# Cosmic strings from pure Yang-Mills theory

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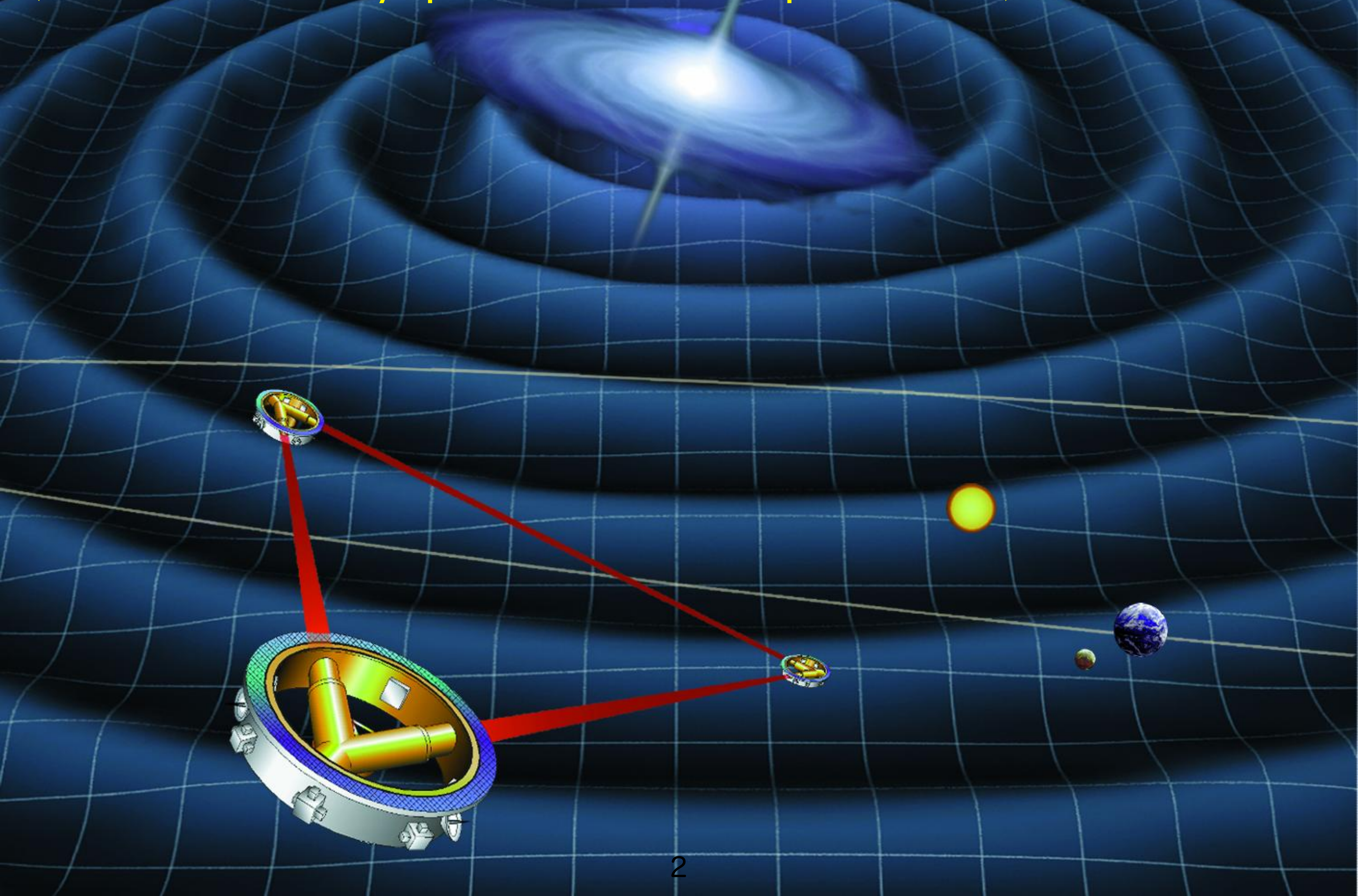
Based on

- [2204.13123], [2204.13125] with Masaki Yamada

# Gravitational wave experiments

(Don't ask me any question about experiments!)



# Introduction

A potential source of gravitational waves: Cosmic string

- 1+1 dimensional cosmic-size object with some energy density.
- A string can exist in field theory and string theory. Even the fundamental string of superstring theory can be very large to be observed in the Universe!
- Depending on its nature, strings may or may not produce observable amount of gravitational waves.

# Introduction

What is the easiest model of cosmic string (without NG boson)?

**Abelian-Higgs model**

$A_\mu$ : $U(1)$ gauge field

$\phi$ : A scalar with charge 1 under the $U(1)$

$$\mathscr{L} = -\frac{1}{4e^2}F^{\mu\nu}F_{\mu\nu} - D_\mu\phi D^\mu\phi^* - V(\phi)$$

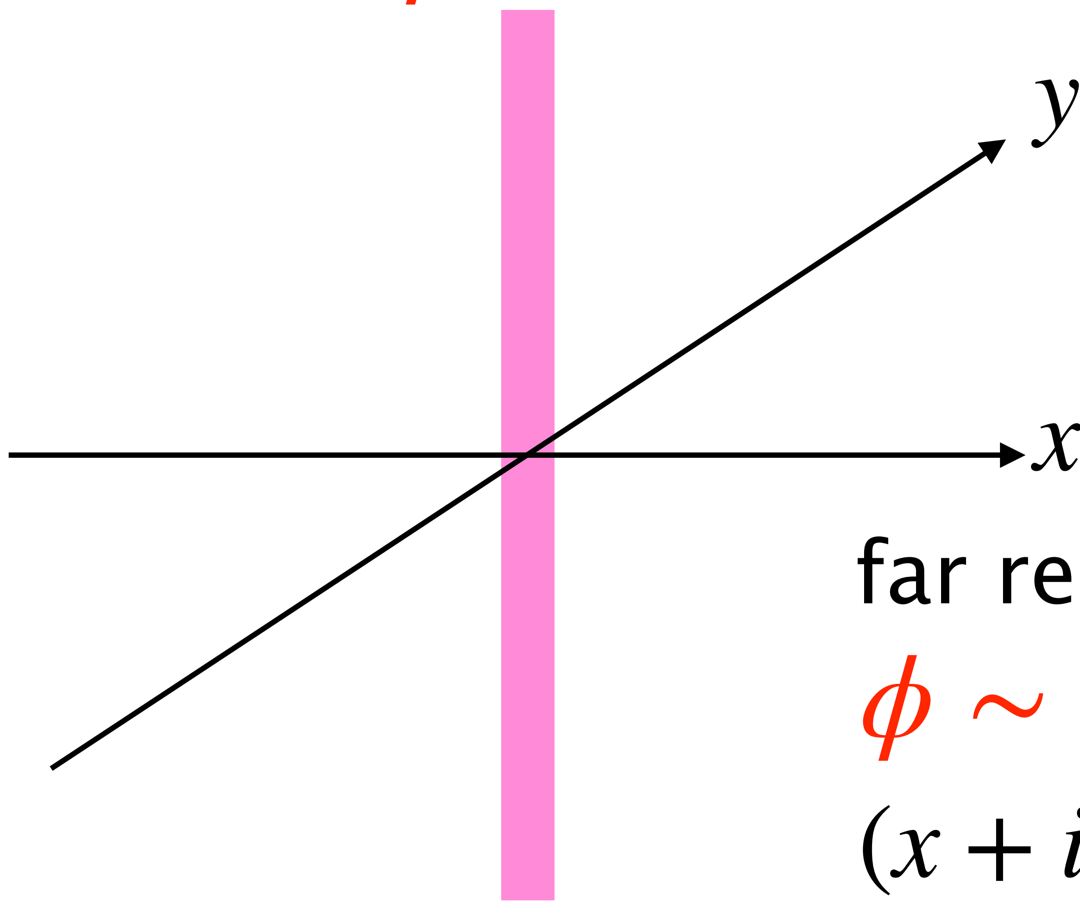
Potential $\quad V(\phi) = \lambda(|\phi|^2 - v^2)^2$

$\langle|\phi|\rangle = v$ : $U(1)$ is spontaneously broken

# Introduction

string core

$\phi \sim 0$




far region

$\phi \sim ve^{i\theta}$

$(x + iy = re^{i\theta})$

# Introduction

Abelian-Higgs model is the easiest to understand for Human.

However, is it the simplest model in Nature?

- The Abelian-Higgs model contains two different kinds of fields $\phi$ and $A_\mu$. One of them is the elementary scalar $\phi$.

- Three parameters : $e$, $\lambda$, $v$

  The dimensionful parameter $v$ is not naturally small:
  Why isn't it of order the Planck scale?

  (The cosmological constant and the SM Higgs might be explained by anthropic principle.)

# Introduction

The simplest model to the best of my knowledge:

**Pure Yang–Mills**

$A_\mu$ : Gauge field for a Lie group $G$ (e.g. $G = SU(2)$ )

$$\mathscr{L} = -\frac{1}{4g^2}\,\mathrm{tr}\, F^{\mu\nu}F_{\mu\nu} \qquad (g : \text{coupling})$$

**Very simple !!!**

- The model contains one kind of field $A_\mu$.
- One parameter $g \to \Lambda$ : just the dynamical scale $\Lambda$.

# Introduction

Q. Is there very strong motivation that pure Yang–Mills should exist at some high energy scale?

A. No. (But it is not restricted to pure Yang–Mills.)

Q. Is there very strong motivation that pure Yang–Mills should not exist at some high energy scale?

A. No. I'm not aware of any strong reason to exclude it.

- String theory can typically contain many gauge fields.
- Yang–Mills is perturbatively massless.
- Its low scale is naturally generated by running coupling.

# Introduction

**The purpose of this talk:**

**To explain why pure Yang–Mills can produce cosmic strings which emit observable amount of gravitational waves.**

<u>Remarks</u>

Most of the content of this talk is known to some experts.
[Witten, Polchinski,...]

However, this possibility seems to be not investigated in detail and not well-recognized in cosmology community.

# Contents

# Color flux tube

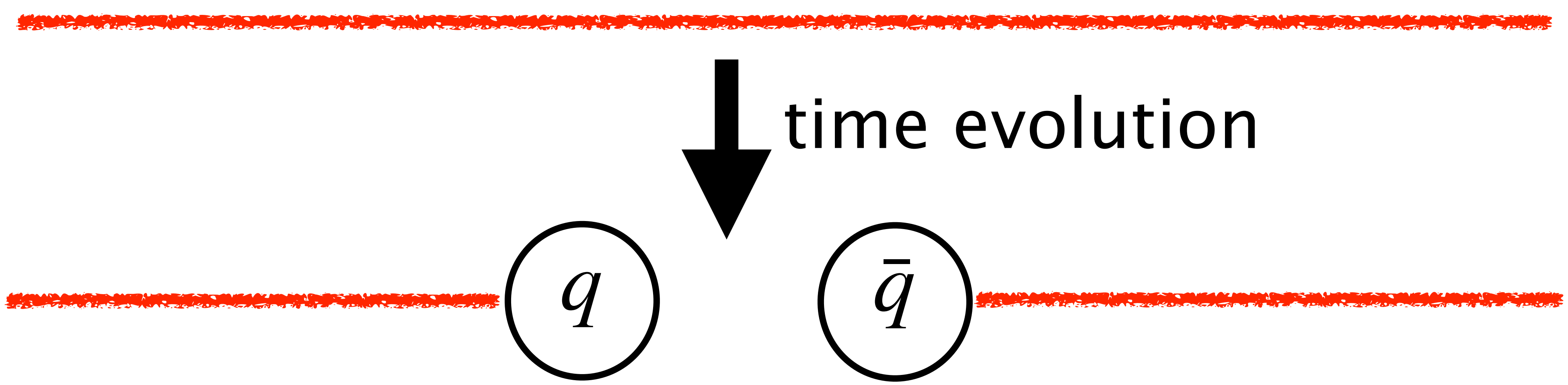
In QCD with quarks, a well-known picture of confinement is by a color flux tube.


If there is no quark:

color flux tube as a dynamical string

# Stability of color flux tube

**If dynamical quarks exist:**




The string decays by a pair creation of quark-antiquark.
(Remark: It is metastable if the quark is heavy.)

**If dynamical quarks do not exist:**

color flux tube is stable

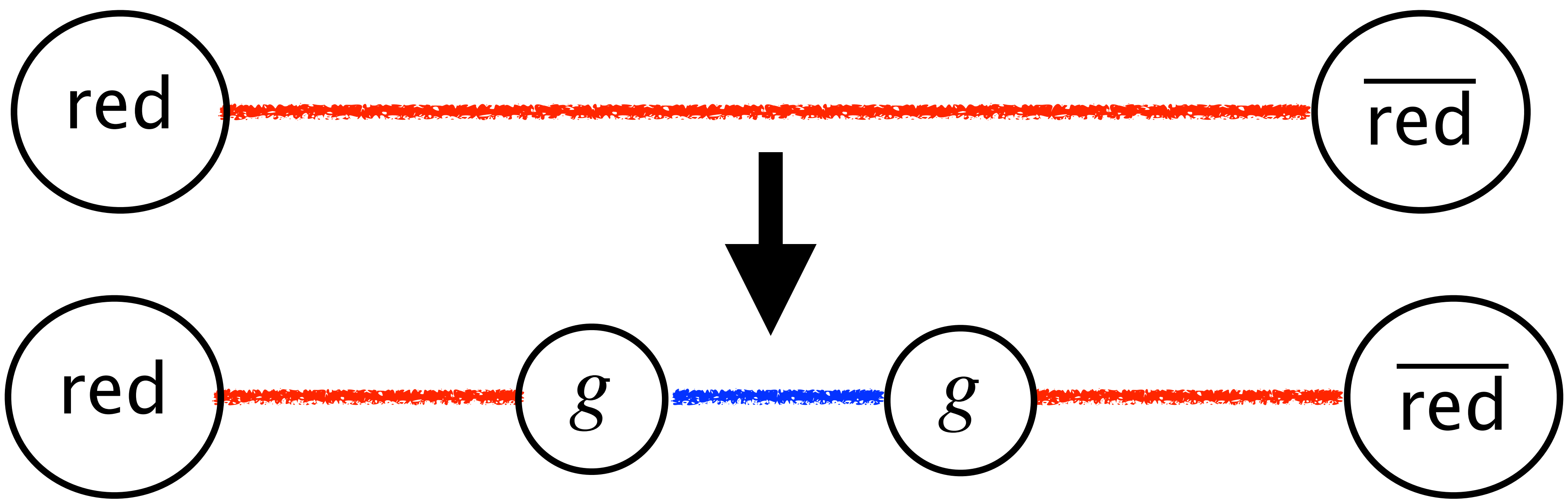
# Types of color flux tubes

Suppose (e.g. for $SU(3)$ )

: Color flux tube created by red charge

: Color flux tube created by blue charge

: Color flux tube created by green charge




Quantum mechanically, flux tubes of different colors mix by creation of gluons $g$.

# Center of gauge group

Classification is conveniently done by focusing on
the center of the gauge group.

$G$ : gauge group. (Assume $\pi_0(G) = \pi_1(G) = 0$ for simplicity.)

Center $C_G \subset G$ : the subgroup of $G$ such that

$$z \in C_G \iff zg = gz \quad (\forall g \in G)$$

Example : $G = SU(N)$

$$C_G = \{e^{2\pi ik/N} I_N \mid k = 0,1,2,\cdots, N-1\}$$

$$= \mathbb{Z}_N$$

# Center of gauge group

Gluon $g$ : neutral under $C_G$, because of the condition

$$z \in C_G \implies zgz^{-1} = g \quad (\forall g \in G).$$

Gluons do not have the charge under $C_G$

**Fact:**

Color flux tubes are determined by charges under $C_G$.

['t Hooft,1979]

Example: $G = SU(3)$

red = blue = green = charge 1 mod 3 under $\mathbb{Z}_3 \subset SU(3)$

# Classification

**Pure Yang–Mills:**

Stable color flux tubes are classified as follows.


$k$ : charge under the center $C_G \subset G$.

Color flux tubes exist as dynamical strings without quarks.

**Generalization:**

If dynamical quarks exists, we take

$$C'_G \subset C_G \subset G$$

such that all dynamical fields are neutral under $C'_G$.

# 1-form symmetry

In modern terminology, there is a symmetry called the 1-form center symmetry. (Details omitted)

[Gaiotto-Kapustin-Seiberg-Willett,2014]

$C'_G \subset C_G \subset G$ ➡ 1-form $C'_G$ symmetry

**Analogy**

- A particle (e.g. dark matter) is stable if it is charged under an unbroken ordinary (0-form) symmetry $C$.
- A string (e.g. cosmic string) is stable if it is charged under an unbroken 1-form symmetry $C^{[1]}$.

# Contents

# Abelian-Higgs

Let's consider again

$$\mathscr{L} = -\frac{1}{4e^2}F^{\mu\nu}F_{\mu\nu} - D_\mu\phi D^\mu\phi^* - V(\phi)$$

$$V(\phi) = \lambda(|\phi|^2 - v^2)^2$$

**Standard fact:**

Cosmic strings are produced by the phase transition from high temperature to low temperature.

# String production

Finite temperature $T$:

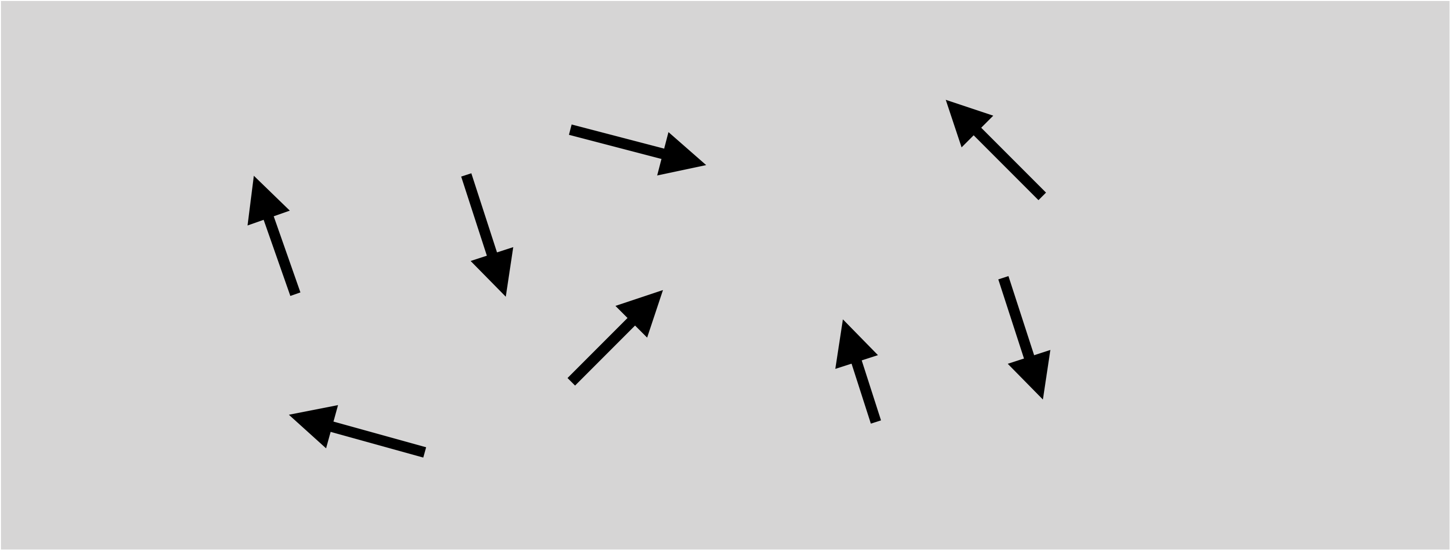
$$V_{\text{eff}}(\phi) = \lambda(|\phi|^2 - v^2)^2 + cT^2|\phi|^2 + \cdots$$

Thermal phase transition

$$\underset{\langle\phi\rangle = 0}{\text{high } T} \;\rightarrow\; \underset{\langle\phi\rangle \neq 0}{\text{low } T}$$

# String production

Right after the phase transition, the complex phase $\arg\langle\phi\rangle$ is random over causally disconnected parts of the Universe.



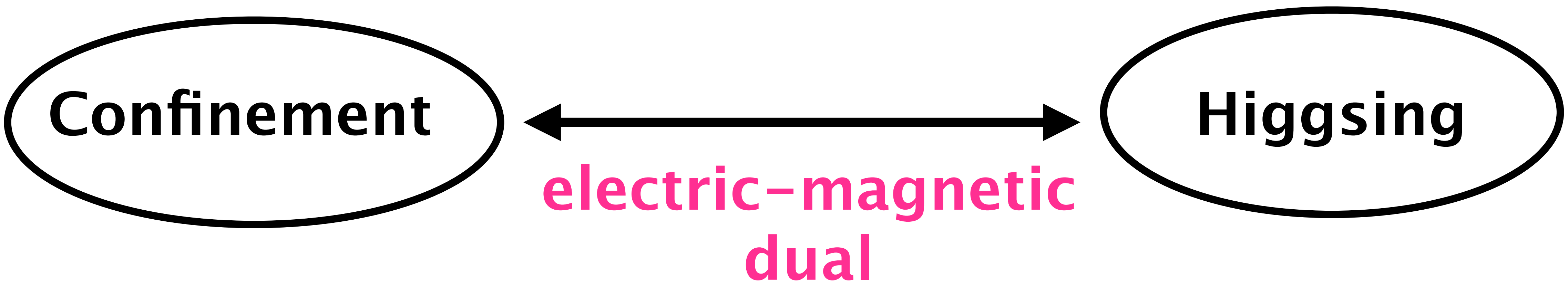
arrows:
schematic picture
for $\arg\langle\phi\rangle$

Strings are produced by the randomness of phase.

[Kibble,1976]

It is not obvious how to extend the argument to color flux tubes of pure Yang–Mills.

# Electric-Magnetic duality




Seiberg–Witten realized this idea very explicitly,
in the case of mass-deformed $\mathcal{N} = 2$ super-Yang-Mills.

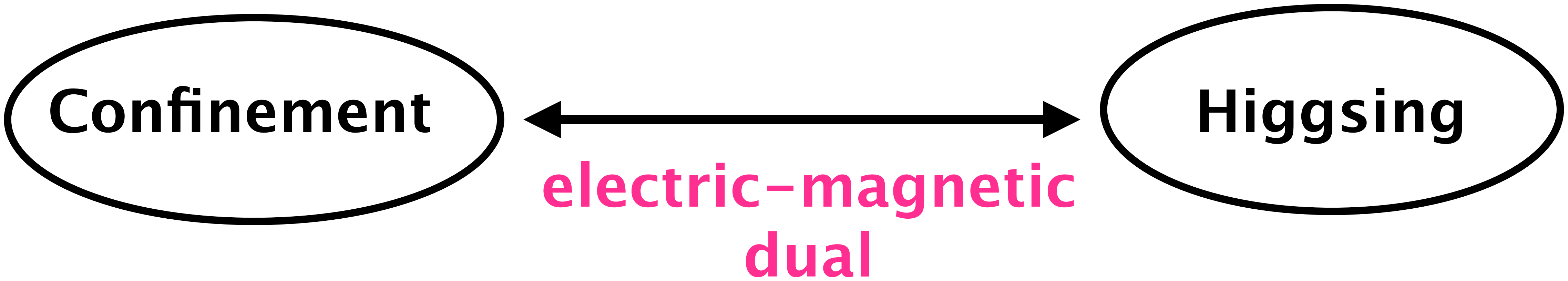
[Seiberg–Witten,1994]

**Mass-deformation:**
Adding some mass terms to super-partners of gluons:
the theory is qualitatively the same as pure Yang–Mills.

# Dual relation

| mass-deformed $\mathcal{N}=2$ $SU(2)$ Yang-Mills | Abelian-Higgs |
|---|---|
| $U(1)_E \subset SU(2)$ | $U(1)_M$ |
| magnetic monopole | charged scalar $\phi$ |
| confinement | higgsing $\langle \phi \rangle \neq 0$ |
| color flux tube | ordinary string |

# Color flux tube production

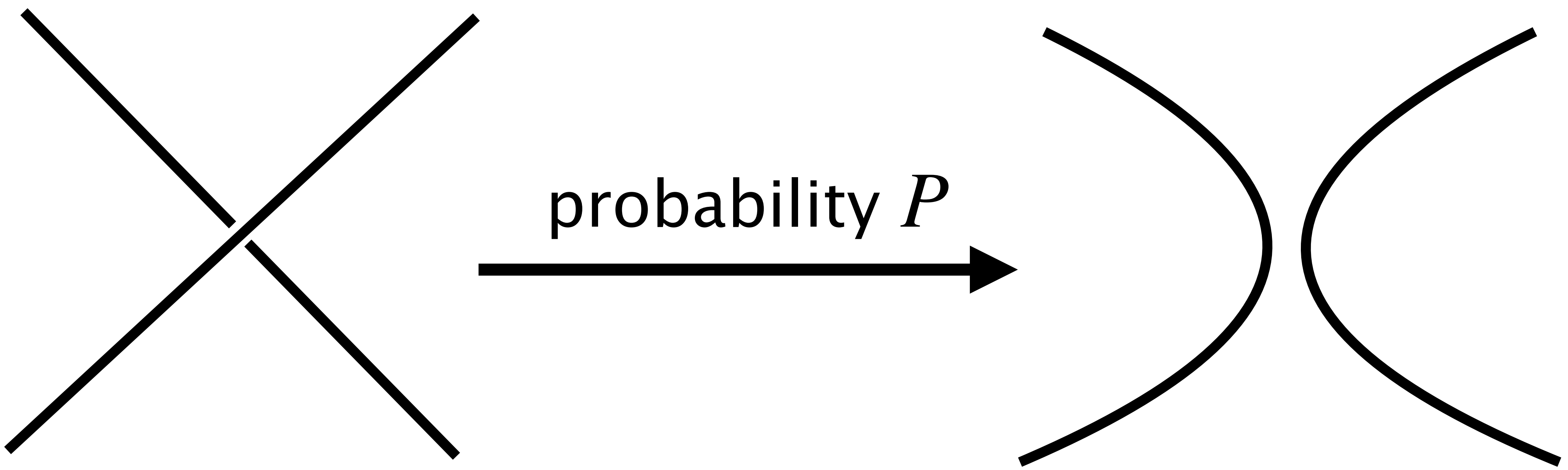



**Conclusion:**
We expect (although cannot prove) cosmic string production in pure Yang-Mills at the phase transition.

# Contents

# Intercommutation probability

Strings annihilate via reconnection.




This intercommutation probability $P$ is important for the evolution of strings:

Small $P$ ⟶ More string density in the Universe

# Large N counting

Let's consider $SU(N)$ gauge group for large $N$.

It is possible to estimate the dependence of $P$ on $N$ field theoretically by large $N$ counting. ['t Hooft,1974]

It may be more intuitive to imagine holographic duals of gauge theories.

- Some theories have explicit holographic duals.
- The results of large $N$ counting are valid even for pure Yang–Mills

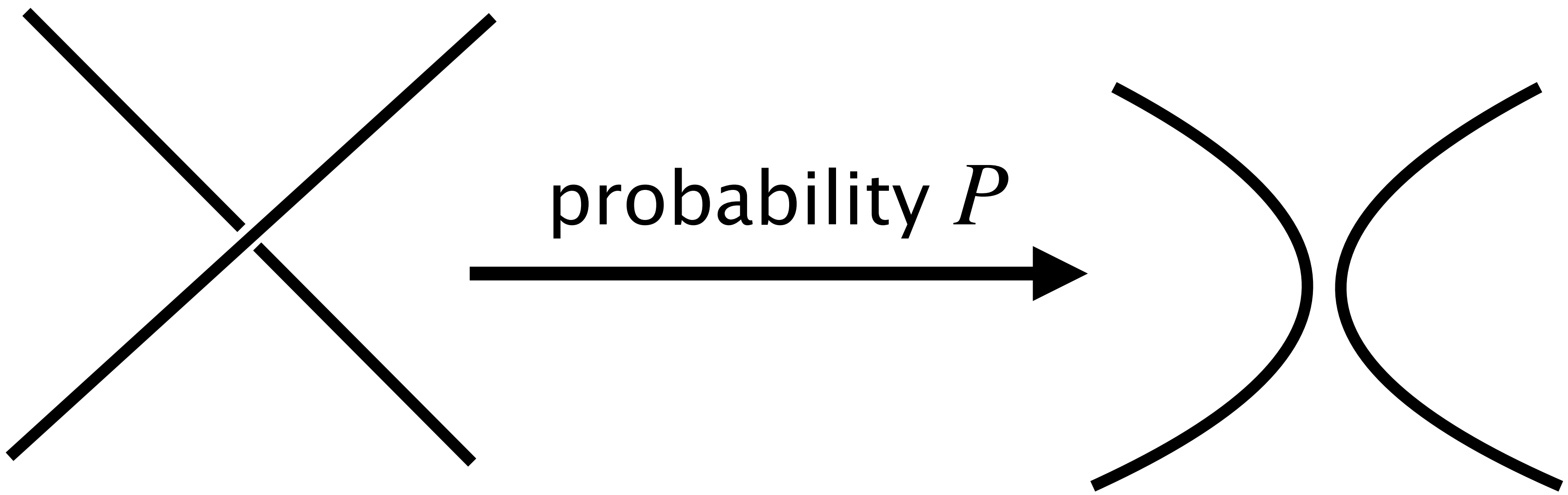
# Holographic duality

| Gauge theory | String theory |
|---|---|
| $\frac{1}{N}$ | String coupling $g_s$ |
| Color flux tubes associated to the fundamental rep. | Fundamental string |

D-strings (D-branes) are also realized by pure Yang-Mills if we consider $Spin(N)$ gauge group. (See our paper.)

# Fundamental string

**fundamental string = color flux tube**




By standard computation (of the worldsheet Euler number)

$$P \sim g_s^2 \sim N^{-2}$$

(Weakly coupled Abelian–Higgs : $P \sim 1$)

This may be regarded as a field theory realization of fundamental cosmic superstring.

# Contents

# Gravitational waves

Let me discuss a very rough sketch of gravitational waves coming from cosmic strings.

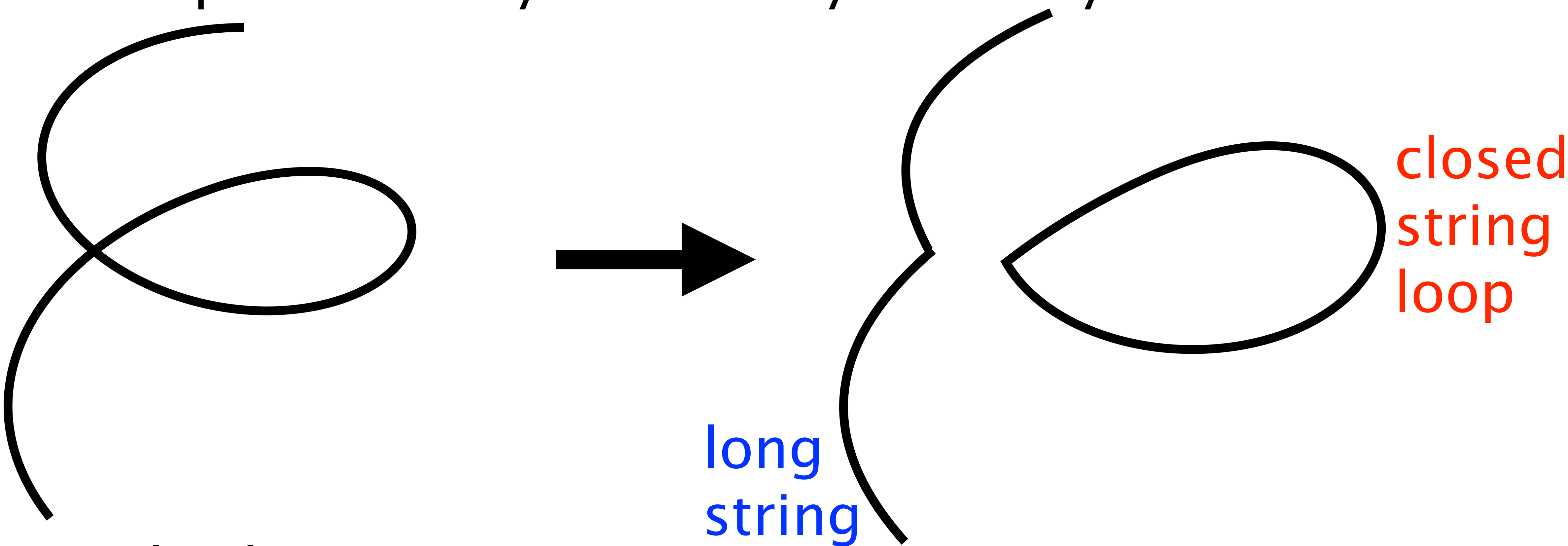
Please ask my collaborator for the details.
(Nontrivial point: pure Yang–Mills can realize $P \ll 1$.)

**Two standard properties:**

- Scaling law
- Energy emission only by gravitational waves

# The scaling law

By reconnection processes, strings try to "annihilate" in the best possible way allowed by causality.




As a result, there are

- $O(1)$ number of long strings per each Hubble volume: the scaling law
- Many small closed string loops

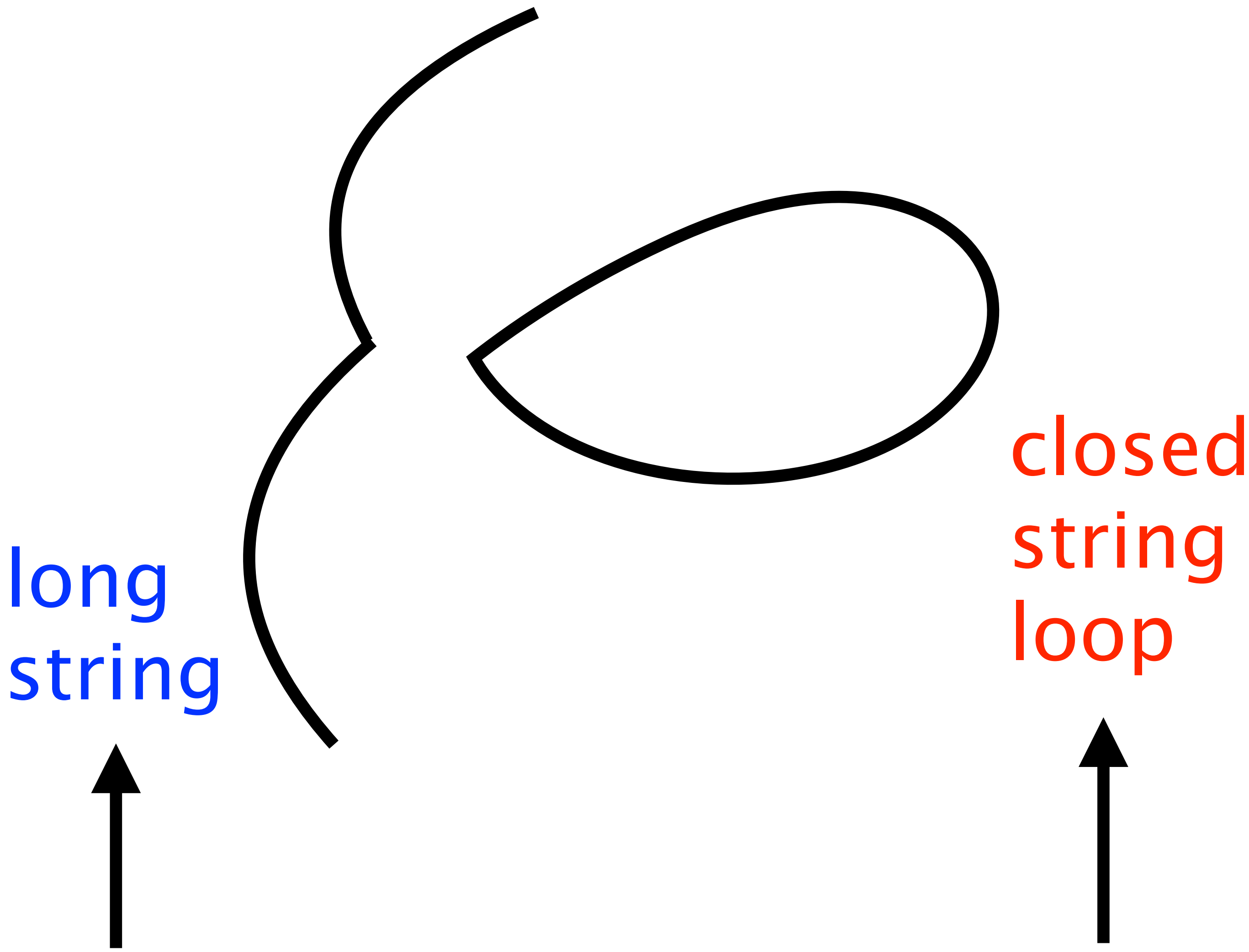
# Energy loss of closed string

All particles, except for gravity, are either

- too heavy to be produced: [mass] $\gg$ [frequency] (glueballs)
- too weakly coupled to the string. (Standard Model particles)

Gravitational wave is the only energy emission mechanism of closed string loops!

# The situation




Always $O(1)$ in the Hubble volume.

Decays only by emitting gravitational waves.

Observable amount of gravitational waves is emitted.

# Enhancement by $N$

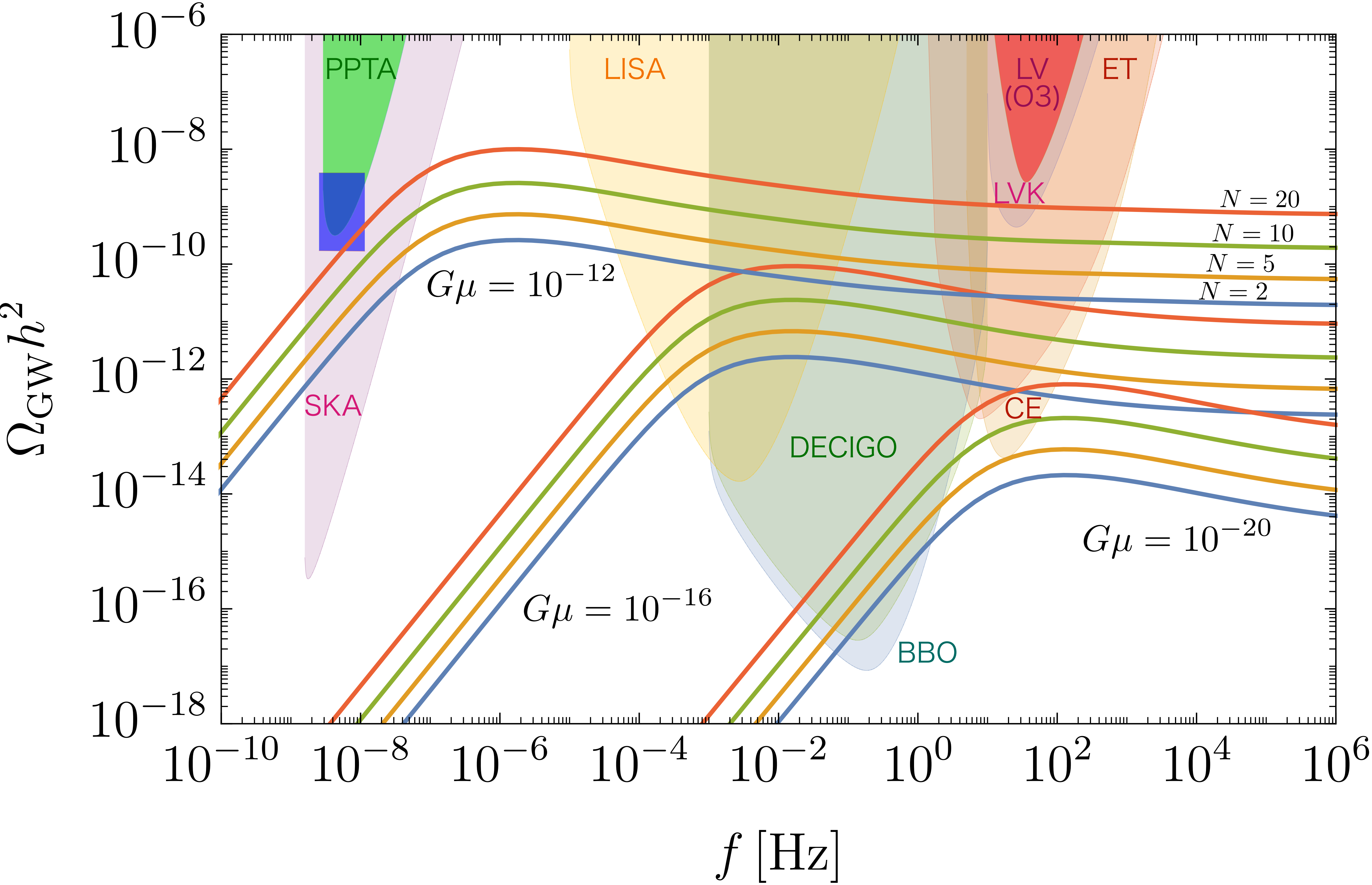
For small intercommutation probability $P$, we get larger string density and gravitational wave density.

The precise enhancement factor is under debate.
One model gives the enhancement factor of order $P^{-1}$.
(Please see our paper.)

Recall $P^{-1} \sim N^2$ for fundamental string in the large $N$ limit.

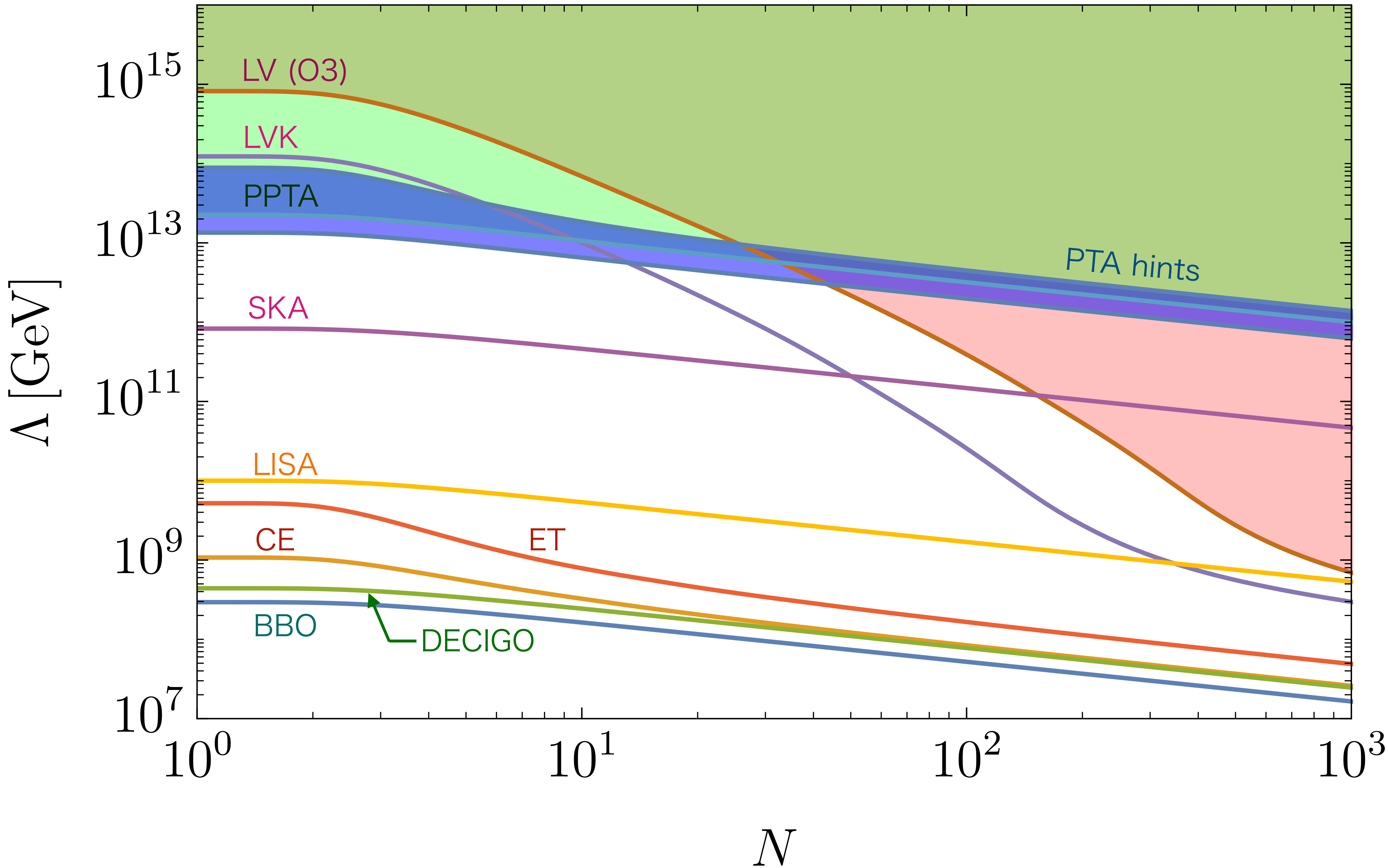
# Result

$SU(N)$ theory
$\mu$ : tension



# Result

$\mu \sim \Lambda^2$ :
dynamical scale



# Contents

# Summary

- Pure Yang–Mills theory is the simplest model which can produce cosmic strings.

- Color flux tube is the string, and its qualitative properties can be seen by various theoretical ideas such as center symmetry, electric–magnetic duality, and holography.

- Observable amount of gravitational waves may be emitted for dynamical scales of order

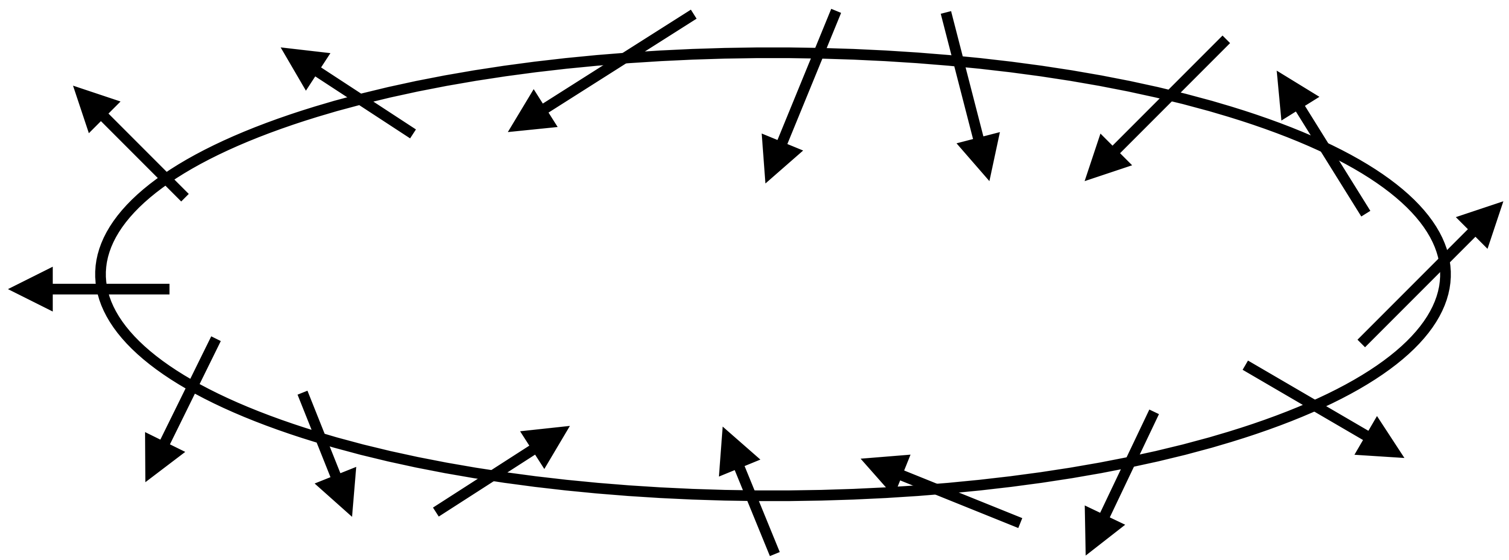
$$10^9\,\mathrm{GeV} \le \Lambda \le 10^{13}\,\mathrm{GeV} \qquad \text{for } N \sim O(1)$$

More enhancement in $N \gg 1$.

# Back up

# String production

Imagine a very large circle in the universe:



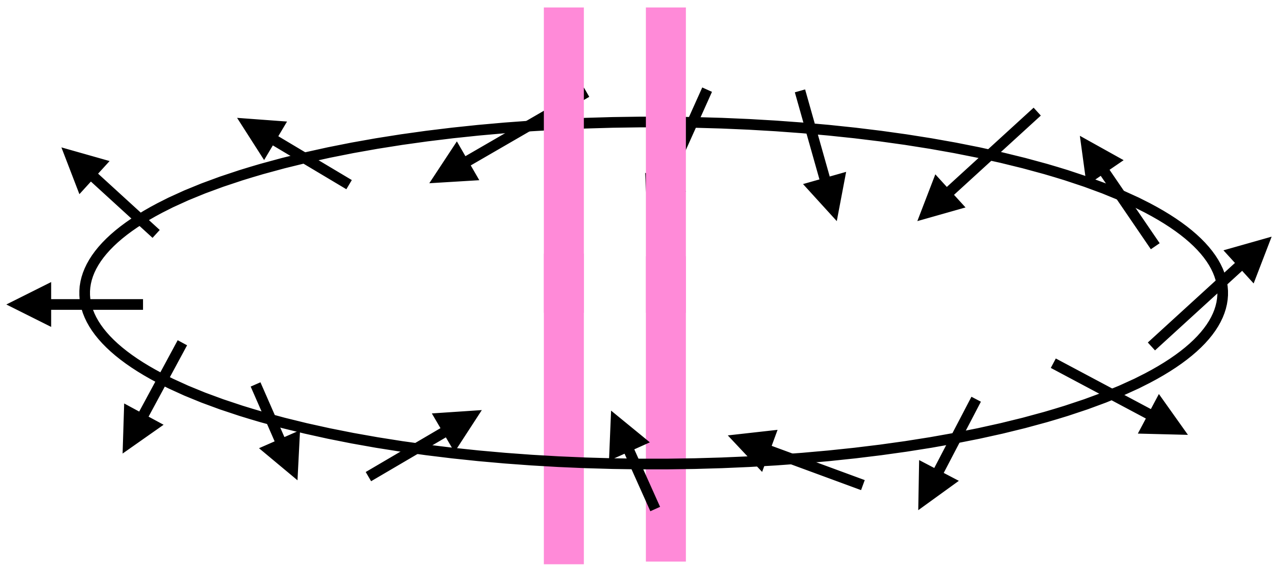
Due to the randomness of $\arg\langle\phi\rangle$, we have

$$\arg\langle\phi\rangle \xrightarrow{\text{around circle}} \arg\langle\phi\rangle + 2\pi N$$

for a random (not too large) integer $N$.

- There are $N$ strings in the region surrounded by the circle.

# String production



**Conclusion:**
In the case of the Abelian Higgs model,
there are at least $O(1)$ strings in a Hubble volume
(the maximal causally connected region).

It is not obvious how to extend the argument
to color flux tubes of pure Yang-Mills.

# D-brane

**D-brane : created by spinor rep. of $\mathbf{Spin(N)}$**

More subtle, but for relative velocity $v \gg N^{-1}$ it seems

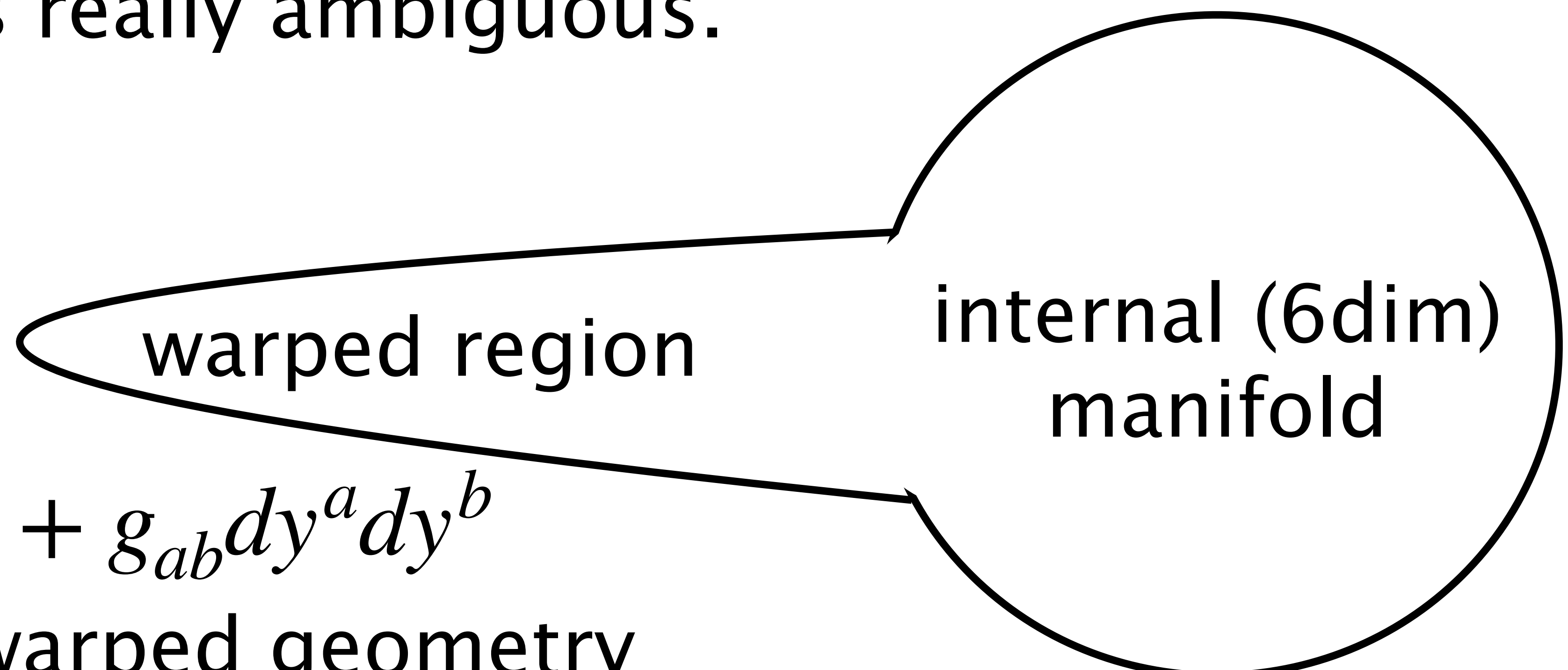
$$P \sim \exp(-O(1) \cdot g_s^{-1}) \sim \exp(-O(1) \cdot N)$$

(Speculative)

In any case, the color flux tube created by spinor rep. may be regarded as a field theory realization of D-branes/D-strings.

# Remark

Sometimes the distinction between field theory strings and superstrings is really ambiguous.



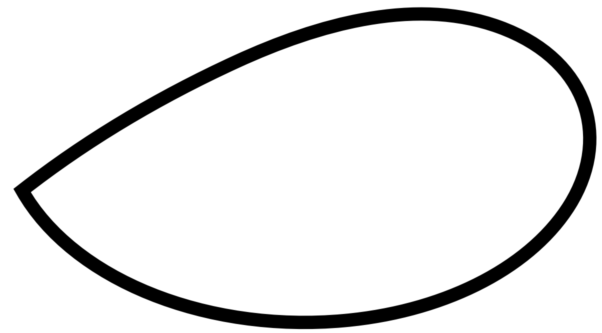

$$ds^2 = h(y)dx^\mu dx_\mu + g_{ab}dy^a dy^b$$

$h(y) \sim \exp(y)$ : warped geometry

Warped geometry near some point of the internal manifold may be regarded as a strongly coupled gauge theory.

[Giddings-Kachru-Polchinski,2001]

# Closed string loop




Very crude order of the size $\ell$ and the frequency $f$ at the time of production is

$$\ell \sim t_{\text{production}}$$ : Hubble size @ production

$$f \sim \ell^{-1}$$

The details of the size $\ell$ is under debate.

# Decay of closed string loop

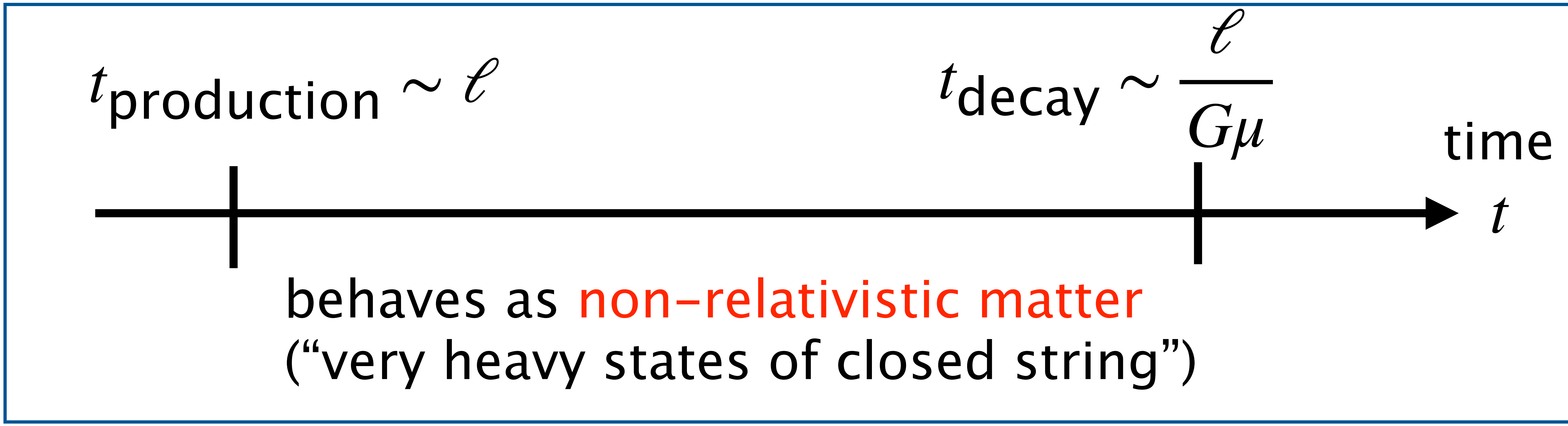
Decay by gravitational wave emission:

$E \sim \mu \ell$ ,  $\mu$ : tension of the string

$\frac{dE}{dt} \sim -G\mu^2$  $G$ : Newton constant

⟶ decay time $t_{\text{decay}} \sim \frac{E}{G\mu^2} \sim \frac{\ell}{G\mu}$

# History of closed string loop




Assuming $t_{\text{decay}} \ll$ [matter-radiation equality],

Enhancement of energy density relative to radiation

$$\left(\frac{t_{\text{decay}}}{t_{\text{production}}}\right)^{1/2} \sim (G\mu)^{-1/2}$$

# Long string energy density

By the scaling law, the density of long strings at the time of production, divided by the total energy is

$$\Omega_{\text{long string}} \sim \frac{\mu H^2}{H^2/G} \sim G\mu \qquad (H : \text{Hubble})$$

Almost all energy of long strings goes into closed string loops to satisfy the scaling law.

# Gravitational wave density

If $t_{\mathsf{decay}} \ll$ [matter-radiation equality],

**Gravitational wave density in the present universe:**

$$\frac{d\Omega_{\mathsf{GW}}}{d\log f} \sim (G\mu)\cdot(G\mu)^{-1/2}\cdot 10^{-5}$$

initial string density

enhancement during matter-like behavior of closed string loops

redshift after matter-radiation equality, etc.

# Remarks

If $t_{\text{decay}} \gg$ [matter–radiation equality],
we need different analysis. (Details omitted.)