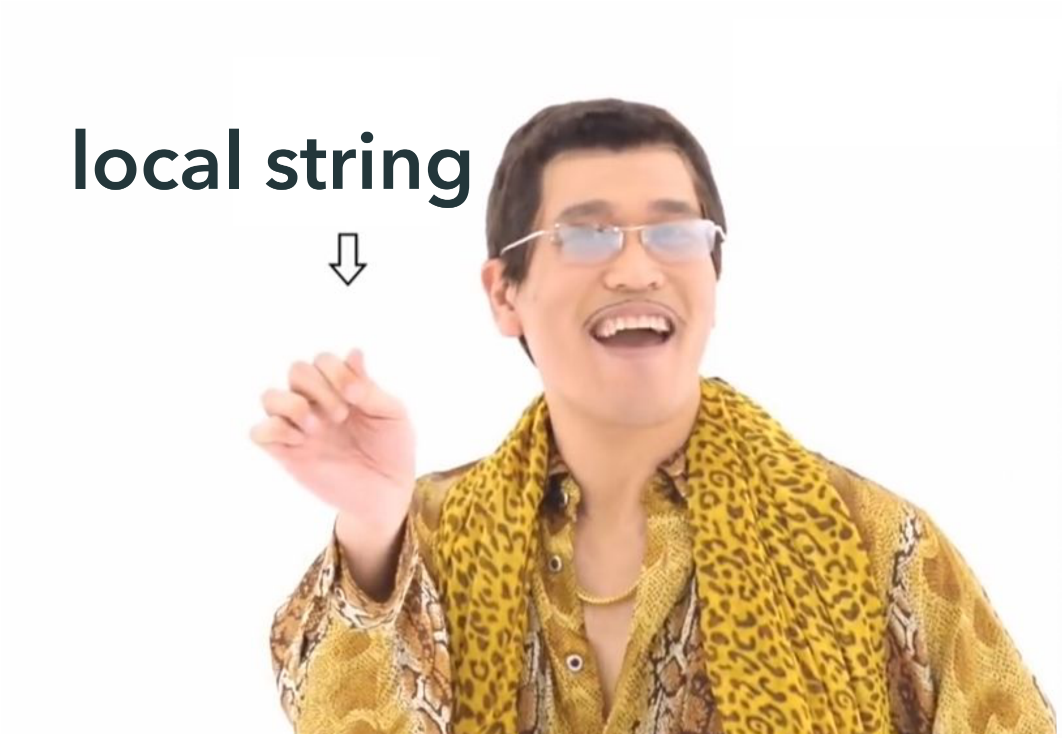
- SSB of **global** $U(1)$ sym → **global** vortex string

  → **w/o magnetic flux**



NG boson phase changes from 0 to 2π

Global vs Local strings



https://www.youtube.com/@PIKOTARO

Global vs Local strings

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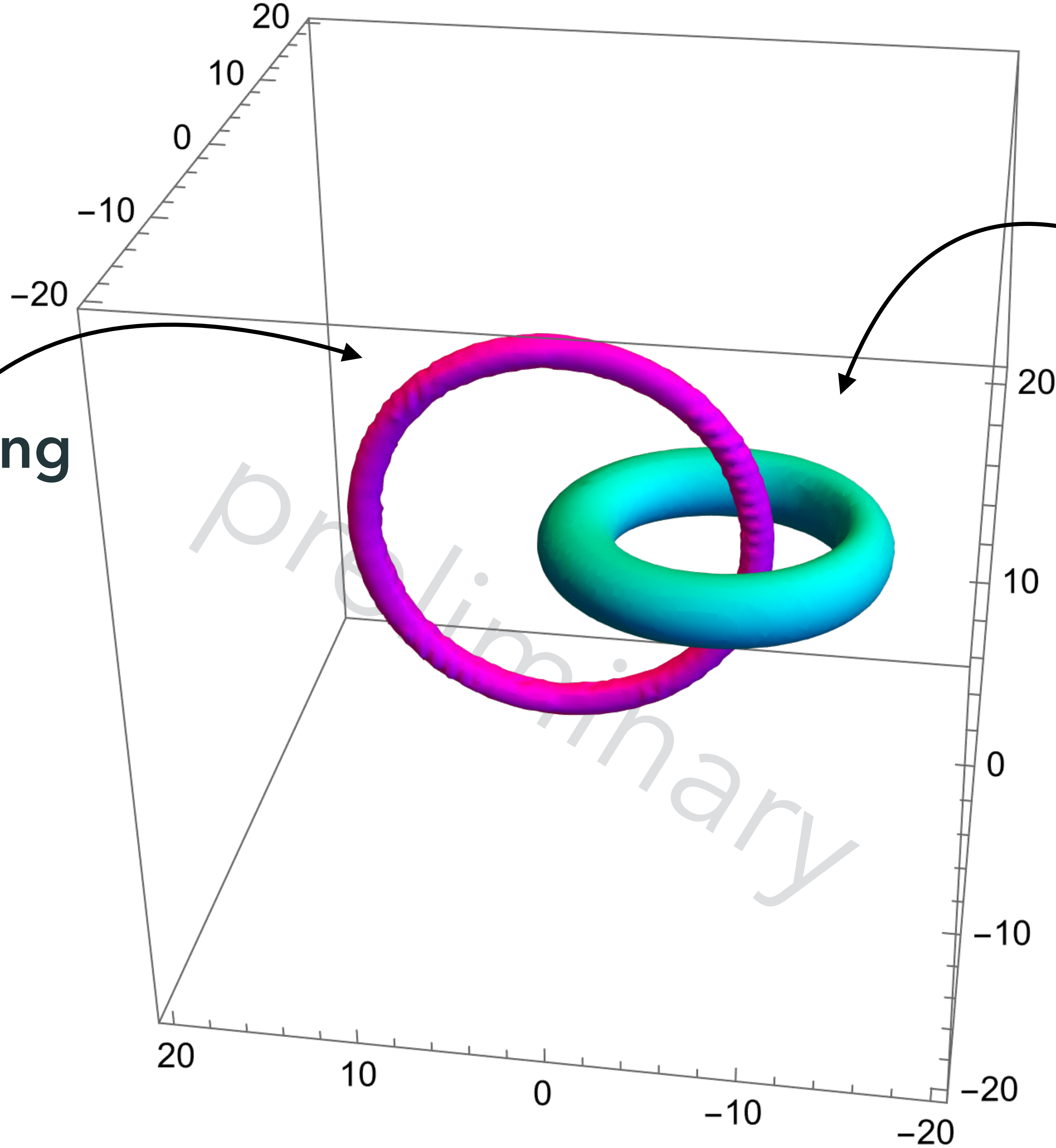
Link soliton

local string



global string

link soliton made of local & global strings!

Link soliton

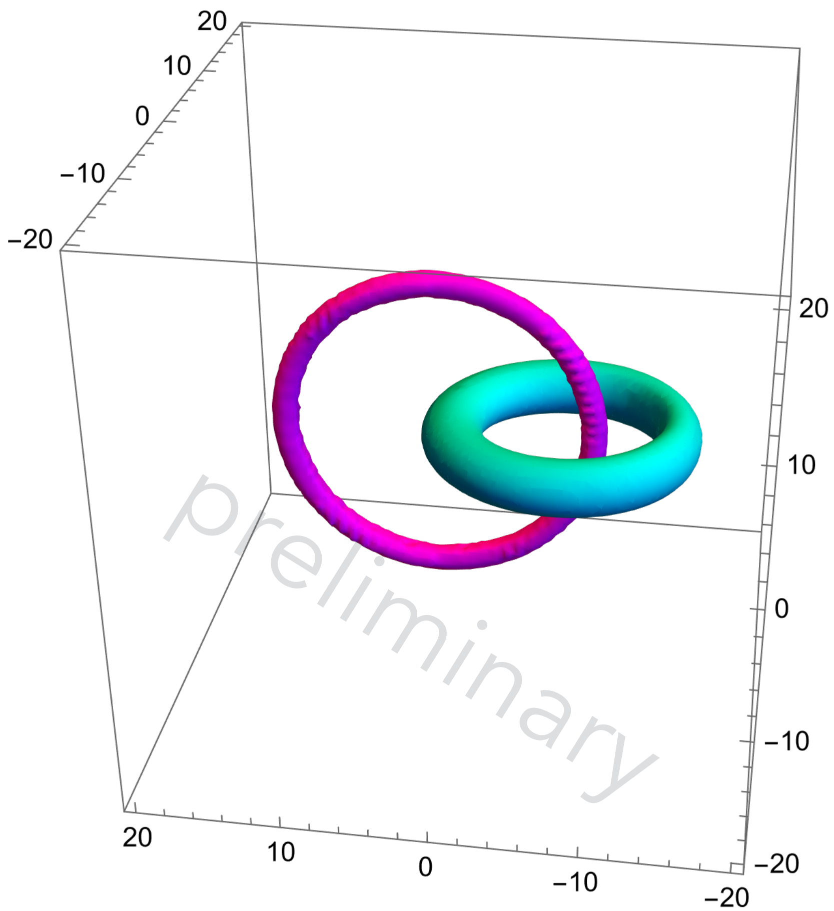
- stable solution of EOM (!)

- Key: Chern-Simons coupling like $\frac{c}{16\pi^2}\int d^4x\, aF\tilde{F}$

- 1st example of link soliton in gauge theory!

- Natural choice:

$$\begin{cases} U(1)_{global} \rightarrow U(1)_{PQ} \\ U(1)_{gauge} \rightarrow U(1)_{B-L} \end{cases}$$



- generally, global string can contain small flux

→linking flux, applicable for baryogenesis

link = origin of matter

Plan of talk

- Introduction
- Vortices w/ CS coupling (review)
- Stability of link soliton
- Baryogenesis
- Summary

Vortices w/ CS coupling (review)

[e.g., Horvathy-Zhang '08]

Abelian-Higgs w/ Chern-Simons

Let's start from 2+1D Abelian-Higgs w/ CS term:

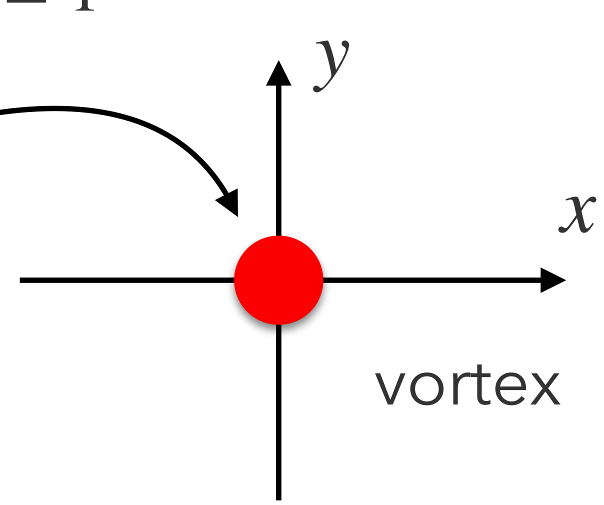
$$\mathscr{L} = |D_\mu \phi|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi) + g^2 c\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$

- For $c = 0$, A_0 is decoupled for static configurations.

→ static solution: $\begin{cases} \phi = v f(r)\, e^{i\theta} \\ A_\theta = a(r)/g \\ A_0 = A_r = 0 \end{cases}$ $\quad \begin{cases} f(0) = 0,\ f(\infty) = 1 \\ a(0) = 0,\ a(\infty) = 1 \end{cases}$

→ quantized magnetic flux $\int d^2x B = 2\pi/g$



Chern-Simons vortex

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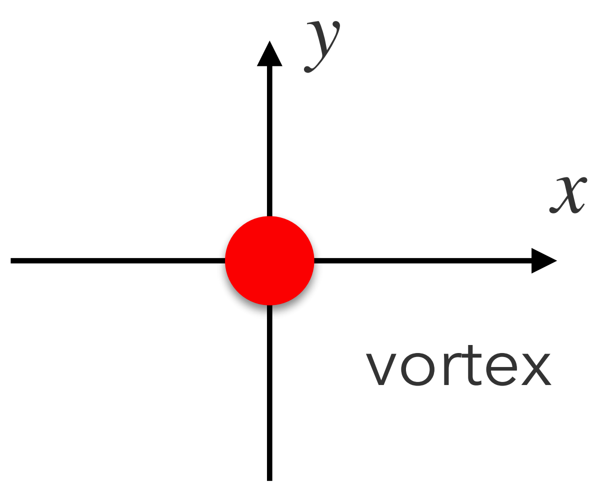
- For $c \neq 0$, A_0 is NOT, due to Gauss law constraint:

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + g^2 c B = 0$$

$$E_i = \partial_i A_0$$

$$J^0 \equiv \phi^\dagger i D^0 \phi + (h.c.)$$

→ **magnetic flux sauces electric field!**



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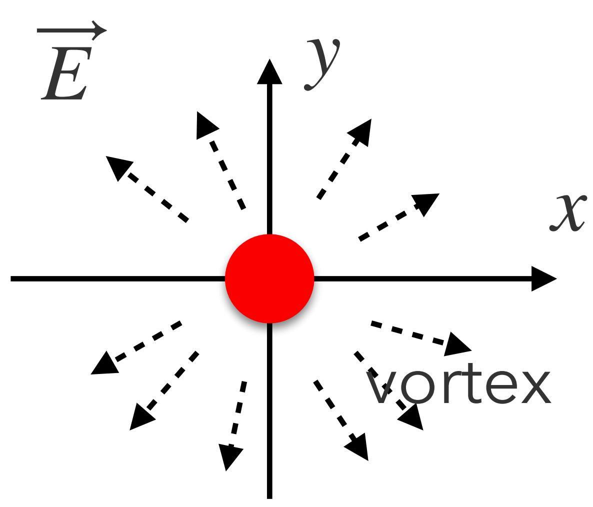

- quantized magnetic flux & electric charge

$$\int d^2x\, B = 2\pi/g \qquad \int d^2x\, J^0 = 2\pi c/g$$

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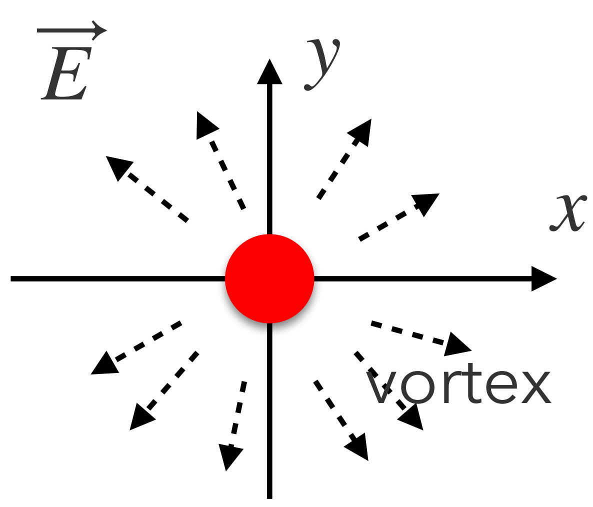

- quantized magnetic flux & electric charge

$$\int d^2x\, B = 2\pi/g \qquad \int d^2x\, J^0 = 2\pi c/g$$

called Chern-Simons vortex

Interaction of Chern-Simons vortices

- Single static solution: $\begin{cases} \phi = v f(r)\, e^{i\theta} \\ A_\theta = a(r)/g \\ A_0 = b(r)/g \end{cases}$ $\quad \begin{cases} f(0) = 0,\ f(\infty) = 1 \\ a(0) = 0,\ a(\infty) = 1 \\ b(0) = 0,\ b(\infty) = 0 \end{cases}$

- Asymptotic behavior at $r \to \infty$

$$1 - f(r) \sim e^{-M_\phi r} \qquad b(r) \sim 1 - a(r) \sim e^{-M_c r}$$

w/ $M_c \equiv gv\left(\frac{1}{2}\sqrt{4+c^2} - \frac{c}{2}\right) \simeq \mathcal{O}(gv/c)$ for $c \to \infty$

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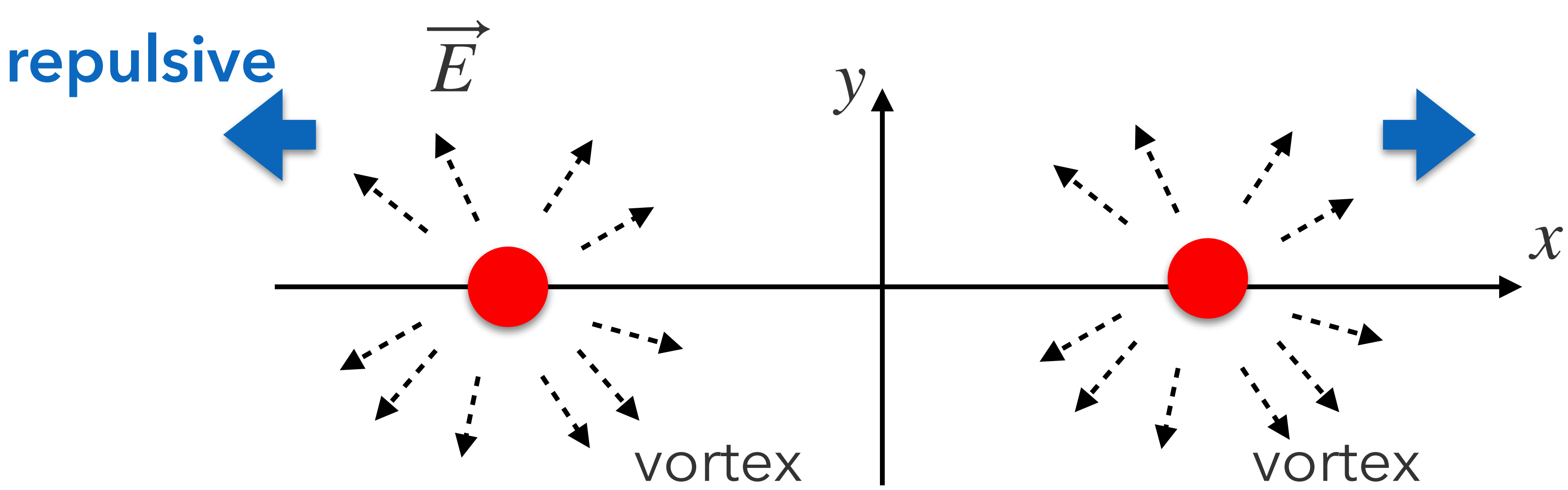
- For large c, **long-range repulsive force!**



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Stability of link soliton

The model

3+1D theory:

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$$

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- Symmetries:

$$U(1)_{gauge} : \phi_1 \to e^{i\theta_1}\phi_1 \qquad U(1)_{global} : \phi_2 \to e^{i\theta_2}\phi_2$$

- For $\kappa > 0$ & $\lambda > 0$, both symmetries are broken at the vacuum:

$$\langle \phi_1 \rangle = v_1, \; \langle \phi_2 \rangle = v_2$$

→ global & local strings

The model

3+1D theory:

CS coupling

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4g^2}F_{\mu\nu}^2 - V(\phi_1, \phi_2) \boxed{+\frac{c}{16\pi^2} aF_{\mu\nu}\tilde{F}^{\mu\nu}}$$

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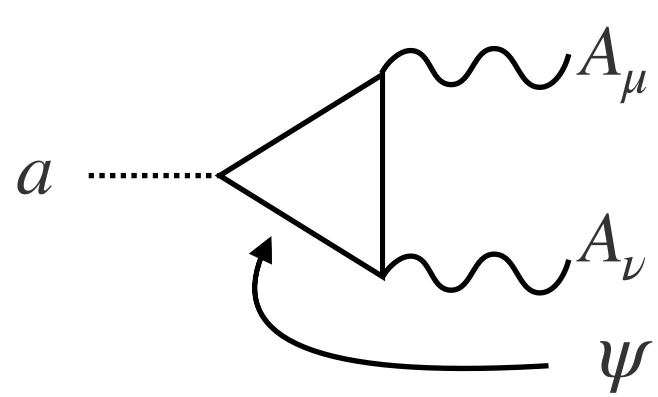
$$a \equiv -i \arg(\phi_2)$$

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- At the broken phase, CS coupling is induced by triangle anomaly.



dependent on matter sector

$$\Rightarrow c = \sum_f Q^f_{global}(Q^f_{gauge})^2$$

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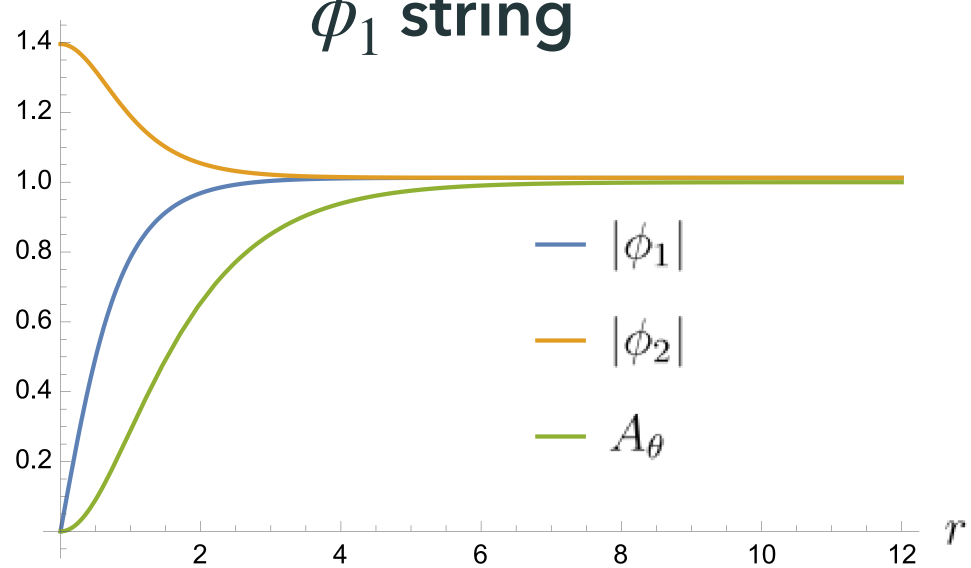
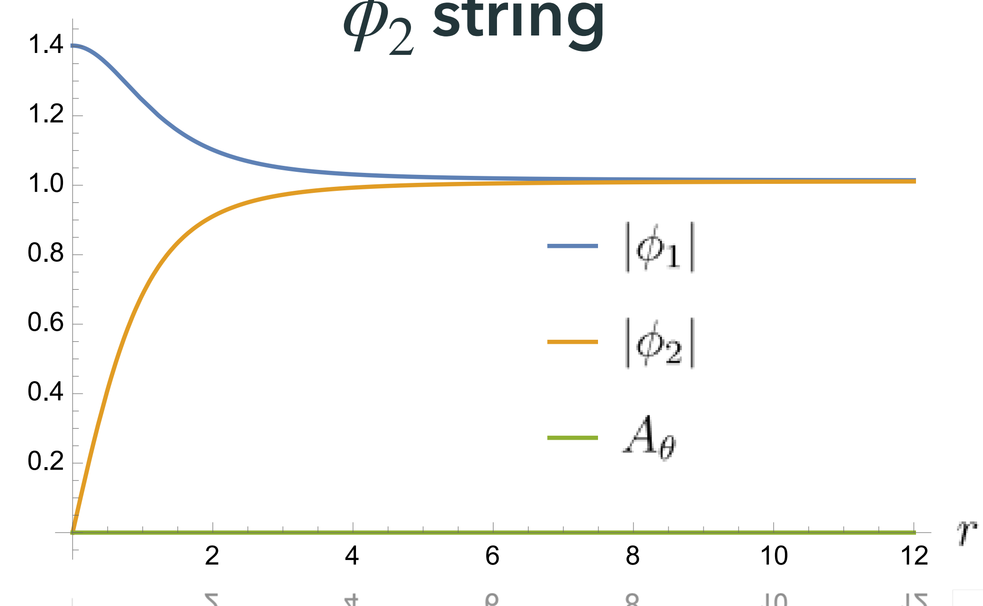
- CS coupling does not affect single strings.

ϕ_1 **string**



ϕ_2 **string**



Charged string

Rewriting CS coupling:

CS coupling

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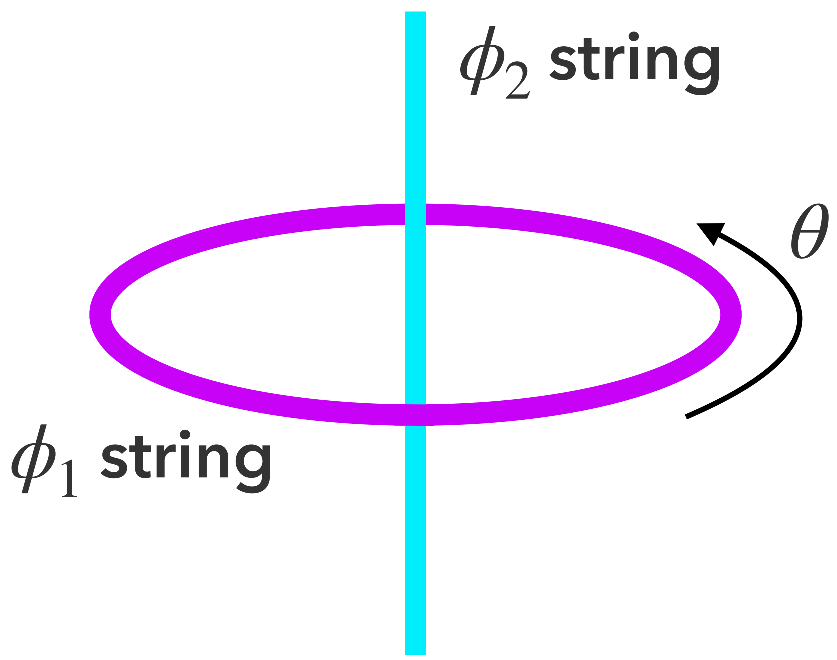

$$a = \theta \quad \Rightarrow \left(\partial_i a\right) A_0 B^i = \frac{1}{R} A_0 |\vec{B}|$$

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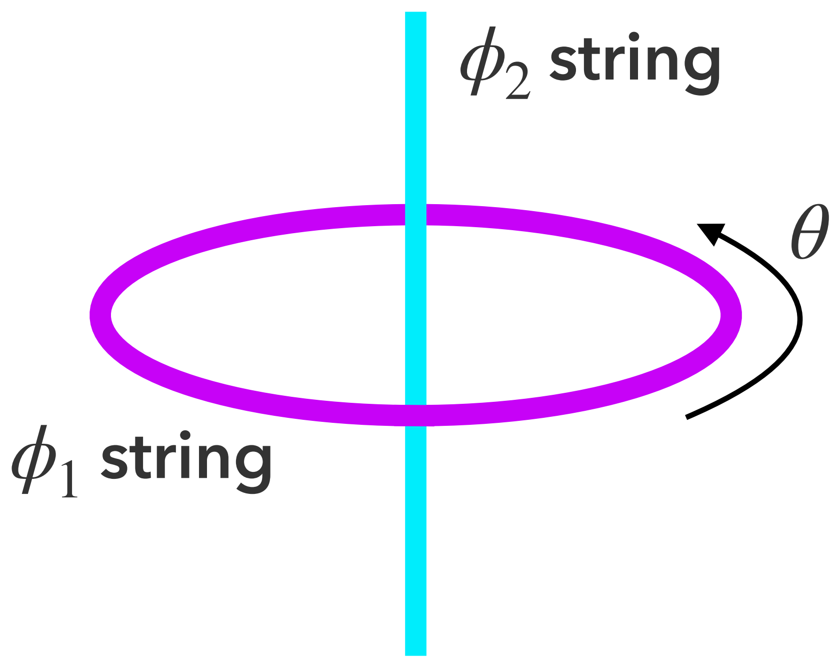

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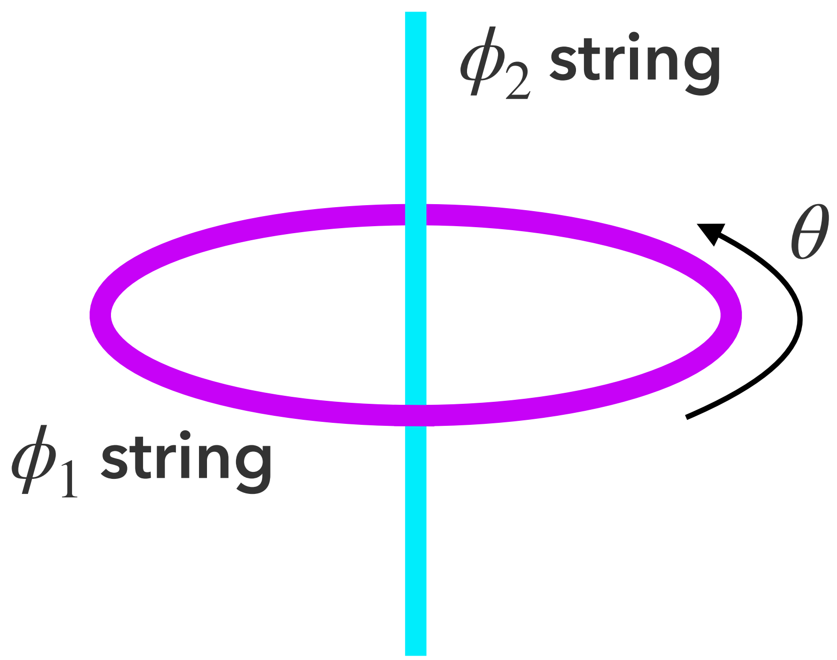

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$$\Rightarrow \int d^3x\, J^0 = 2\pi R \int d^2x \frac{c}{16\pi^2 R} |\vec{B}| = \frac{c}{4g}$$

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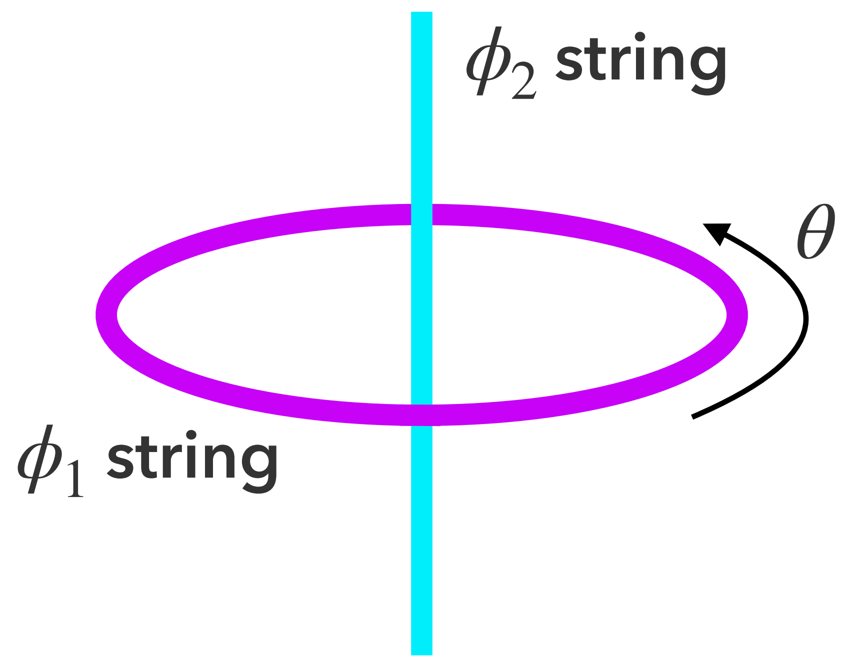

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ϕ_1 string is electrically charged! → doesn't shrink

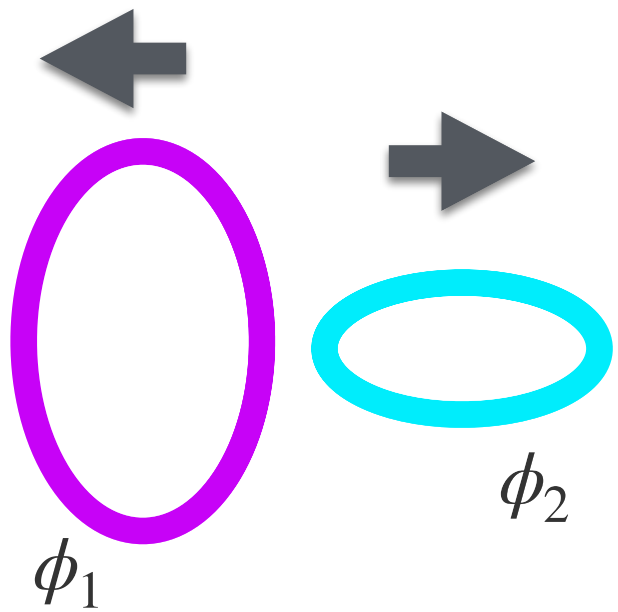
Stability

- Delinking by passing through each other?

  → prevented by taking $\lambda \gg g^2, \kappa, \chi$

  Overlap of strings ($\phi_1 = \phi_2 = 0$) cost large energy

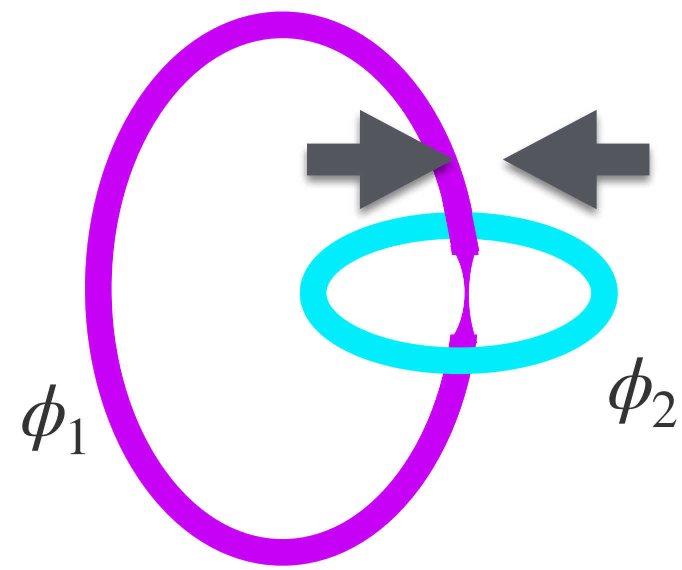
$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$



- ϕ_2 string is not charged and thus can shrink ?

  → prevented by taking $v_2/v_1 \ll 1$

  ϕ_2 string is too light to pinch ϕ_1 string



Numerical calculation

Energy:

$$\mathscr{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1, \phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2$$

$$-g^2|\phi_1|^2 A_0^2 - \frac{1}{2g^2}(\partial_i A_0)^2 - \frac{c}{16\pi^2}\, aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Not positive definite → remove A_0 by solving Gauss law:

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial^2 A_0 - 2g^2|\phi_1|^2 A_0 + \frac{g^2 c}{16\pi^2}(\vec{\nabla} a)\cdot \vec{B} = 0$$

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$\sim M_c^2 A_0 \sim \mathscr{O}(g^2 v_1^2/c^2)\, A_0$ for large c

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$\sim M_c^2 A_0 \sim \mathcal{O}(g^2 v_1^2/c^2)\, A_0$ for large c

$$\therefore A_0 \approx \frac{g^2 c}{16\pi^2}\frac{(\vec{\nabla} a)\cdot\vec{B}}{2g^2|\phi_1|^2}$$

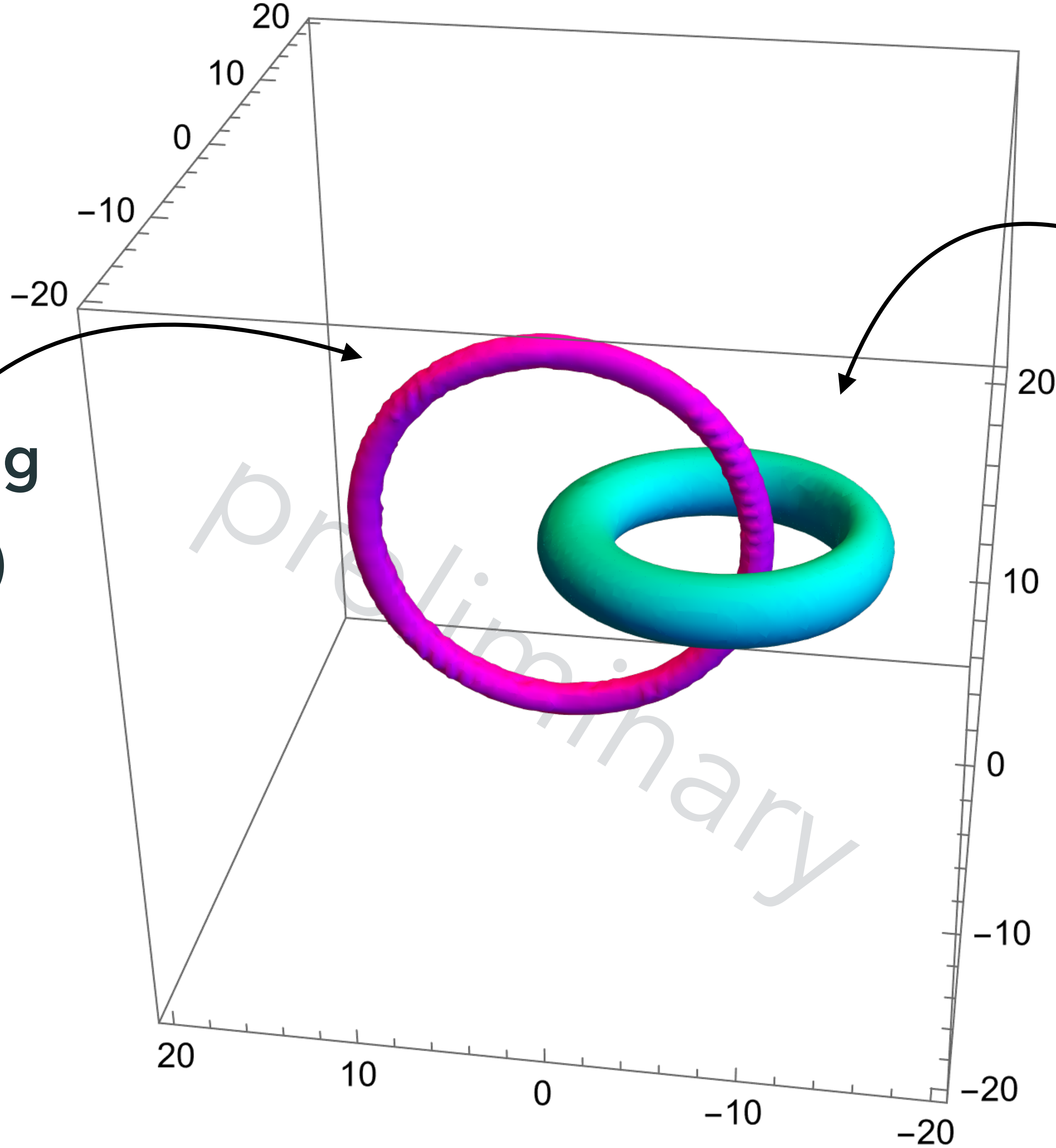
Numerical calculation

Energy:

$$\mathscr{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1,\phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2 + \left(\frac{g^2 c}{16\pi^2}\right)^2 \frac{\left((\vec{\nabla} a)\cdot\vec{B}\right)^2}{2g^2|\phi_1|^2}$$

- positive definite -> no obstacle
- Minimizing energy via non-linear conjugate gradient method
- CPU 400-cores parallelizing on YITP computer cluster
- lattice spacing $=0.4/gv_1$, $N = 100^3$, converged w/ O(1) hours

Numerical solution



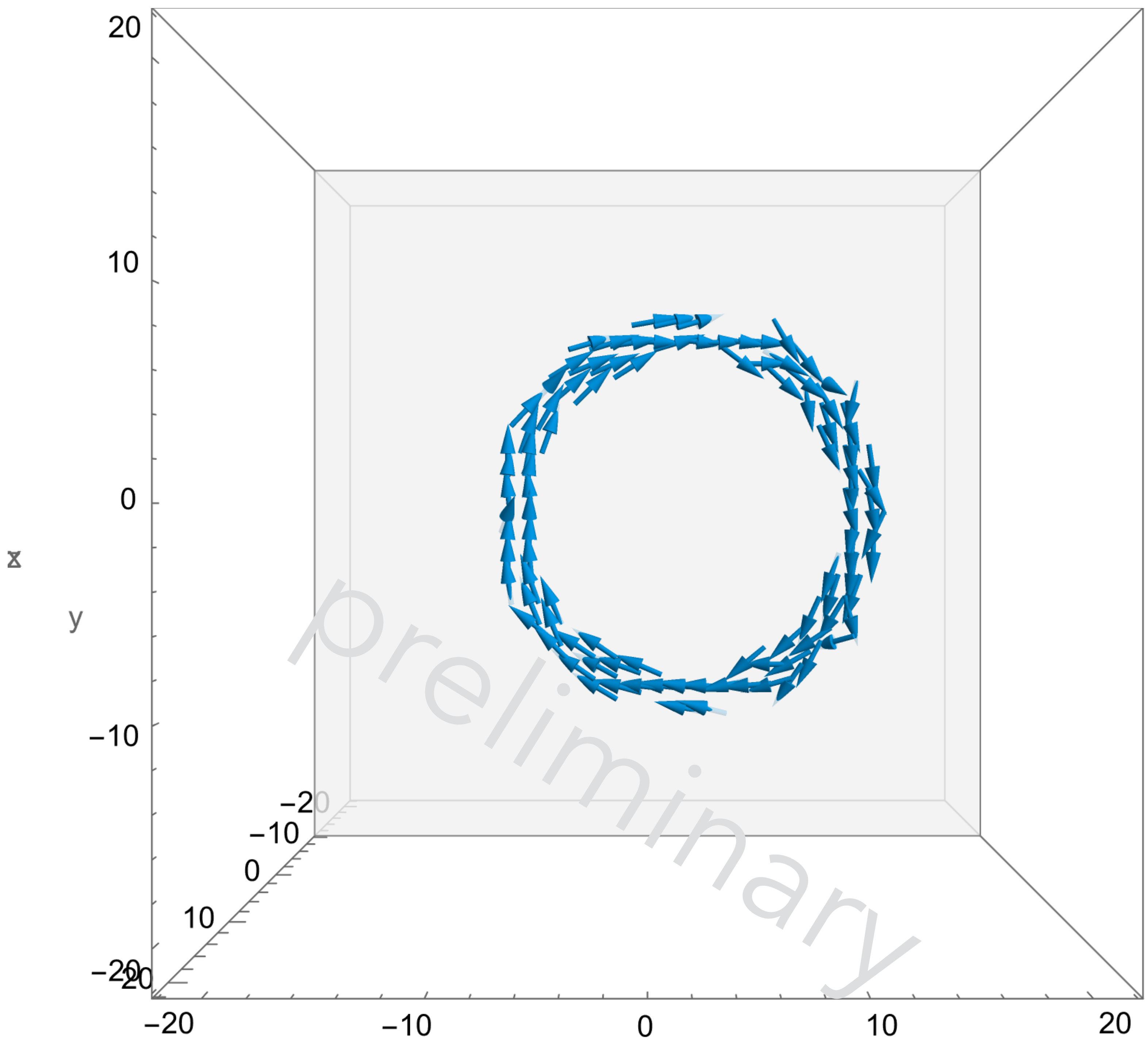
ϕ_1 **string (local)**

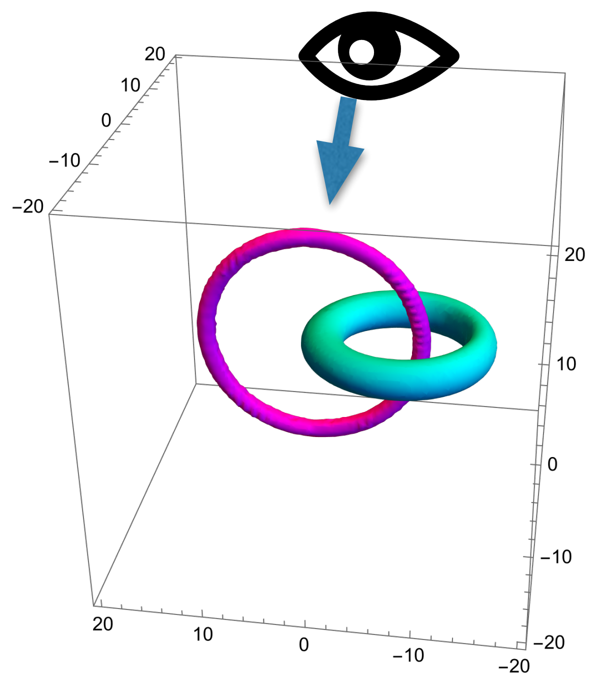
ϕ_2 **string (global)**

$\lambda/g^2 = 100,\ \chi/g^2 = 19.5,\ \kappa/g^2 = 0.1,\ v_2/v_1 = 0.05 \qquad g^2 c/(16\pi^2) = 16$

Magnetic field

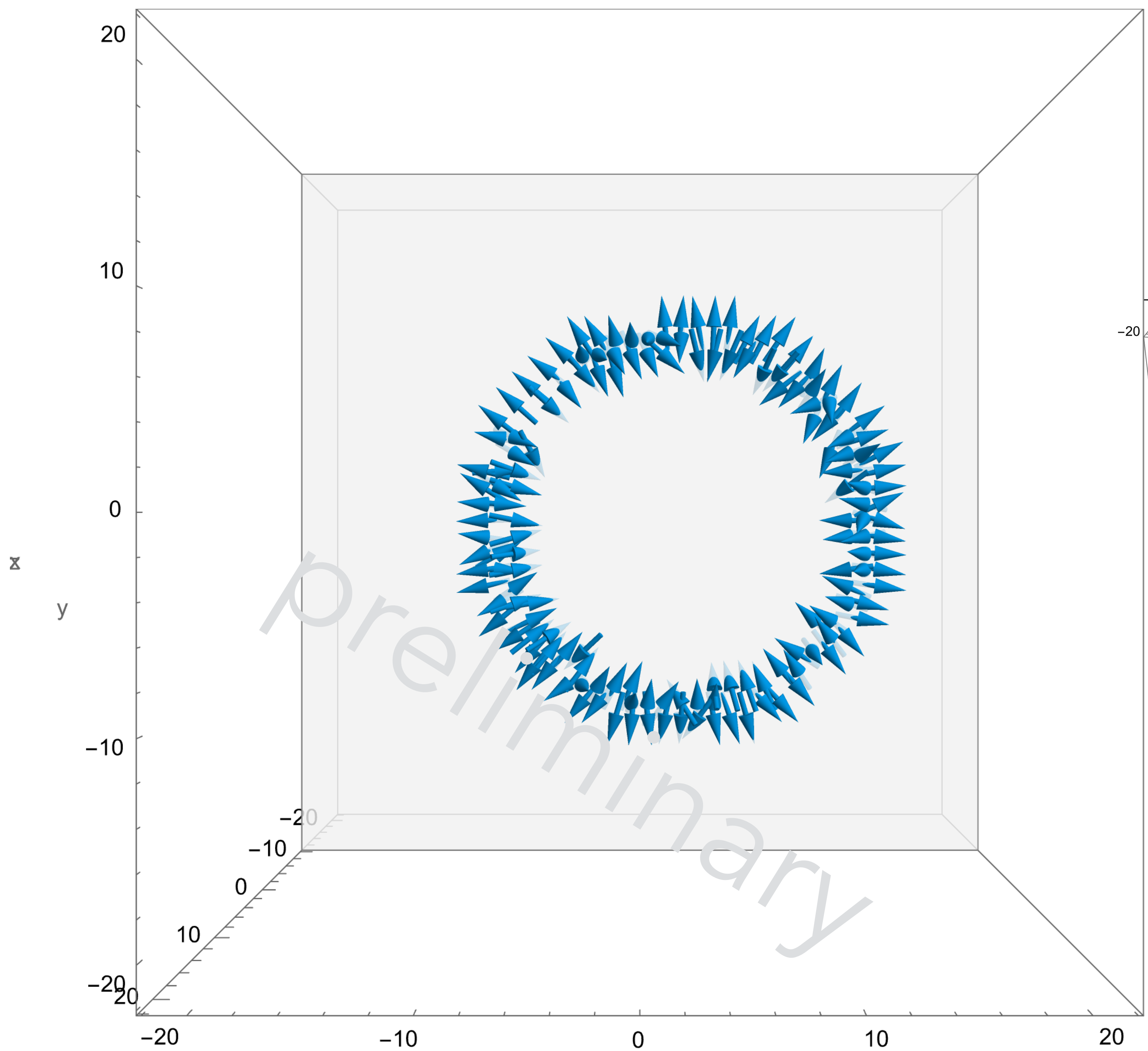
$\vec{B}$

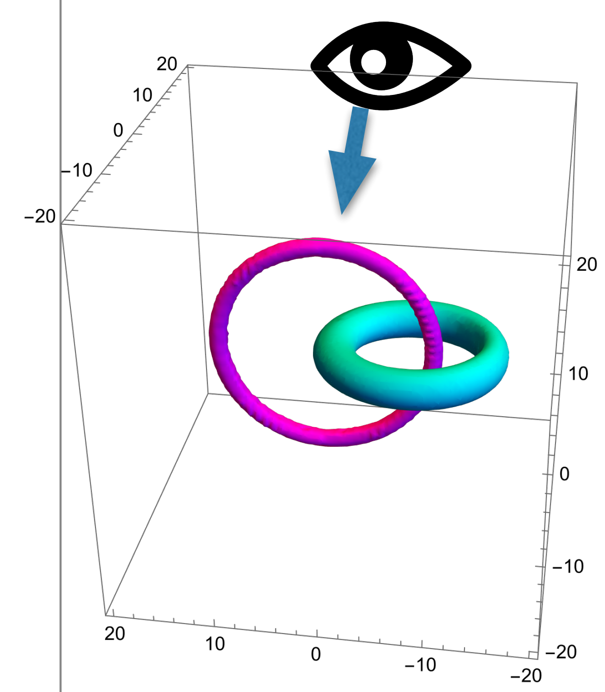




Electric field

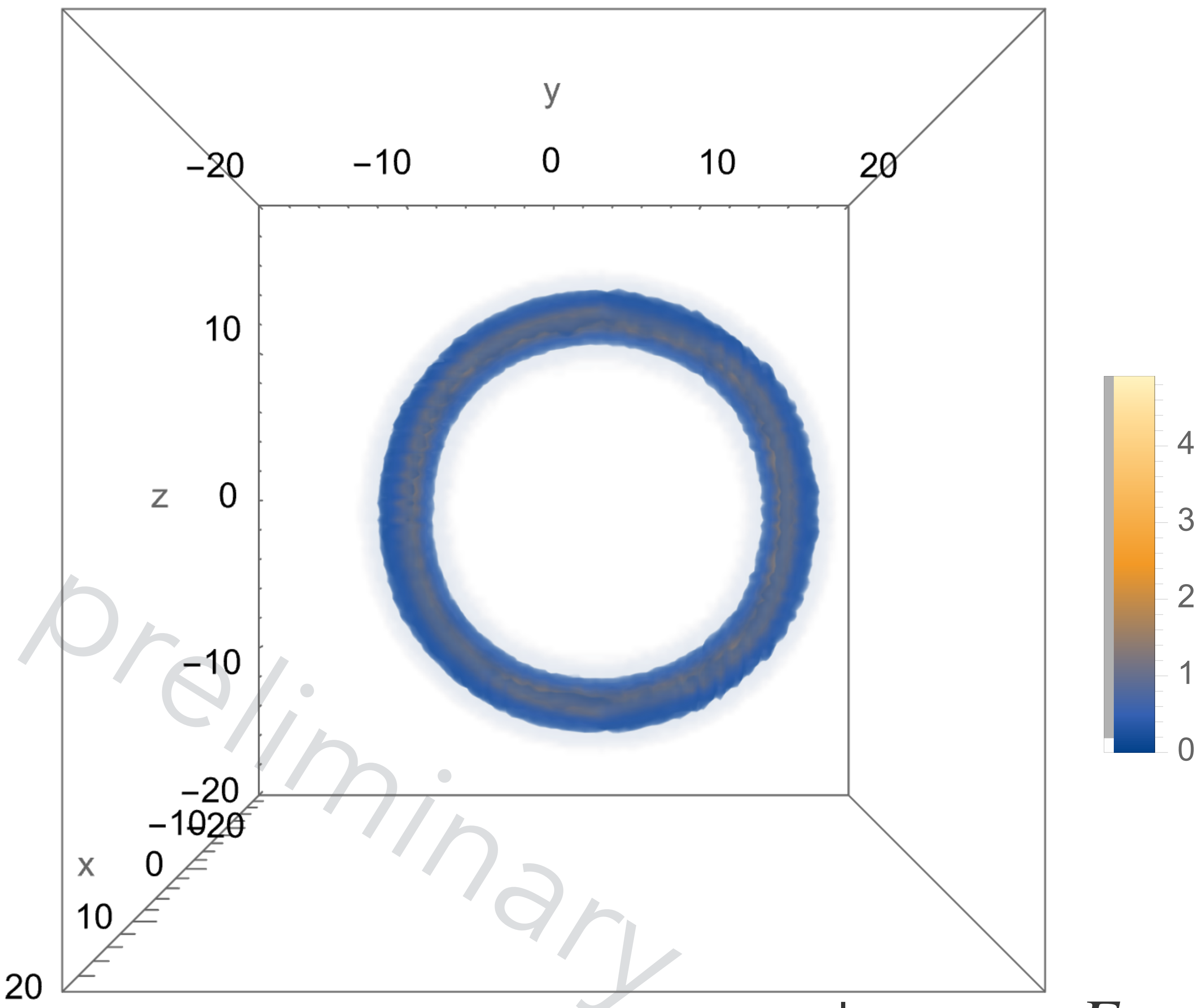
$$\vec{E} = \vec{\nabla} A_0$$

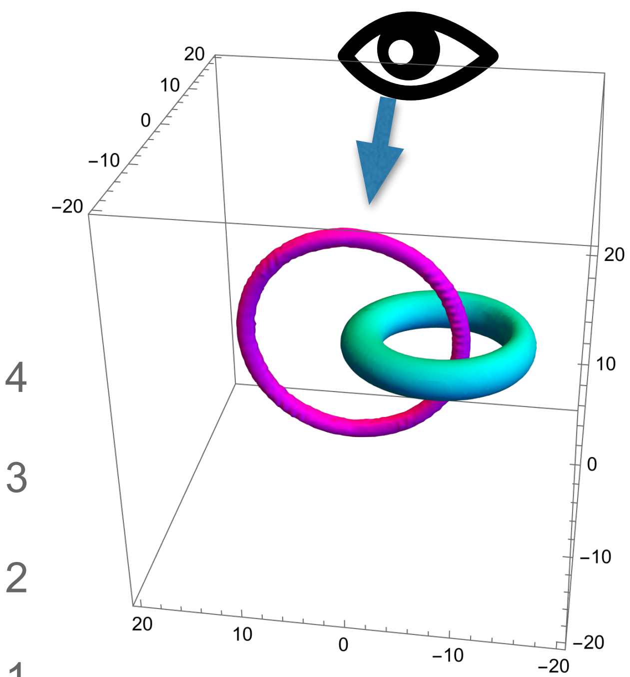




Energy

Energy is dominated by ϕ_1 string





total energy: $E \sim 401 g v_1$

General setup

More general charge assignment:

$$\mathscr{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$D_\mu \phi_1 = (\partial_\mu - igq_1 A_\mu)\phi_1 \qquad D_\mu \phi_2 = (\partial_\mu - igq_2 A_\mu)\phi_2 \qquad q_2/q_1 \ll 1$$

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- Symmetries:

$$U(1)_{gauge} : \begin{cases} \phi_1 \to e^{iq_1\theta_1}\phi_1 \\ \phi_2 \to e^{iq_2\theta_1}\phi_2 \end{cases} \qquad U(1)_{global} : \begin{cases} \phi_1 \to e^{-iq_2\theta_1}\phi_1 \\ \phi_2 \to e^{iq_1\theta_1}\phi_2 \end{cases}$$

General setup

More general charge assignment:

$$\mathscr{L} = |D_\mu\phi_1|^2 + |D_\mu\phi_2|^2 - \frac{1}{4g^2}F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$D_\mu\phi_1 = (\partial_\mu - i\underline{gq_1}A_\mu)\phi_1 \qquad D_\mu\phi_2 = (\partial_\mu - i\underline{gq_2}A_\mu)\phi_2 \qquad q_2/q_1 \ll 1$$

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- Symmetries:

$$U(1)_{gauge} : \begin{cases} \phi_1 \to e^{iq_1\theta_1}\phi_1 \\ \phi_2 \to e^{iq_2\theta_1}\phi_2 \end{cases} \qquad U(1)_{global} : \begin{cases} \phi_1 \to e^{-iq_2\theta_1}\phi_1 \\ \phi_2 \to e^{iq_1\theta_1}\phi_2 \end{cases}$$

→ Also ϕ_2 string contains magnetic flux, but the solution is almost the same.

General setup

More general charge assignment:

$$\mathscr{L} = |D_\mu\phi_1|^2 + |D_\mu\phi_2|^2 - \frac{1}{4g^2}F_{\mu\nu}^2 - V(\phi_1, \phi_2) \ + \frac{c}{16\pi^2} aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$D_\mu\phi_1 = (\partial_\mu - i\underline{gq_1}A_\mu)\phi_1 \qquad D_\mu\phi_2 = (\partial_\mu - i\underline{gq_2}A_\mu)\phi_2 \qquad q_2/q_1 \ll 1$$

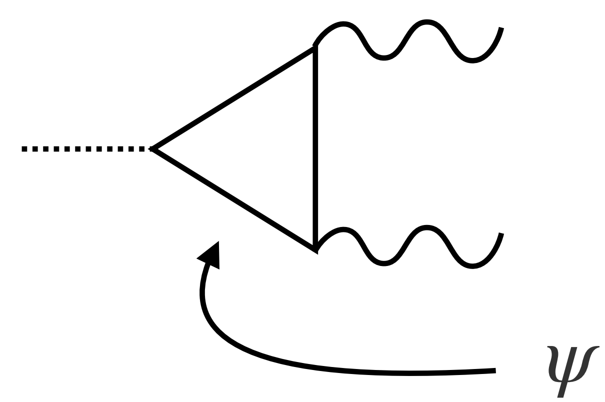
$$V(\phi) = \lambda\left(|\phi_1|^2 + |\phi_2|^2 - \mu^2\right)^2 - \kappa|\phi_1|^2|\phi_2|^2 + \chi|\phi_2|^4$$

- Definition of "axion" is more complicated

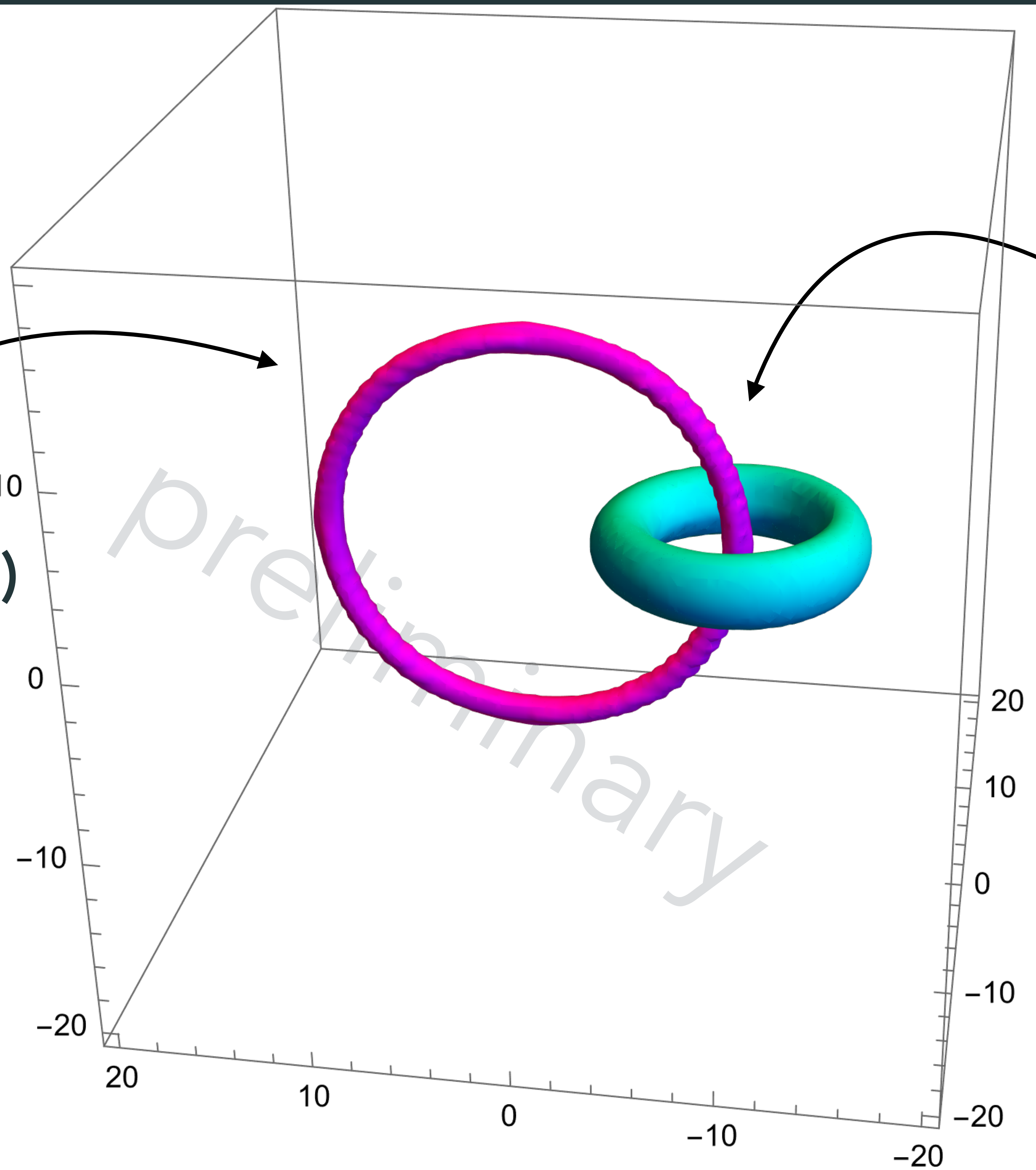
$$a \equiv \frac{1}{i\sqrt{q_2^2v_1^2 + q_1^2v_2^2}} \left(-q_2v_1\arg(\phi_1) + q_1v_2\arg(\phi_2)\right)$$

- Triangle anomaly is also complicated

  → c will be taken as free parameter.



Numerical solution

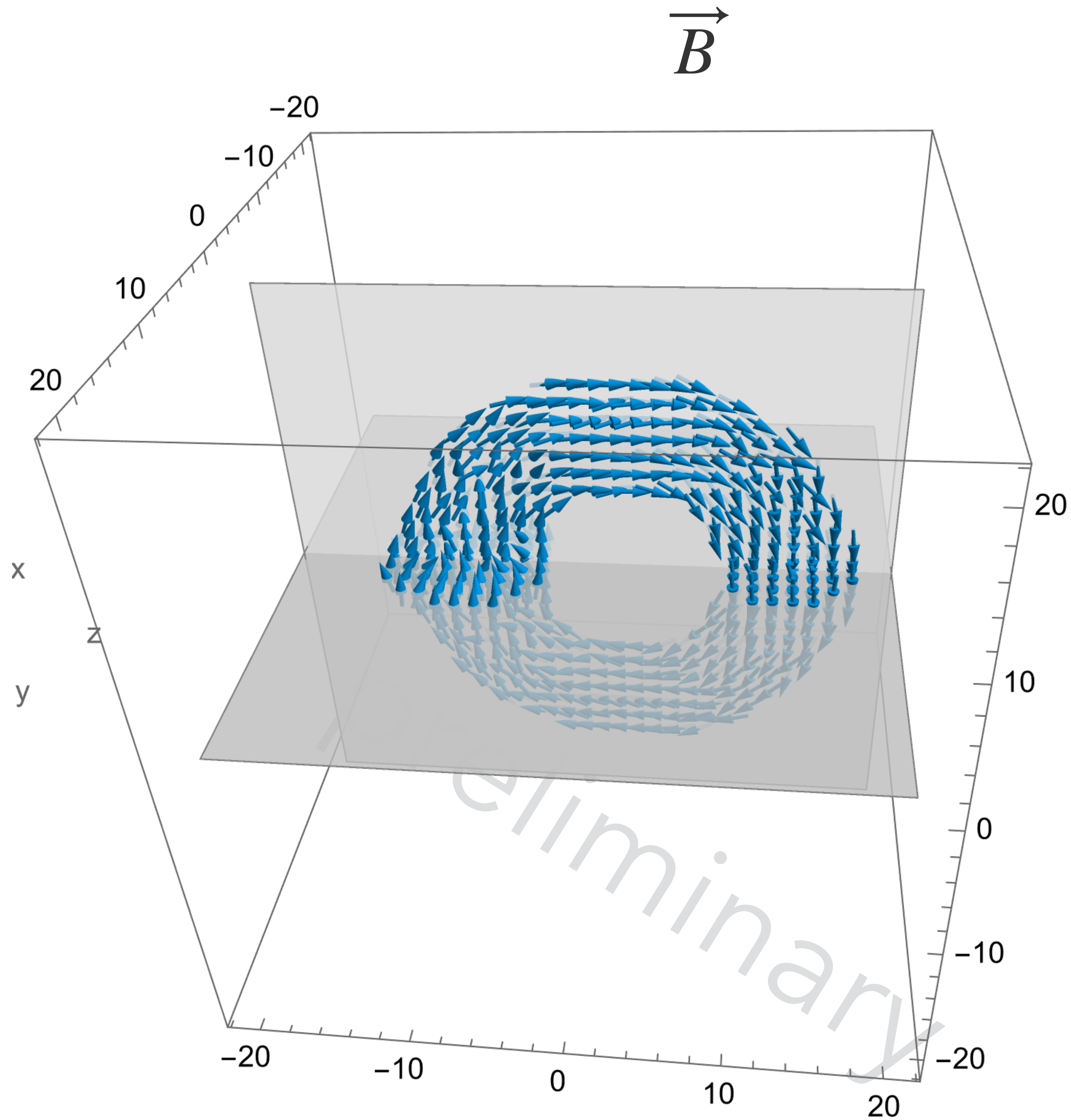


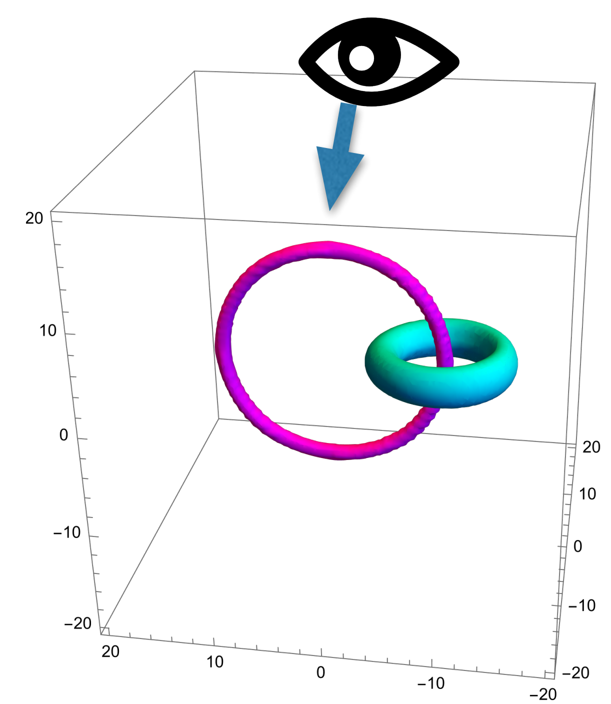
ϕ_1 string
(almost local)

ϕ_2 string
(almost global)

$\lambda/g^2 = 100,\ \chi/g^2 = 19.5,\ \kappa/g^2 = 0.1,\ v_2/v_1 = 0.05$ $\quad g^2 c/(16\pi^2) = 16$ $\quad q_2/q_1 = 0.1$

Magnetic field





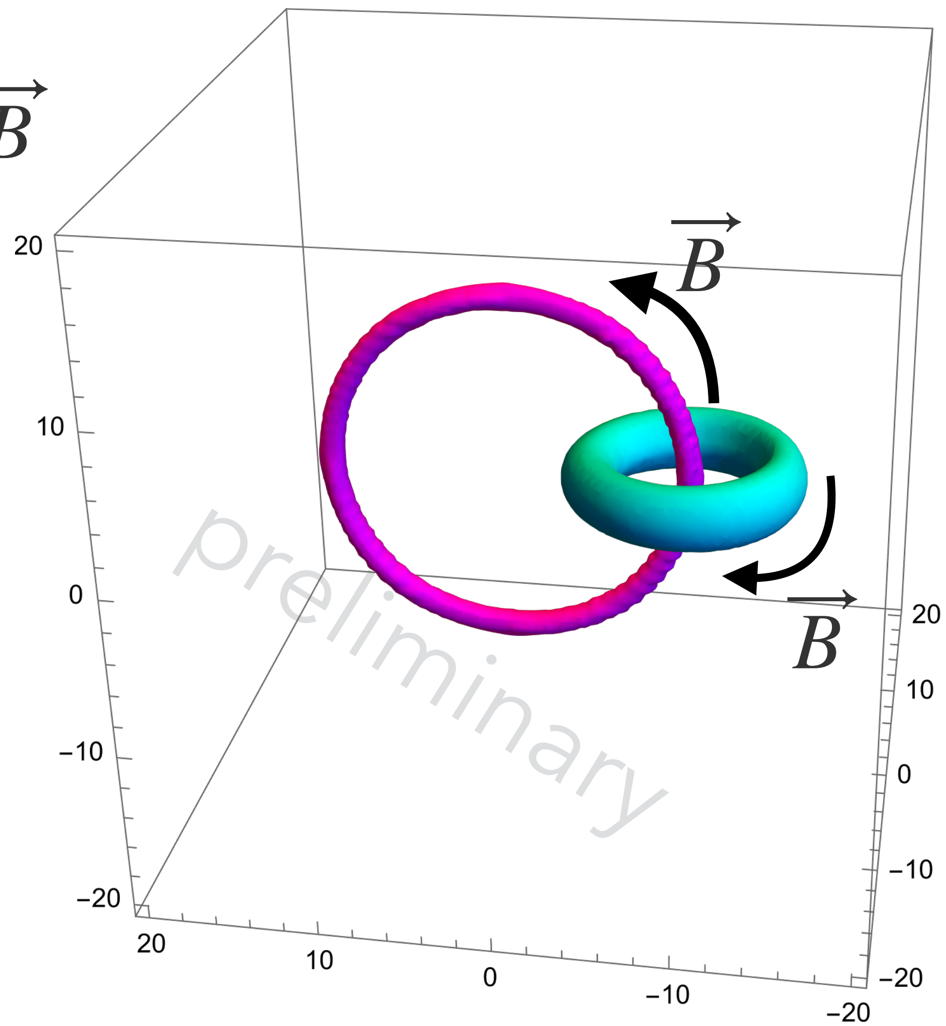
Helical magnetic field

- Since the magnetic fluxes are linked, this soliton has finite helicity (Chern-Simons number):

$$N_{CS}[A] \equiv \frac{1}{16\pi^2}\int d^3x\, AdA = \frac{1}{16\pi^2}\int d^3x\, \vec{A}\cdot\vec{B}$$

For $q_1 = 1,\ q_2 = 0.1, \quad N_{CS}[A] \simeq 0.28$

$(N_{CS} \simeq 0$ for $q_1 = 1,\ q_2 = 0)$



can be used for baryogenesis!

(cf: baryogenesis by helical $U(1)_Y$ field) [Kamada-Long '16]

Plan of talk

- Introduction
- Vortices w/ CS coupling (review)
- Stability of link soliton
- Baryogenesis
- Summary

Baryogenesis (work in progress)

The model

$$\mathscr{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2) \; + \frac{c}{16\pi^2} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- Natural setup: $U(1)_{gauge} = U(1)_{B-L}$ & $U(1)_{global} = U(1)_{PQ}$

  Type-I seesaw → ν-mass

  QCD axion → strong CP & Dark matter

  $\Rightarrow v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$

- Axion quality problem can be avoided by gauged PQ mechanism.

  $\Rightarrow q_1 = 1,\ q_2 = 0.1$ [Fukuda-Ibe-Suzuki-Yanagida '17]

- Assume kinetic mixing with $U(1)_Y$ in SM: $\mathscr{L} \supset \frac{\epsilon}{2} Y_{\mu\nu} F^{\mu\nu}$

  → contains $U(1)_Y$ helicity: $N_{CS}[Y] \simeq \epsilon^2 N_{CS}[A]$

Fate of link soliton

- produced by Kibble mechanism or thermal fluctuation at $T \sim v_1, v_2$

- classically stable but can decay by quantum tunneling

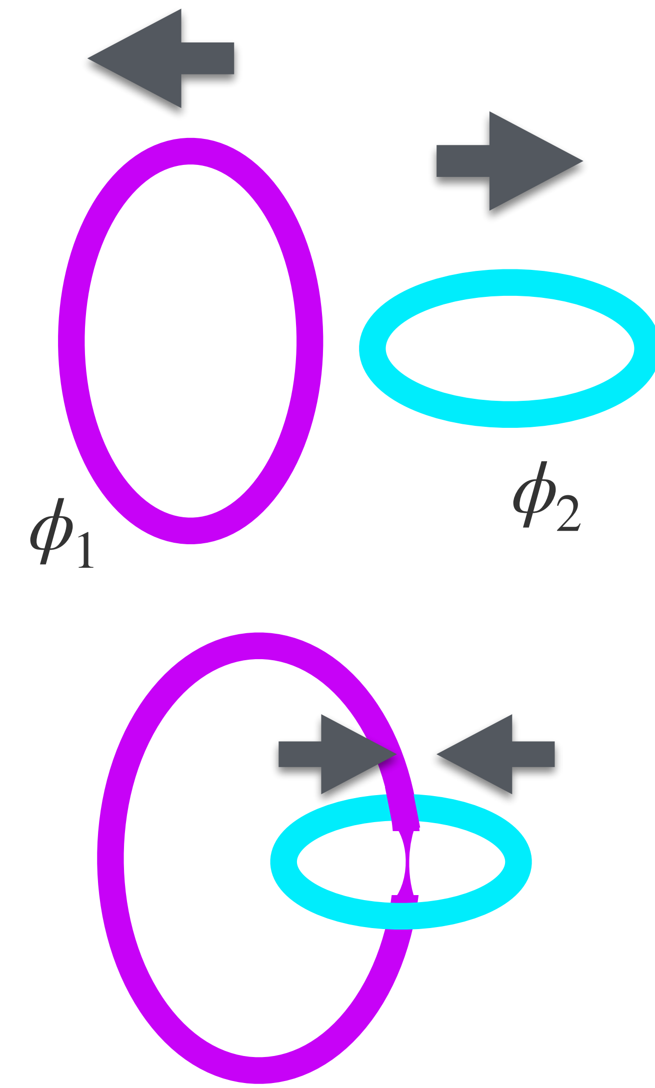
- decay after electroweak phase transition

  → change of helicity: $\Delta N_{CS}[Y] \simeq \epsilon^2 N_{CS}[A]$

  → baryon # is generated through chiral anomaly:

$$\boxed{\Delta B = \epsilon^2 N_{CS}[A] \text{ per link}}$$

$$(\partial_\mu J^\mu_B \sim Y\tilde{Y} + \mathrm{tr}\, W\tilde{W})$$



Baryon # from link

- Naively, anti-link is also produced → $n_{link} - n_{\overline{link}} = 0$?
- need "chemical potential" μ (discussed later) when produced:

$$\frac{n_{link} - n_{\overline{link}}}{s} \simeq \frac{\mu}{T}\frac{n_{link}}{s} \simeq \frac{\mu}{T}10^{-6}$$

We have used $n_{link} \sim 10^{-4}\,T^3$

[Vachaspati '84]

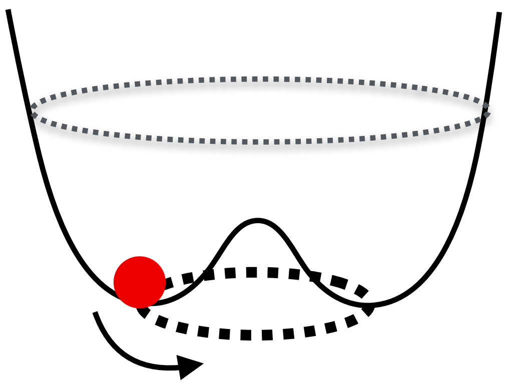
→ generated total baryon # due to decay:

$$\boxed{\eta_B = \frac{n_B}{s} \sim 10^{-10}\left(\frac{\epsilon}{0.1}\right)^2\left(\frac{\mu/v_1}{0.1}\right)}$$

Origin of chemical potential?

- Simplest choice: rotating pseudo scalar (axion-like particle)

$$\Delta\mathscr{L} = \frac{c}{16\pi^2} a' F_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{c}{16\pi^2} \underbrace{(\partial_0 a')}_{\equiv \mu_{eff}} AdA$$



(cf: Affleck-Dine mechanism, axiogenesis [Co-Harigaya '19])

$$\boxed{\eta_B = \frac{n_B}{s} \sim 10^{-10}\left(\frac{\epsilon}{0.1}\right)^2 \left(\frac{\dot{a}'|_{T\sim v_1}}{0.1 v_1}\right)}$$

Once the link asymmetry is produced, it remains until decay.

→ **later & earlier dynamics of ALP are irrelevant.**

Testability

- Before the links decay, they dominate the energy density of universe.

$$\left.\frac{\rho_{link}}{\rho_\gamma}\right|_{T\sim T_{EW}} = \left.\frac{M_{link} n_{link}}{T^4}\right|_{T\sim T_{EW}} \simeq 10^{-2}\,\frac{v_1}{v_{EW}} \gg 1$$

- The entropy production due to decay cannot be ignored.

  → distorts spectrum of primordial gravitational wave

  → **probed by primordial gravitational wave?**

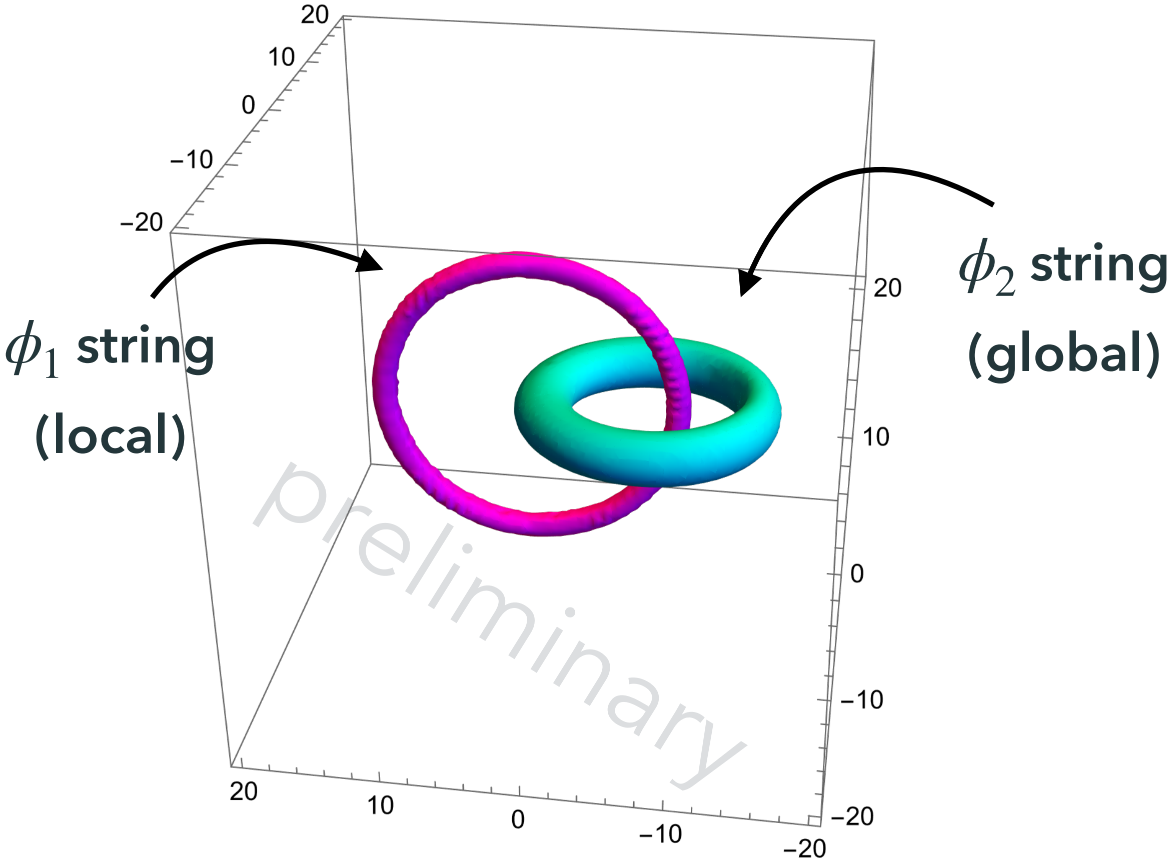
Summary

- 1st example of link soliton in realistic models

- Key: CS coupling $\frac{c}{16\pi^2}\int d^4x\, aF\tilde{F}$

- Natural setup

$$\begin{cases} U(1)_{global} \to U(1)_{PQ} \\ U(1)_{gauge} \to U(1)_{B-L} \end{cases}$$



→linking flux, applicable for baryogenesis

link = origin of matter